INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2008

Mathematics Contest (Second Round) in Taiwan

Short Answer Problems

Name:________ School:________ Grade:______ ID number:__________

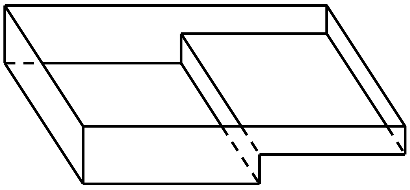
Short Answer: there are 20 questions, fill in the correct answers in the answer sheet. Each correct answer is worth 2 points. Time limit: 60 minutes.

1. Jack wrote the word **MINES** on a frosty window. From the other side of the window it appears as :

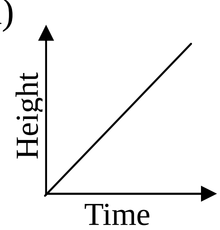
(A)  (B)  (C)  (D)  (E) 

ANS: (E)

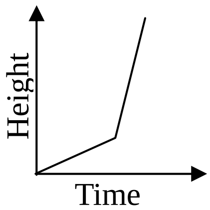
2. The swimming pool is filled with water at a constant rate. Which graph below best shows the increase in height of the water with the passing of time?



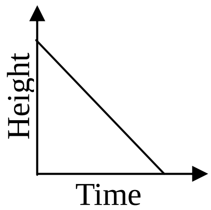
(A)



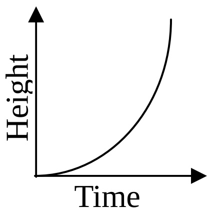
(B)



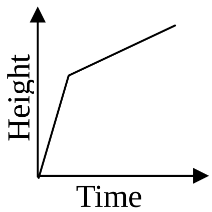
(C)



(D)



(E)



The depth of the water will be steadily increased until the top of the lower section of the pool; then it will increase steadily at a slower rate until it is full. So graph (E) is the best graph.

ANS: (E)

3. How many two-digit numbers have the property of being equal to 7 times the sum of their digits?

Suppose that a two-digit number is written $\overline{ab}$. Then a is one of 1, 2, 3, 4, 5, 6, 7, 8 and 9, while b is one of 1, 2, 3, 4, 5, 6, 7, 8 and 9. The number written $\overline{ab}$ is then $10a+b$. Thus we require the numbers $\overline{ab}$ which satisfy $10a+b=7(a+b)$. This equation is equivalent to $10a+b=7a+7b$ and so to $3a=6b$, or $a=2b$. Therefore the required numbers are the numbers $\overline{ab}$ such that $a=2b$, which are 21, 42, 63 and 84. There are totally 4 such numbers.

ANS: 4

4. Two snails, Gastero and Pod, were completing in an exciting marathon. Gastero set off first, and rested for a day every third day. Pod started a week later, covered twice as much as ground in a day as Gastero did, but rested every second day. At least how many days was Pod overtake Gastero?
The diagram below shows how far the snails have travelled by the end of days 1, 2, 3, 4 and so on. The unit of distance used is the distance travelled by Gastero on any one day when he is moving. Gastero started moving on day 1, but Pod did not move until 7 days had elapsed and so her position is first given for the end of day 8.

Distance	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Gastero		1	2	4	5	7	8	10	11	13	14	16	17	19	20	22			
			3		6		9		12		15		18		21				
Pod			8		10		12		14		16		18		20		22		
			9		11		13		15		17		19		21				

We see that both had moved the same distance, 14 units, by the end of day 20, when Pod caught up with Gastero but did not overtake him. They both rested on day 21 and then, immediately after the start of day 22, Pod galloped clear.

ANS: 22 days

5. Jane and her dog Jimmy have a favorite walk in which they cover a total of 32 km between them. On the outward journey, Jimmy runs four times as far as Jane walks, and on the return he covers twice the distance she does. Jane comes back by the same route as she followed on the outward journey. How far does she walk?

	Outward journey	Return journey
Jane	↔	↔
Jimmy	↔↔↔↔	↔↔

Total distance covered between them is eight times one of Jane's journeys. Since we are told this is 32 km, then each of Jane's journeys=4 km. Hence Jane travels 8 km in total.

