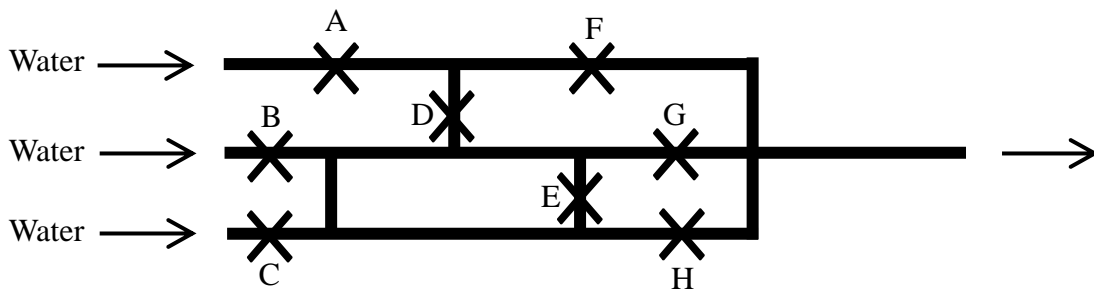


**INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD
FOR PRIMARY SCHOOLS (IMSO) 2008
Mathematics Contest in Taiwan, Exploration Problems**

Name: _____ School: _____ Grade: _____ ID number: _____

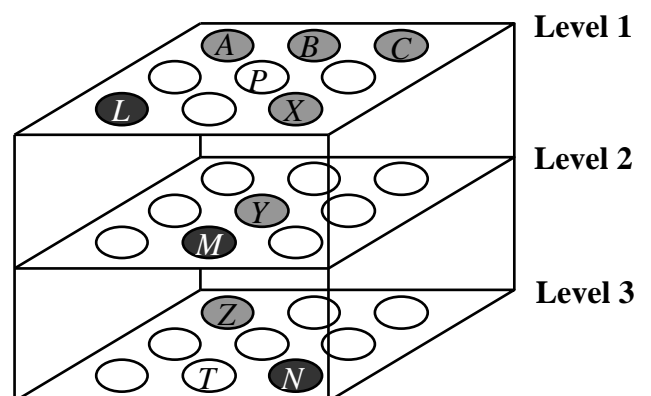
Answer the following 5 questions. Write down your answer in the space provided after each question. Each question is worth 8 points. Time limit: 60 minutes.

1. The diagram shows a network of water pipes, with eight taps, each shown by a symbol \times , which are at positions A, B, C, D, E, F, G and H. Water enters along three of the pipes at the left, as shown by the symbol \rightarrow , and is prevented from making any progress past a tap which is in the "off" position. Taps are either "on" or "off". How many ways of taps in the "on" position will result in water coming out of the pipe on the extreme right, assuming that no more than three taps can be in the "on" position at the same time?

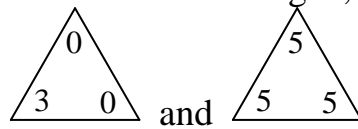


2. The diagram shows a $3 \times 3 \times 3$ frame with 27 holes in which marbles can be placed. $A-B-C$, $L-M-N$ and $X-Y-Z$ are some possible lines of three marbles.

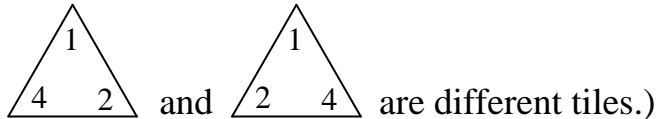
- (a) From A, how many different lines of three marbles are possible? (3 points)
- (b) What is the total number of different possible lines of three marbles in this frame? (5 points)



3. Triominoes is played with a set of tiles, all sides of which are equal in length. Each vertex of every tile is marked with one of the digits from 0 to 5. Different vertices on the same tile can have the same digits, so that



are both in the set. All possible tiles with duplicate or triplicate digits, like those illustrated, are included. How many of all possible tiles are there? (The tiles



4. At the end of the season, the total prize money in a football tipping competition is to be shared among the 5 place getters according to the following rules:
- (i) All prizes are different and each is a positive whole number of dollars;
 - (ii) First prize is the sum of second and third prizes;
 - (iii) Second prize is the sum of fourth and fifth prizes;
 - (iv) The higher the placing, the larger the prize. (Assume that there are no ties among the place getters.)
- (1) Find the smallest amount of total prize money possible for which these rules may apply. (3 points)
 - (2) If the total prize money available is \$72, find all possible ways in which the \$72 may be distributed among the place getters. (5 points)
5. An n -dragon is a set of n consecutive positive integers. The first two-thirds of them is called the *tail*, the remaining one-third is called the *head* and the sum of the numbers in the tail is equal to the sum of the numbers in the head. For example, the 9 consecutive integers 2, 3, 4, 5, 6, 7, 8, 9 and 10 form a 9-dragon.
- Its tail is 2, 3, 4, 5, 6 and 7, i.e. six numbers with sum 27.
 - Its head is 8, 9 and 10, i.e. three numbers with sum 27.
- (1) Find a 21-dragon. (3 points)
 - (2) Find the sum of the tail of a 99999-dragon. (5 points)