3rd International Young Mathematicians' Convention (IYMC)

2008 Team Contest – Junior level

Team : ____________________________

Problem 1.

Show that, for all positive real numbers *p*, *q*, *r*, *s*,

$$\left(p^2+p+1\right)\left(q^2+q+1\right)\left(r^2+r+1\right)\left(s^2+s+1\right)\geq 81pqrs$$

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Problem 2.
In a troop of 2008, 12 are on patrol duty every night. Prove that it is impossible to draw up a schedule according to which every 2 are on duty together exactly once.

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Problem 3.
Find all integers that satisfy the equation:

$$x^2 - 2xy + 2x - y + 1 = 0.$$

ANSWER : ______________________

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Problem 4.

Markers are to be placed in some squares of a 4×4 chessboard.

(a) Place 7 markers so that if the markers on any two rows and any two columns are removed, at least one marker remains on the board.

(b) Prove that no matter how 6 markers are placed on the board, then it is always possible to choose two rows and two columns so that no markers remain on the board when all markers in these rows and columns are removed.

(a)

ANSWER : ____________________

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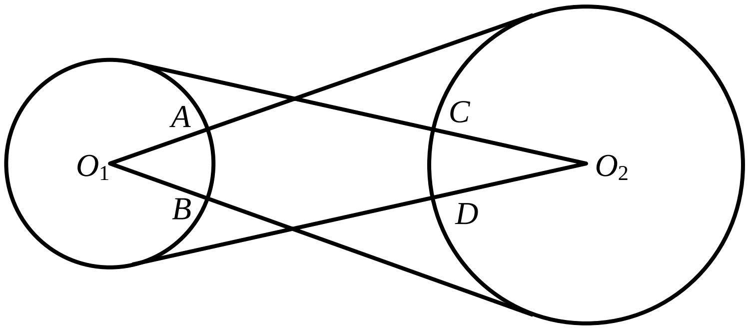
Problem 5.
From the centers of two "exterior" circles draw the tangents to the other circle. Prove that *AB*=*CD*.



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Problem 6.

Find all possible integers *N* satisfying the following properties:

(i) *N* has at least two prime divisors, and

(ii) $N = d_1^2 + d_2^2 + d_3^2 + d_4^2$, where d_1, d_2, d_3 and d_4 are the first four positive divisors of *N*.

ANSWER : ____________________