ANS: 8 km

6. When it is $\frac{1}{5}$ full of water (by weight) a jug weights 560 grams. When it is $\frac{4}{5}$ full, the jug weights 740 grams. What does the jug weigh when it is empty?
The difference between the two weights is the weight of three fifths of a jugful of water and is 180 grams. Hence one fifth of a jugful weights 60 grams. Since the weight of the jug + one fifth of a jugful weights 560 grams, the jug itself must weight 500 grams.

ANS: 500 grams

7. How many integers from 1 to 1200 are not divisible by 2, 3 or 5?
Let n be a positive integer. If n is divisible by 2, 3 or 5 then so is n+30. If n+30 is divisible by 2, 3 or 5 then so is n. The integers between 1 and 30 which are not

divisible by 2, 3 or 5 is 1, 7, 11, 13, 17, 19, 23 and 29. Since 1200=40×30, there are 40×8=320 integers between 1 and 1200 which are not divisible by 2, 3 or 5.

ANS: 320

8. A benefactor left a sum of money to be divided equally amongst a number of charities. From the full amount of money, Oxfam received \$4000 plus one ninth of the remainder. Next, from what was left, Save the Children received \$6000 plus one ninth of the remainder. Then what was left was distributed amongst the other charities. How many charities eventually benefited?

Suppose that the total sum of money in the benefaction was \$$N$, that Oxfam received \$$X$ and that Save the Children received \$$S$. Then

$$X = 4000 + \frac{1}{9}(N - 4000) = \frac{1}{9}(N + 32000)$$

and

$$S = 6000 + \frac{1}{9}(N - X - 6000)$$

$$= 6000 + \frac{1}{81}(9N - N - 32000 - 54000)$$

$$= \frac{1}{81}(400000 + 8N)$$

Since X=S, we get 9N+288000=400000+8N and so N=112000. Therefore $X = \frac{1}{9}(112000 + 32000) = 16000$. Since all the charities received the same total sum of money, the number of charities which eventually benefited was 112000÷16000=7.

ANS: 7

9. The pages of a book are numbered consecutively: 1, 2, 3, 4 and so on. No pages are missing. If in the page numbers the digit 3 occurs exactly 99 times, what is the number of the last page?

In the integer 1 to 99 there are 20 threes. We can see that by nothing that there are 10 threes in the units places (in 3, 13, 23, 33, 43, 53, 63, 73, 83 and 93) and other 10 in the tens places (in 30, 31, 32, 33, 34, 35, 36, 37, 38 and 39).

Similarly there are 20 threes in 100 to 199 and other 20 in 200 to 299. So up to page 299 we have 60 threes in total.

In 300 to 309 there are 10+1 threes, i.e. 11. Similarly in 310 to 319 and 320 to 329. Thus the grand total is 60+33=93 by page 329. From then the total goes as follows:

Page	329	330	331	332	333
Total	93	95	97	99	102

Thus, since the next page would bring the total to 102, the number of the last page must be 332.

ANS: 332

10. For admission to the school play, adult were charged \$130 each and students \$65 each. A total of \$30225 was collected, from fewer than 400 people. What was the smallest possible number of adults who paid?

Let x be the number of adults and y the number of students. Then

$$30225 = 130x + 65y \text{ and } x + y < 400.$$

Dividing the first equation by 65 we get

$$465 = 2x + y = (x + y) + x$$

which shows that

$$x > 465 - 400 = 65.$$

For $x=66$ we get $y=333$ giving a total attendance of 399. Hence the smallest possible number of adults was 66.

ANS: 66

11. *Peter*: How many scouts attended the jamboree last weekend?

Paul : I cannot remember, but I know that when the organizers attempted to group us into "threes" there were 2 scouts left over. However, when they attempted to group us into "fives" there were 3 scouts left over. Also the number of scouts present was odd and less than 1000.

Peter: I need more information!

Paul : I remember two other facts. The total present was a palindromic number (i.e. it reads the same backwards as forwards). Also, when they attempted to divide us into "sevens" there were 5 scouts left over.

Peter: Now I know the answer.

How many scouts were present at the jamboree?

