# *3rd International Young Mathematicians' Convention (IYMC)*

## 2008 Team Contest – Senior level

Team : ____________________________

**Problem 1.**

Prove that $\sqrt{4n+1} < \sqrt{n} + \sqrt{n+1} < \sqrt{4n+2}$, where $n$ is a positive integer.

# $3^{rd}$ *International Young Mathematicians' Convention (IYMC)*

## 2008 Team Contest – Senior level

Team : ___________________________

**Problem 2.**

Two players take turns choosing one number at a time (without replacement) from the set $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$. The first player to obtain three numbers (out of three, four, or five) which sum to 0 wins. Does either player have a forced win?

# *3rd International Young Mathematicians' Convention (IYMC)*

## 2008 Team Contest – Senior level

Team : ____________________________

**Problem 3.**

Let $n$ be a positive integer and $x_1$, $x_2$, …, $x_{n-1}$ and $x_n$ be integers such that

$$x_1 + x_2 + \cdots + x_n = 0 \quad \text{and} \quad x_1 x_2 \cdots x_n = n.$$

Prove that $n$ is a multiple of 4.

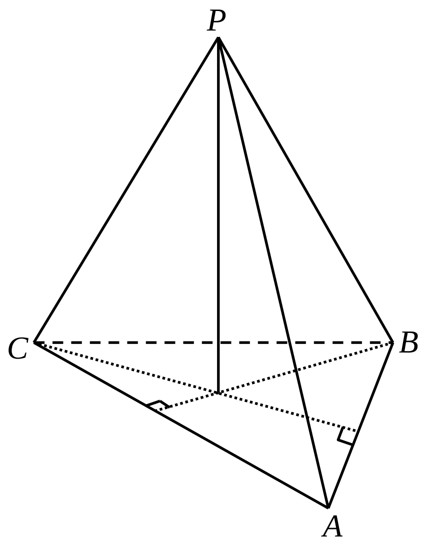
# *3rd International Young Mathematicians' Convention (IYMC)*

## 2008 Team Contest – Senior level

Team : ____________________________

**Problem 4.**

In the tetrahedron *PABC* the altitude from *P* passes through the orthocenter of triangle $\Delta ABC$. Find the ratio of the areas of triangles $\Delta PAB$ and $\Delta PAC$ if $PC=6-\sqrt{2}$, $PB=6+\sqrt{2}$, $BC=2\sqrt{19}$.




ANSWER : $\dfrac{S_{\Delta PAB}}{S_{\Delta PAC}} =$ ____________________

# *3rd International Young Mathematicians' Convention (IYMC)*

## 2008 Team Contest – Senior level

Team : ___________________________

**Problem 5.**

Your calculator is not working properly—it cannot perform multiplications. But it can add (and subtract) and it can compute the reciprocal $\frac{1}{x}$ of any number $x$. Can you nevertheless use this defective calculator to multiply numbers?

# *3rd International Young Mathematicians' Convention (IYMC)*

## 2008 Team Contest – Senior level

Team : ___________________________

**Problem 6.**

Given an arbitrary convex quadrilateral *ABCD* and the centers *P*, *Q*, *R*, *S* of the external squares on the sides *AB*, *BC*, *CD*, *DA*, respectively. Show that the area of quadrilateral *PQRS* is $\frac{1}{2} \times QS \times QS$