INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2009

Mathematics Contest in Taiwan

Name:________ School:____________ Grade:_____ Number:______

Short Answer: there are 12 questions, fill in the correct answers in the answer sheet. Each correct answer is worth 10 points. Time limit: 90 minutes.

1. Find the largest possible divisor of the three numbers, 480608, 508811 and 723217, so that the reminder is the same in each case.

【***Solution***】

Since the reminder is the same in each case, the largest possible divisor must be a common factor of

$508811 - 480608 = 28203 = 3 \times 7 \times 17 \times 79$,

$723217 - 508811 = 214406 = 2 \times 23 \times 59 \times 79$,

$723217 - 480608 = 242609 = 37 \times 79 \times 83$.

Hence the answer is 79.

ANS:79

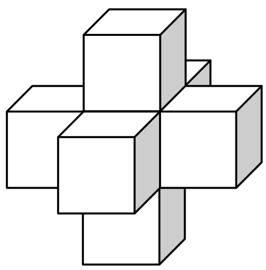
2. In a small group of people it was found that all of the following relationships were present: father, mother, son, daughter, brother, sister, cousin, nephew, niece, uncle and aunt. What is the smallest group of people for which this is possible?

【***Solution***】

Since father and son must be 2 different men and mother and daughter must be 2 different women, there are at least 4 people. We can find the following situation can satisfy the conditions: A brother and a sister. The brother has a son. The sister has a daughter.

ANS: 4 people

3. Seven cubes are glued together face to face as shown in the diagram. The volume of the solid formed in this way is 189 cm^3. Find the surface area of the solid.



【***Solution***】

Since the volume of the solid formed in this way is 189 cm^3, the volume of a cube is $189 \div 7 = 27$ cm^3 and hence the area of a face is 9 cm^2. The number of the faces of the solid is $6 \times 6 - 6 = 30$. So the surface area of the solid is $9 \times 30 = 270$ cm^2.

ANS: 270 cm^2

4. Jack said to Jim: "If I give you 6 pigs for one horse, then you will own twice as many animals as I own." Dan said to Jack: "If I give you 14 sheep for one horse, then you will own three times as many animals as I own." Jim said to Dan: "If I give you 4 cows for one horse, then you'll own six times as many animals as I own." How many animals in total do Jack, Jim and Dan own?

【*Solution*】

Assume Jack has a animals, Jim has b animals, and Dan has c animals. Thus we have the following equations:

$$\begin{cases} 2(a-6+1)=b+6-1 \\ 3(c-14+1)=a+14-1 \\ 6(b-4+1)=c+4-1 \end{cases} \Leftrightarrow \begin{cases} 2a-10=b+5 \\ 3c-39=a+13 \\ 6b-18=c+3 \end{cases} \Leftrightarrow \begin{cases} 2a-15=b \\ 3c-52=a \\ 6b-21=c \end{cases}$$

Hence a=11, b=7, and c=21. So there are 11+7+21=39 animals in total.

ANS: 39

5. By adding brackets in various ways to the expression 1÷3÷5÷7÷11÷13, what is the maximum number of different values which the expression can have?

【*Solution*】

No matter how the brackets are added, 1 is always part of the numerator and 3 is always part of the denominator. Each other number may be in either the numerator or the denominator, we will have 2^4=16 different values.

ANS:2^4=16

6. Replace the asterisks with digits so that the multiplication below is correct:

$$\begin{array}{cccccccccc}
 & & & & & \ast & \ast & \ast & \ast & \ast \\
\times & & & & & & \ast & \ast & \ast & \ast \\
\hline
 & & & & & 3 & 3 & 3 & 3 & 7 \\
 & & & \ast & \ast & \ast & \ast & \ast & \ast & \\
 & & \ast & \ast & \ast & \ast & \ast & \ast & & \\
 & \ast & \ast & \ast & \ast & \ast & \ast & & & \\
\hline
 & \ast & \ast & \ast & \ast & 2 & 0 & 0 & 9 & \ast
\end{array}$$

What is the product?