Numbers which have remainder 2 on division by 3, and 3 on division by 5, are 8, 23, 38, 53, 68, 83, 98, 113 and so on, in step of 15 ($=3\times5$). The first such number having remainder 5 on division by 7 is 68 and the others satisfying this condition will go up in step of 105 ($=3\times5\times7$):

68, 173, 278, 383, 488, 593, 698, 803, 908

are all the values less than 1000. Of these, only 383 is palindromic. Hence 383 scouts were present at the jamboree.

ANS: 383

12. Alistair and I travelled by car to my cousin's wedding, which was due to begin at 12 noon in a village an exact number of miles away. Alistair had slept in, so we did not leave until 7.30 a.m., later than planned. On the first 15 miles of road we averaged 40 miles per hour. Then, on a main road, we covered an exact number of ninths of the total distance at an average of 49 miles per hour, until we had to turn into a country lane for the final one-seventh of the journey. Here we were held up, first by a tractor and later by a flock of sheep, but eventually we reached our destination just before the bridge arrived at the church, with the clock striking 12. What was our average speed on the last frustrating leg of our journey?

Let d be the total distance and k the number of ninths of the total distance for the 'main road' section. Then d and k are positive integers and $k<8$, since one seventh of the distance is covered in final section. From all this we get

$$15 + \frac{kd}{9} + \frac{d}{7} = d \quad \text{or} \quad 7kd = 9(6d - 105)$$

from which we can see that k must be odd. The possible values of k are thus 1, 3, 5, 7. When $k=1$ the second equation gives $47d=945$, which contradicts the fact that d is positive integer. Similarly k=3 gives 11d=315 and k=5 gives 19d=945 which also give the same contradiction. Thus k=7 and d=189. Armed with this, we have that on the main road the distance covered was $\frac{7\times 189}{9}=147$ miles and on the last seventh of the journey the distance covered was $\frac{189}{7}=27$ miles. The first part of the journey took $\frac{15}{40}$ hours (i.e. 22.5 minutes). The middle part took $\frac{147}{49}=3$ hours. The total time was 4 hours and 30 minutes, so the time taken on the last part of the journey was 1 hour 7.5 minutes. Since 27 miles were covered the average speed on this stretch was $27\times\frac{60}{67.5}=24$ miles per hour.

ANS: 24 miles per hour

13. How many integers between 1 and 1,000,000 contain the digit 5 at least twice?

Think of all the integers between 1 and 999999 inclusive written as 6-digit numbers, with initial zeros to pad them out where appropriate. There are 9^6-1 which do not contain a 5, and 6×9^5 which contains exactly one 5. Hence the number containing at least two fives is $999999-(15\times 9^5)+1=114265$.

ANS: 114265

14. A certain positive integer n has its digits all equal to 3 and is exactly divisible by 383. Find the last five digits in the quotient n÷383.

Let n=333…333=383×q. Then we can write $q=a_0+10a_1+10^2a_2+10^3a_3+\cdots$, where a_0, a_1, a_2, … are digits. Then

$$383\times q=\left(3+8\times 10+3\times 10^2\right)\times\left(a_0+10a_1+10^2a_2+10^3a_3+\cdots\right).$$

(i) The unit digit of n, 3, is obtained by $3\times a_0$, so $a_0=1$.

(ii) The tens digit of n is given by the units digit in $8\times a_0+3\times a_1=8+3a_1$, using $a_0=1$. Therefore, $3a_1$ must end in 5, and hence $a_1=5$ and $8+3a_1=23$, giving a carry of 2 into the hundreds.

(iii) The hundreds digit of n is the units digit in $3a_0+8a_1+3a_2+2=3a_2+45$. So $3a_2$ ends in 8. So $3a_2=18$ and hence $a_2=6$. Also $3a_2+45=63$, giving a carry of 6 into the thousands.

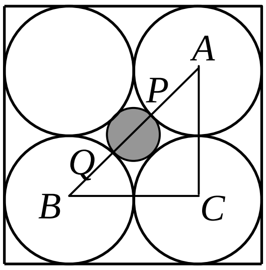
(iv) The thounds digit of n is the units digit in $3a_1+8a_2+3a_3+6=3a_3+69$. So $3a_3$ ends in 4. So $3a_3=24$ and hence $a_3=8$. Also $3a_3+69=93$, giving a carry of 9 into the next place.

