

**INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD
FOR PRIMARY SCHOOLS (IMSO) 2009
Mathematics Contest (Second Round) in Taiwan
Short Answer Problems**

Name: _____ School: _____ Grade: _____ ID number: _____

Short Answer: there are 20 questions, fill in the correct answers on the answer sheet. Each correct answer is worth 2 points. Time limit: 60 minutes.

1. Henry was given a certain number to multiply by 507, but he placed the first figure of his product by 5 below the second figure from the right instead of below the third. The result of Henry's mistake was that his answer was wrong by 382050. What was the multiplicand?

【Solution】

Assume the number is a . Then we can get the following equation from the situation: $507a - 57a = 382050$. Hence $a = 849$.

ANS:849

2. Mice have 4 legs, ants have 6 legs and spiders have 8 legs. Eddy has twice as many mice as spiders, and three times as many ants as spiders. The number of legs adds up to 68. How many spiders does he have?

【Solution】

Since Eddy has twice as many mice as spiders and three times as many ants as spiders, we can assume he has x spiders and hence has $2x$ mice and $3x$ ants. Thus we have $8x + 6 \times 3x + 4 \times 2x = 68$, i.e. $34x = 68$. So $x = 2$.

ANS:2

3. There are 100 nuts in five bags. In the first and second bags, there are altogether 42 nuts; in the second and third bags, there are 43 nuts; in the third and fourth bags, there are 34 nuts; in the fourth and fifth bags, there are 30 nuts. How many nuts are there in the first bag?

【Solution】

Assume there are a nuts in the first bag, b nuts in the second bag, c nuts in the third bag, d nuts in the fourth bag and e nuts in the fifth bag. Thus we have the following equations:

$$\left\{ \begin{array}{l} a+b+c+d+e=100 \\ a+b=42 \\ b+c=43 \\ c+d=34 \\ d+e=30 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} a+b+c+d+e=100 \\ a+2b+2c+2d+e=149 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} b+c+d=49 \\ a+e=51 \end{array} \right.$$

Hence $d=49-43=6$, $e=30-d=24$, $a=51-e=27$, $b=42-a=15$, $c=43-b=28$.

ANS:27

4. A student had to multiply 169 by a two-digit number whose second digit is twice as big as the first digit. Accidentally he changed the places of the two digits and obtained a product that differed from the correct one by 4563. Find the two-digit number.

【Solution】

Assume the number is \overline{ab} , where $2a=b$. Thus we can get the following equation from the situation: $(10b+a) \times 169 - (10a+b) \times 169 = 4563$, i.e. $b - a = 3$. So $a=3$ and $b=6$ and hence the number is 36.

ANS: 36

5. How many days is it from Wednesday the 1st August to the first Saturday in September? (Inclusive of both dates)

【Solution】

Since there are 31 days in August and the 1st August is Wednesday, the 31st August is Friday and hence the 1st September is Saturday. So the answer is $31+1=32$ days.

ANS:32

6. If 6 cats can catch 6 rats in 6 minutes, how many cats are needed to catch 12 rats in 12 minutes?

【Solution】

Since 6 cats can catch 6 rats in 6 minutes, 1 cat catches 6 rats in 36 minutes and hence 1 cat catches 1 rat in 6 minutes. So 1 cat can catch 2 rats in 12 minutes, i.e. 6 cats can catch 12 rats in 12 minutes.

ANS:6 cats

7. A collection of sheep and turkeys have a total of 99 heads and legs between them. There are twice as many turkeys as there are sheep. How many turkeys are there?

【Solution】

We know that a turkey has a total of 3 heads and legs and a sheep has a total of 5 heads and legs. Assume there are a turkeys and b sheep, then we have the following equations:

$$\begin{cases} 3a + 5b = 99 \\ a = 2b \end{cases}$$

Hence $a=18$ and $b=9$.

ANS: 18 turkeys

8. Find the smallest possible number that leaves a remainder of 1 when divided by 2, 3, 4, 5 or 6, and which can be divided by 7 exactly.

【Solution】

Since L.C.M [2, 3, 4, 5, 6]=60, the numbers that leave a remainder of 1 when divided by 2, 3, 4, 5 or 6 are all of the form $60k+1$ for some k . They are 1, 61, 121, 181, 241, 301, 361,...The smallest number which can be divided by 7 exactly is 301.

ANS:301

9. Each of the letters A, B, C, D, E is used to represent a single digit in these two statements.(Same letter = same digit.) $A + B = C, C + D = \overline{EA}$ (Note \overline{EA} is a 2-digit number) What is the value of $B + D$?

【Solution】

Because $10E+A=C+D=A+B+D, B+D=10E$.

Since B and D are both single digits, $B+D < 8+9=17$. So $E=1$ and hence the only possible value of $B + D$ is 10.

ANS:10

10. Ann, Ben and Carol each have some money. If Ann gave Ben \$30, then Ben would have twice as much as Ann. If Ben gave Carol \$30, then Carol would have twice as much as Ben. If Carol gave Ann \$30, then they would both have the same amount. How much money did Ann have?