【*Solution*】

Since 33337 is not divisible by 2 and 3, we can set the multiplication as following:

$$\begin{array}{cccccccccc}
 & & & & & 3 & 3 & 3 & 3 & 7 \\
\times & & & & & & A & B & C & 1 \\
\hline
 & & & & & 3 & 3 & 3 & 3 & 7 \\
 & & & \ast & \ast & \ast & \ast & \ast & D & \\
 & & \ast & \ast & \ast & \ast & \ast & \ast & & \\
 & \ast & \ast & \ast & \ast & \ast & \ast & & & \\
\hline
 & \ast & \ast & \ast & \ast & 2 & 0 & 0 & 9 & 7
\end{array}$$

Thus D must be 6 and hence C must be 8. We can set the multiplication again as following:

$$\begin{array}{cccccccccc}
 & & & & & 3 & 3 & 3 & 3 & 7 \\
\times & & & & & & A & B & 8 & 1 \\
\hline
 & & & & & 3 & 3 & 3 & 3 & 7 \\
 & & & 2 & 6 & 6 & 6 & 9 & 6 & \\
 & & \ast & \ast & \ast & \ast & \ast & E & & \\
 & \ast & \ast & \ast & \ast & \ast & \ast & & & \\
\hline
 & \ast & \ast & \ast & \ast & 2 & 0 & 0 & 9 & 7
\end{array}$$

Thus E must be 8 and hence B must be 4. We can set the multiplication again as following:

$$\begin{array}{r} 3\ 3\ 3\ 3\ 7 \\ \times \qquad A\ 4\ 8\ 1 \\ \hline 3\ 3\ 3\ 3\ 7 \\ 2\ 6\ 6\ 6\ 9\ 6\ \ \\ 1\ 3\ 3\ 3\ 4\ 8\ \ \ \ \\ *\ *\ *\ *\ *\ F\ \ \ \ \ \ \\ \hline *\ *\ *\ *\ 2\ 0\ 0\ 9\ 7 \end{array}$$

Thus F must be 5 and hence A must be 5. We can get the multiplication as following:

$$\begin{array}{r} 3\ 3\ 3\ 3\ 7 \\ \times \qquad 5\ 4\ 8\ 1 \\ \hline 3\ 3\ 3\ 3\ 7 \\ 2\ 6\ 6\ 6\ 9\ 6\ \ \\ 1\ 3\ 3\ 3\ 4\ 8\ \ \ \ \\ 1\ 6\ 6\ 6\ 8\ 5\ \ \ \ \ \ \\ \hline 1\ 8\ 2\ 7\ 2\ 0\ 0\ 9\ 7 \end{array}$$

Hence the product is 182720097.

ANS: 182720097

7. Tom has a contract to dig out some foundations and it must be done in 30 days. His own machine, which he wishes to use as much as possible, would take 48 days to do all the work. He can hire a bigger machine which would do the complete job in 21 days, but it costs $300 a day. There is only enough room for one machine at a time. What is the least number of days for which he will have to hire the larger machine?

【*Solution*】

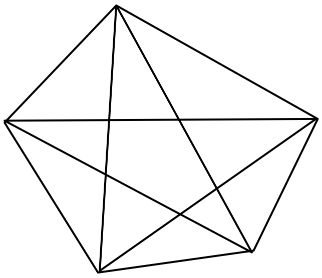
Set the amount of work to 1. Thus Tom's machine does $\frac{1}{48}$ of the work per day and the larger machine does $\frac{1}{21}$ of the work per day. Hence Tom must hire the larger machine for at least $(1-\frac{1}{48}\times 30)\div(\frac{1}{21}-\frac{1}{48})=14$ days. ANS:14 days

8. Four different right-angled triangles all have sides which are of integral length and their perimeters are the same length. Find the smallest perimeter for which this is possible.

The right-angled triangles with smaller integral sides have sides of length: (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (13, 84, 85) (15, 112, 113), (20, 21, 29),…. Their perimeters are 12, 30, 56, 40, 90, 132, 182, 240, 70,….We can find right-angled triangles (60, 80, 100), (40, 96, 104), (48, 90, 102), (15, 112, 113) with same perimeter. Hence the smallest perimeter is 240.