(v) Finally, we need the units digit in $3a_2+8a_3+3a_4+9=3a_4+91$. So $3a_4$ ends in 2. So $3a_3=12$ and hence $a_4=4$.

The last 5 digits of the quotient is 48651.

ANS: 48651

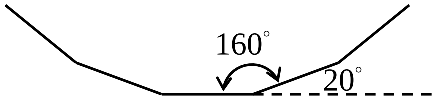
15. The square alongside has sides of length 8 units. The four identical circles fit tightly inside the square. What is the radius of the largest circle that will fit in the central hole?

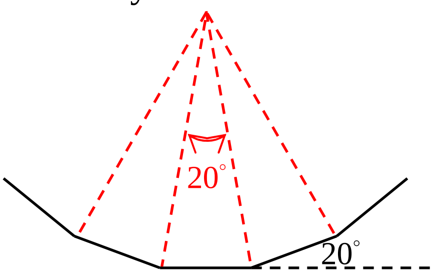


Let *A*, *B* and *C* be the centres of three of the large circles as shown. Since the large circles touch and fit tightly into the square, the diameter of each circle is 8÷2=4. Thus *AC* and *BC* have length 4, and, by Pythagoras' Theorem, AB has length equal to the square root of 32. By symmetry, the centre of the largest circle that will fit in the central hole is on *AB* and its diameter *PQ* has length equal to *AB*—*AP*—*QB*=*AB*—2—2 since *AP* and *QB* are radii of the larger circles. Thus, the radii of the largest circle that will fit is $2\sqrt{2}-2$

ANS: $2\sqrt{2}-2$

16. The Big Wheel at a fairground is a regular polygon. Here is a part of it. How many sides does it have?

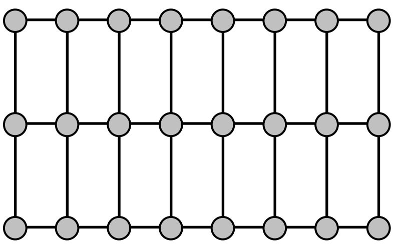


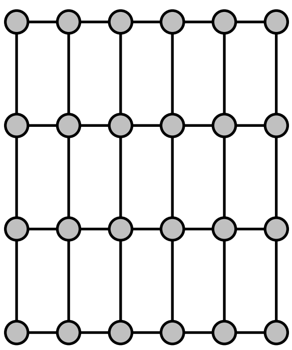


The external angle shown is 20°. The number of sides is 360°÷20°=18. Another approach is to use the 160° of the polygon, giving 20° at the centre. Then, again, the number of sides is 360°÷20°=18.

ANS: 18

17. I have a collection of Mathematical Challenge posters, all on A3 paper, so that they are rectangular in shape and have the same measurements. I have some drawing pins which I am going to use to pin the posters on my wall. Each poster must have a pin at each of its four corners, but adjacent posters can share a pin by allowing them to overlap slightly. I want to arrange them so that they cover a rectangular area of the wall, with their longer sides vertical.

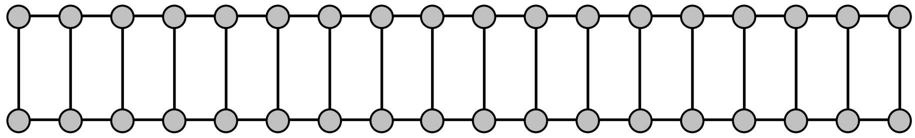




In the diagrams, 24 pins are used for 14 posters and 15 posters respectively. What is the greatest number of posters that I can pin up using 36 drawing pins?
To fill a rectangular area of wall the pins must be arranged in some rectangular pattern. The 36 pins can be arranged as: (a) 2 rows of 18, (b) 3 rows of 12, (c) 4

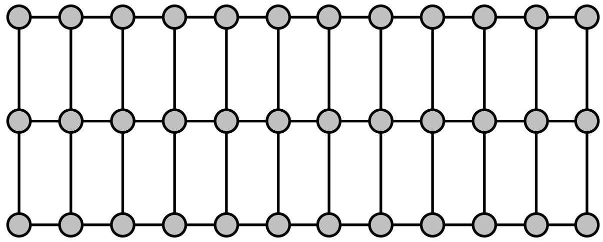
rows of 9, or (d) 6 rows of 6.

