Problem 1 Given three reals a_1, a_2, $a_3 > 1$, $S = a_1 + a_2 + a_3$. Provided $\frac{a_i^2}{a_i - 1} > S$ for every $i = 1, 2, 3$, prove that

$$\frac{1}{a_1 + a_2} + \frac{1}{a_2 + a_3} + \frac{1}{a_3 + a_1} > 1$$

Problem 2 Each grid point of a cartesian plane is colored with one of three colors, whereby all three colors are used. Show that one can always find a right-angled triangle, whose three vertices have pairwise different colors.

Problem 3 Prove that for any real numbers we have

$$\sum_{i<j} |a_i - a_j| + \sum_{i<j} |b_i - b_j| \leq \sum_{i,j=1,n} |a_i - b_j|$$

Problem 4 Integers $x > 2$, $y > 1$, $z > 0$ satisfy an equation $x^y + 1 = z^2$. Let p be the number of different prime divisors of x, q be the number of different prime divisors of y. Prove that $p \geq q + 2$.

Problem 5 Find all pairs of integers a, b for which there exists a polynomial $P(x) \in Z[X]$ such that product $(x^2 + ax + b) \cdot P(x)$ is a polynomial of the form

$$x^n + c_{n-1}x^{n-1} + ... + c_1 x + c_0$$

where each of $c_0, c_1, ..., c_{n-1}$ is equal to 1 or -1.

Problem 6 Given a triplet we perform on it the following operation. We choose two numbers among them and change them into their sum and product, left number stays unchanged. Can we, starting from triplet $(3, 4, 5)$ and performing above operation, obtain again a triplet of numbers which are lengths of right triangle?

Problem 7 Find all real values of α, for which there exists a unique function $f : R \to R$ and satisfying the equation

$$f(x^2 + y + f(y)) = f(x)^2 + \alpha y \quad \forall x, y \in R$$

Problem 8 100 people from 25 countries, four from each countries, stay on a circle. Prove that one may partition them onto 4 groups in such way that neither two countrymen, nor two neighbors will be in the same group.