ANS:240

9. The diagram is of an irregular pentagon with all 5 of its diagonals drawn in. How many distinct triangles (not necessarily different) can be found, using only the lines (or parts of lines) shown in the diagram?



【***Solution***】

There are 5 triangles with two edges of the pentagon,
$5\times4=20$ triangles with exactly one edge of the pentagon,
5 triangles with exactly one full diagonal and without edges of the pentagon, and
5 triangles without full diagonals and without edges of the pentagon. There are in total $5+20+5+5=35$ distinct diagonals.

ANS:35

10. I have a rectangular picture whose edges are each an exact number of centimeters in length. At a quick glance it could be mistaken for a square, but it is not a square. It is placed inside a black border which is 3 cm wide all the way around the picture. The area of the border is exactly equal to the area of the picture. What is the area, in cm^2, of the picture alone?

【***Solution***】

Assume the length of the picture is a and the width of the picture is b. We can suppose that $b\geq a$. Thus we can get the following equation:

$$ab=4\times3\times3+2\times3a+2\times3b=36+6a+6b$$
$$ab-6a-6b+36=36+36$$
$$a(b-6)-6(b-6)=72$$
$$(a-6)(b-6)=72$$

Since a and b are positive integers and at a quick glance it could be mistaken for a square, there is only one possible factorization of 72 which is 8×9 and hence the solution is $a=14$ and $b=15$, the area of the picture alone is 210 cm^2.

ANS:210 cm^2

11. A combination lock on a safe needs a 6-letter sequence to open the safe. This is made from the letters A, B, C, D, E, F with none of them being used twice. Here are three guesses at the combination

C B A D F E
A E D C B F
E D F A C B

In the FIRST guess only ONE letter is in its correct place. In the SECOND guess only TWO letters are in their correct places and those two correct places are not next to each other. In the THIRD guess only THREE letters are in their correct places. Each of the 6 letters is in its correct place once. What is the correct combination?

【***Solution***】

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	C	B	A	D	F	E
SECOND	A	E	D	C	B	F
THIRD	E	D	F	A	C	B

There are 5 mistakes in the FIRST guess, 4 mistakes in the SECOND guess, and 3 mistakes in the THIRD guess.

(a) If the letter C is on the correct place in the FIRST guess, then the positions (4) and (5) can't be C. Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	C	~~B~~	~~A~~	~~D~~	~~F~~	~~E~~
SECOND	~~A~~	E	D	~~C~~	B	F
THIRD	~~E~~	D	F	A	~~C~~	B

Hence E is on (2) and A is on (4). Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	C	~~B~~	~~A~~	~~D~~	~~F~~	~~E~~
SECOND	~~A~~	E	D	~~C~~	B	F
THIRD	~~E~~	~~D~~	F	A	~~C~~	B

Now there are 3 mistakes in the THIRD guess, so we can finish the table as follows:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	C	~~B~~	~~A~~	~~D~~	~~F~~	~~E~~
SECOND	~~A~~	E	~~D~~	~~C~~	B	~~F~~
THIRD	~~E~~	~~D~~	F	A	~~C~~	B

Thus B is on position (5) and (6), which is impossible!

(b) If the letter B is on the correct place in the FIRST guess, then the positions (5) and (6) can't be B. Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	B	~~A~~	~~D~~	~~F~~	~~E~~
SECOND	A	~~E~~	D	C	~~B~~	F
THIRD	E	~~D~~	F	A	C	~~B~~

Hence D is on (3), C is on (5)and E is on (1). Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	B	~~A~~	~~D~~	~~F~~	~~E~~
SECOND	~~A~~	~~E~~	D	~~C~~	~~B~~	F
THIRD	E	~~D~~	~~F~~	A	C	~~B~~

Thus we got one possible answer, EBDACF.