(a)



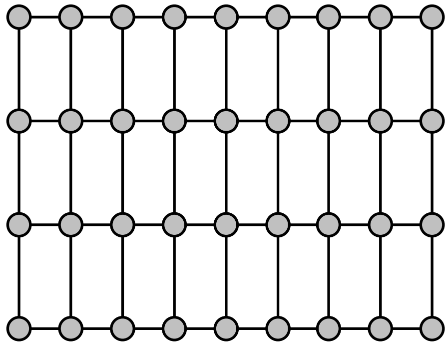
17 posters

(b)



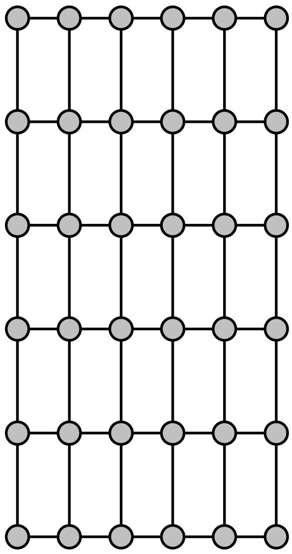
22 posters

(c)



24 posters

(d)

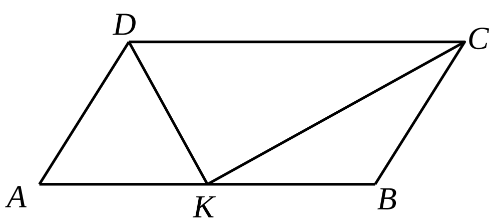


25 posters

Hence the greatest number of posters that can be pinned up using 36 pins is 25.

ANS: 25

18. In a parallelogram *ABCD*, *AB*=2*AD*. *K* is the midpoint of *AB*. Find the size of $\angle DKC$.



Since *AB*=2*AD* and *K* is the midpoint of *AB*, *AK*=*AD* and *BK*=*BC*. Hence $\angle ADK=\angle AKD$ and $\angle BKC=\angle BCK$.

Because *AD*//*BC* and *AB* is a straight segment, $\angle ADC+\angle BCD=180°$ and $\angle AKD+\angle DKC+\angle BKC=180°$.

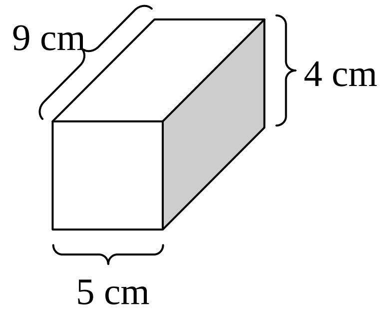
Hence $\angle ADK + \angle KDC + \angle KCD + \angle BCK = \angle AKD + \angle DKC + \angle BKC$

or $\angle KDC + \angle KCD = \angle DKC$.

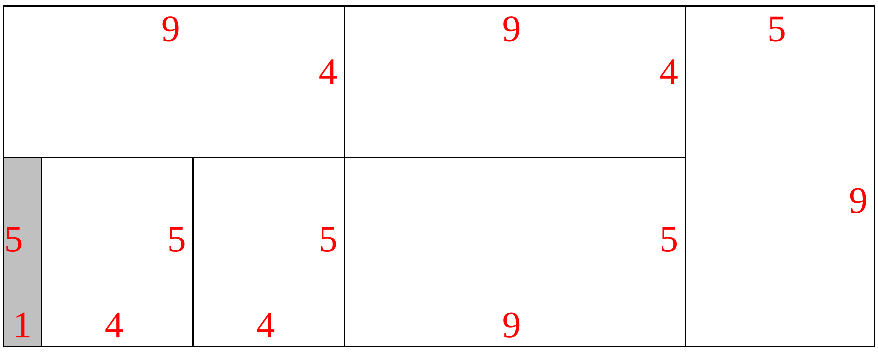
Since $\angle KDC + \angle KCD + \angle DKC = 180°$, $\angle DKC = 90°$

