INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2008

Mathematics Contest (Second Round) in Taiwan, Essay Problems

Name:_______ School:_______ Grade:______ ID number:________

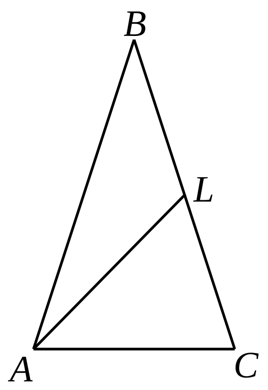
Answer the following 10 questions, and show your detailed solution in the space provided after each question. Each question is worth 4 points.
Time limit: 60 minutes.

1. Find which is the greatest, 2^{9972}, 65^{1662} or 33^{1995}.

2. Sarah and her young brother Stephen are earning pocket-money by mowing lawns for their neighbors. Sarah uses the lawn mower and hopes to make $5 an hour for herself. Stephen helps by taking away the cuttings and tidying up. They agree that he will receive 20% of the total payment made for doing the work. It will take them four hours to mow the lawn in the garden of a large house nearby. How much will the owner pay if Sarah is to earn for herself exactly $5 an hour?

3. John cycles to Jane's house and back along a flat straight road, giving him a round trip of 32 km. During the whole of the time both the speed and direction of the wind are constant. On the way out, the wind is directly behind John and so may be added to his normal bicycle speed to give his speed for this part of the journey, but on the way back the wind opposes him and its speed must be subtracted from his bicycle speed. When John arrives back home he finds that his total travelling time has been 2 hours and that the homeward part of his journey has taken twice as long as the outward part. Find the speed of the wind, in km/hour.

4. In a triangle *ABC*, *AB*=*BC*. *L* is the point on *BC* such that *AL* bisects $\angle BAC$. If *AL*=*AC*, find the size of the angle of the triangle *ABC*.



5. In the first stage of a baseball competition, the teams are divided into grounds of four. Each team in a group plays the three other teams once each, and the results are drawn up in a “group table”, which records the number of wins, draws and defeats for each team, and the number of runs scored and against. Three points are allocated for a win, one for a draw and none for a defeat. Here is part of a group table

Team	Runs		Points
	Scored	Against	
A	2	2	4
B	4	4	6
C	2	1	4
D	1	2	2

Find the results and the scores in all the matches played in the group.

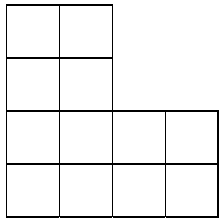
A:*B*= : ; *A*:*C*= : ; *A*:*D*= : ; *B*:*C*= : ; *B*:*D*= : ; *C*:*D*= :

6. The number 132 has three digits, no two of which are equal. It has the property of being equal to the sum of all the different 2 digit numbers made up from its three digits, viz.

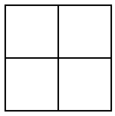
$$132=13+12+21+23+31+32$$

Find all other such 3 digit numbers.

7. Color the twelve small squares in the diagram, using three colors altogether, in such a way that

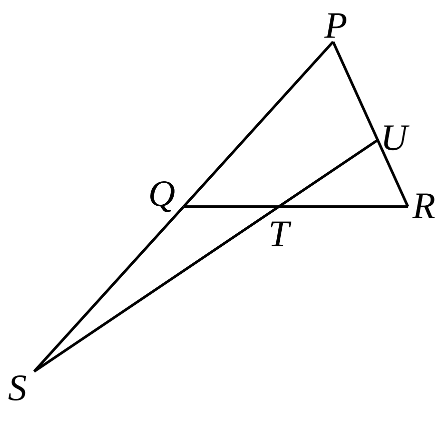


(a) No two squares which have an edge in common have the same color.
(b) Each of the three colors is used in exactly four of the small squares.
(c) In any four small squares forming a block as part of the diagram, all three colors are used.

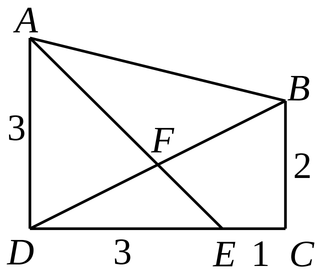


(Note: You must not mix the colors or use different colors in the same small square. Using B, G, R as three colors)

8. *PQR* is a triangle. *PQ* is extended to *S* so that *PQ*=*QS* and *U* is a point on *PR* such that *PU*:*UR*=3:2. *T* is the point of intersection of the lines *QR* and *SU*. Find *QT*:*QR*.



9. In the quadrilateral $ABCD$ the sides AD and BC are parallel and AD is perpendicular to DC. The lengths of AD and BC are 3 and 2, respectively. E is the point on DC between D and C such that DE has length 3 and EC has length 1, as shown. The lines AE and BD meet at F. Find the exact area of triangle ABF.



10. Equilateral triangles and squares, each with sides of unit length, can be used to construct convex polygons. For example, two triangles and a square can be put together to form a hexagon, and three triangles and two squares to form a 7 side polygon, as shown in the diagrams. (The region enclosed by the polygon must be covered exactly by the triangles and squares used in the construction.) How the process can be used to construct a convex polygon with 11 sides?