(c) If the letter A is on the correct place in the FIRST guess, then the positions (1) and (4) can't be A. Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	A	~~D~~	~~F~~	~~E~~
SECOND	~~A~~	E	~~D~~	C	B	F
THIRD	E	D	~~F~~	~~A~~	C	B

Hence D is on (2) and F is on (6). Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	A	~~D~~	~~F~~	~~E~~
SECOND	~~A~~	~~E~~	~~D~~	C	B	F
THIRD	E	D	~~F~~	~~A~~	C	~~B~~

Now there are 3 mistakes in the THIRD guess, so we can finish the table as follows:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	A	~~D~~	~~F~~	~~E~~
SECOND	~~A~~	~~E~~	~~D~~	C	~~B~~	F
THIRD	E	D	~~F~~	~~A~~	C	~~B~~

Thus C is on position (4) and (5), which is impossible!

(d) If the letter D is on the correct place in the FIRST guess, then the positions (3) and (2) can't be D. Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	D	~~F~~	~~E~~
SECOND	A	E	~~D~~	~~C~~	B	F
THIRD	E	~~D~~	F	~~A~~	C	B

Hence A is on (1) , F is on (3)and C is on (5). Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	D	~~F~~	~~E~~
SECOND	A	E	~~D~~	~~C~~	~~B~~	~~F~~
THIRD	~~E~~	~~D~~	F	~~A~~	C	B

But the correct places of A and E are next to each other in the SECOND guess, hence it is not an answer.

(e) If the letter F is on the correct place in the FIRST guess, then the positions (6) and (3) can't be F. Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	~~D~~	F	~~E~~
SECOND	A	E	D	C	~~B~~	~~F~~
THIRD	E	D	~~F~~	A	~~C~~	B

Hence B is on (6) and C is on (4). Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	~~D~~	F	~~E~~
SECOND	A	E	D	C	~~B~~	~~F~~
THIRD	E	D	~~F~~	~~A~~	~~C~~	B

There are 3 mistakes in the THIRD guess, so we can finish the table as:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	~~D~~	F	~~E~~
SECOND	~~A~~	~~E~~	D	C	~~B~~	~~F~~
THIRD	E	D	~~F~~	~~A~~	~~C~~	B

Thus D is on position (3) and (2), which is impossible!

(f) If the letter E is on the correct place in the FIRST guess, then the positions (2) and (1) can't be E. Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	~~D~~	~~F~~	E
SECOND	A	~~E~~	D	C	B	~~F~~
THIRD	~~E~~	D	F	A	C	~~B~~

Hence B is on (5) and F is on (3). Thus we have:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	~~D~~	~~F~~	E
SECOND	A	~~E~~	~~D~~	C	B	~~F~~
THIRD	~~E~~	D	F	A	~~C~~	~~B~~

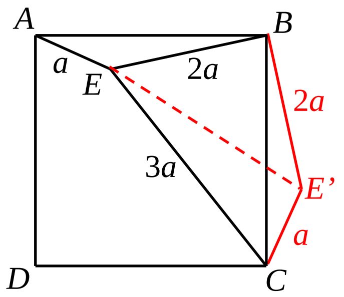
Now there are 3 mistakes in the THIRD guess, so we can finish the table as follows:

	(1)	(2)	(3)	(4)	(5)	(6)
FIRST	~~C~~	~~B~~	~~A~~	~~D~~	~~F~~	E
SECOND	A	~~E~~	~~D~~	~~C~~	B	~~F~~
THIRD	~~E~~	D	F	A	~~C~~	~~B~~

Thus A is on position (1) and (4), which is impossible!

ANS: EBDACF

12. Given that *ABCD* is a square and the lengths *EA*, *EB*, *EC* are in the ratio *EA*:*EB*:*EC*=1:2:3, determine the size of the angle *AEB*, in degree.



【***Solution***】

Rotate △*AEB* to △*CE'B* with center *B*, as the right figure. Connect *EE'*.

Thus △*BEE'* is an isosceles right triangle and hence ∠*BE'E*=45° and the length of *EE'* is $2\sqrt{2}a$. Hence △*CEE'* is a right triangle by Pythagorean theorem and we get ∠*CE'E*=90°.

We have ∠*AEB*=∠*BE'C*=∠*CE'E*+∠*BE'E* =90°+45°=135°.

ANS: 135°