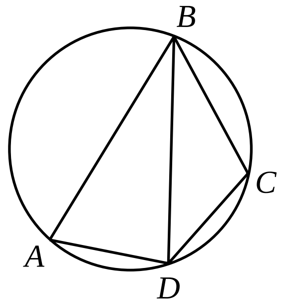
3^{rd} International Young Mathematicians' Convention (IYMC)

2008 Individual Contest –Senior level

Problem 1.

The diagonal *BD* of the inscribed quadrilateral *ABCD* is the bisector of $\angle ABC$. Find the area of the quadrilateral if $BD = 10$ cm and $\angle ABC = 60^\circ$.



Problem 2.

The function $f(x)$ satisfies the equation

$$f(2008^x) + xf(2008^{-x}) = 2008$$

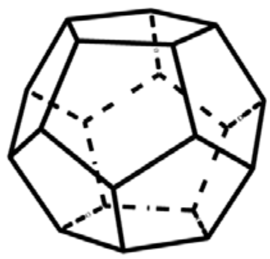
for all values of *x*. What is the value of $f(2008)$?

Problem 3.

Let *a* be the sum of the digits of an arbitrary 2008-digit multiple of 9. Let *b* be the sum of the digits of *a*, and *c* be the sum of the digits of *b*. Determine *c*.

Problem 4.

A desk calendar consists of a regular dodecahedron with a different month on each of its twelve pentagonal faces. How many essentially different ways are there of arranging the months on the faces?



Problem 5.

Find an integer x satisfying the following equation.

$$\left[\frac{x}{1!}\right] + \left[\frac{x}{2!}\right] + \left[\frac{x}{3!}\right] + \cdots + \left[\frac{x}{10!}\right] = 2008,$$

where $[x]$ denotes the greatest integer which is less than or equal to x.

Problem 6.

It's given that $4x^2 + 25y^2 + 196z^2 = 144$. Find the maximal possible value of $4x + 5y - 28z$.

Problem 7.

Determine a constant *k* such that the polynomial

$$P(x, y, z) = x^5 + y^5 + z^5 + k(x^3 + y^3 + z^3)(x^2 + y^2 + z^2)$$

has the factor $x + y + z$.

Problem 8.

A given convex pentagon *ABCDE* has the property that the area of each of the five triangles *ABC*, *BCD*, *CDE*, *DEA* and *EAB* is 1. Find the area of the pentagon.