ANS: 90°

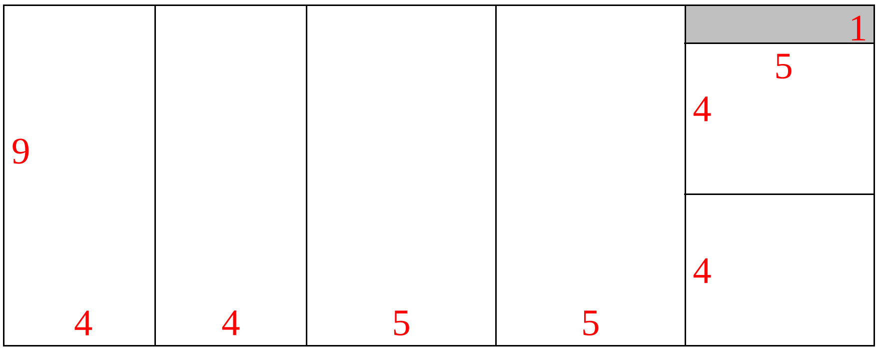
19. To cover the Christmas present (shown alongside) with sticky backed paper, you will require six rectangular pieces. What are the dimensions of the smallest single rectangular piece of sticky backed paper from which you could cut out the six pieces with the minimum of wastage?



Because the covering is sticky backed paper, we require two 9×5 pieces, two 9×4 pieces and two 5×4 pieces. Total area is therefore 202 cm^2. A rectangle measuring 9×23 i.e. 207 cm^2 gives one possible rectangular piece of paper with 5 cm^2.

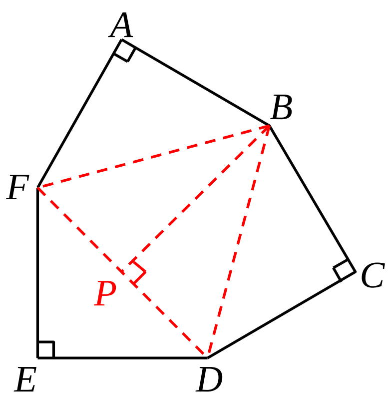


Another arrangement using the same size of paper is



ANS: 23 cm × 9 cm

20. A hexagon *ABCDEF* has each side 6 cm in length. The angles at *A*, *C*, *E* are all right angles, and the angles at *B*, *D*, *F* are obtuse angles. Find the area of the hexagon.



The area of the hexagon is the sum of the areas of the right-angled isosceles triangles *ABF*, *BCD* and *DEF*, together with the equilateral triangle *BDF* (see the figure). Since *AB*=*AF*=6 cm, and $\angle FAB$ is a right angle, then, using the fact that the area of a triangle is equal to $\frac{1}{2}$×base×perpendicular height, the area of triangle *ABF* is $\frac{1}{2}\times 6\times 6=18\,\text{cm}^2$. Similarly, triangles *BCD* and *DEF* also haave area 18cm^2. The length of each side of the equilateral triangle *BDF* is $6\sqrt{2}$ cm. If *P* is the foot of the perpendicular from *B* to *DF* then *P* is the mid-point of *DF* and so, by Pythagora's Theorem, $BP^2 = BD^2 - DP^2 = (6\sqrt{2})^2 - (3\sqrt{2})^2 = 72 - 18 = 54$ cm^2. Hence the height *BP* of triangle *BDF* is $\sqrt{54} = 3\sqrt{6}$ cm and so the area of this triangle is $\frac{1}{2}\times DF \times BP = \frac{1}{2}\times 6\sqrt{2}\times 3\sqrt{6} = 18\sqrt{3}\,\text{cm}^2$. It follows that the area of hexagon *ABCDEF* is $3\times 18 + 18\sqrt{3} = 54 + 18\sqrt{3}\,\text{cm}^2$.

ANS: $54 + 18\sqrt{3}\,\text{cm}^2$