【Solution】

Assume Ann has \$ a , Ben has \$ b , and Carol has \$ c . Thus we have the following equations:

$$\begin{cases} 2(a - 30) = b + 30 \\ 2(b - 30) = c + 30 \\ c - 30 = a + 30 \end{cases} \Leftrightarrow \begin{cases} 2a - 60 = b + 30 \\ 2b - 60 = c + 30 \\ c - 30 = a + 30 \end{cases} \Leftrightarrow \begin{cases} 2a - 90 = b \\ 2b - 90 = c \\ c - 60 = a \end{cases}$$

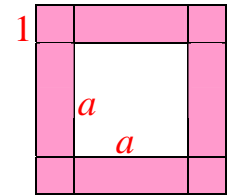
Hence $a=110, b=130, \text{ and } c=170$. Ann has \$110.

ANS:\$110

11. A square lawn has a path 1 m wide which goes around the outside of all the four edges. The area of the path is 40 m^2 . What is the area of the lawn?

【Solution】

Assume the length of an edge of the lawn is a . From the right figure, we know that $(1 \times 1 + 1 \times a) \times 4 = 40$, i.e. $a = 9$. Hence the area of the lawn is $9 \times 9 = 81 \text{ m}^2$.



ANS: 81 m^2

12. Four consecutive odd numbers add up to a total of 80. What is the smallest of those four numbers.

【Solution】

Assume the four consecutive odd numbers are $a - 3$, $a - 1$, $a + 1$, and $a + 3$. Thus we have $4a = 80$, i.e. $a = 20$. So the four consecutive odd numbers are 17, 19, 21 and 23, the smallest number is 17.

ANS: 17

13. A cube with an edge length of 10 cm is resting on a horizontal table. An insect starts crawling from the table at an angle of 30 degrees to the horizontal. How far will it have crawled on the cube by the time it gets to the top?

【Solution】

Since the angle is 30° and the height the insect increases is 10 cm, the insect has crawled $10 \times 2 = 20 \text{ cm}$.

ANS: 20 cm

14. On this diagram you may start at any square and move up or down or across (but NOT diagonally) into the next square. No square may be used twice. The digits in each square are written down in the order they are used to form a number. What is the largest number that can be made?

5	9	1
8	4	7
3	6	2

【Solution】

If you start at the red squares, then the number of the digits of your number is at most 8 since the number of red squares is less than the number of white squares.

If you start at the white squares, then the number of the digits of your number is at most 9. So we start at 5 and get the number 594836271

5	9	1
8	4	7
3	6	2

ANS: 594836271

15. A new monument is to be made in the shape of a cuboid. Only three of the faces are to be decorated. To allow for this: one face has to have an area of 48 m^2 ; another is to have an area of 72 m^2 ; and another of 96 m^2 . What will be the volume of the monument?

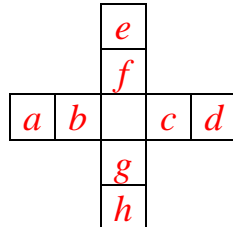
【Solution】

Let the sides of the cuboid be a , b and c . Since $ab = 48 = 2^4 \times 3$, $bc = 72 = 2^3 \times 3^2$ and

$ac=96=2^5 \times 3$, $a^2b^2c^2=48 \times 72 \times 96=2^{12} \times 3^4$, hence the volume abc is $2^6 \times 3^2 = 576$.

ANS:576 m³

16. Arrange the numbers 1 to 9, using each number only once and placing only one number in each cell so that the totals in both directions (vertically and horizontally) are the same. How many different sums are there?



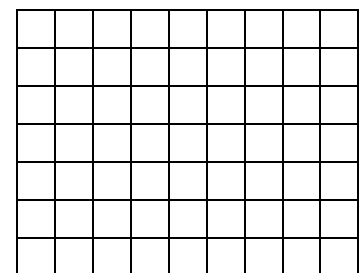
【Solution】

Since the totals in both directions are the same, $a+b+c+d=e+f+g+h$, i.e. the sum of the eight numbers, $a+b+c+d+e+f+g+h$, must be an even number. Because $1+2+3+4+5+6+7+8+9=45$ is an odd number, the number in the center square must be an odd number and hence there are 5 possibilities:

- (i) If 1 is placed in the center square, then the sum is 23 and we can divide the other eight number into two sets, {2, 3, 8, 9} and {4, 5, 6, 7}.
- (ii) If 3 is placed in the center square, then the sum is 24 and we can divide the other eight number into two sets, {1, 4, 7, 9} and {2, 5, 6, 8}.
- (iii) If 5 is placed in the center square, then the sum is 25 and we can divide the other eight number into two sets, {1, 2, 8, 9} and {3, 4, 6, 7}.
- (iv) If 7 is placed in the center square, then the sum is 26 and we can divide the other eight number into two sets, {1, 3, 6, 9} and {2, 4, 5, 8}.
- (v) If 9 is placed in the center square, then the sum is 27 and we can divide the other eight number into two sets, {1, 2, 7, 8} and {3, 4, 5, 6}.

ANS:5

17. How many distinct squares (not necessarily different in size) can be traced out following only the lines of the grid drawn on the right?



【Solution】

There are $9 \times 7 = 63$ unit squares, $8 \times 6 = 48$ 2×2 squares, $7 \times 5 = 35$ 3×3 squares, $6 \times 4 = 24$ 4×4 squares, $5 \times 3 = 15$ 5×5 squares, $4 \times 2 = 8$ 6×6 squares, and $3 \times 1 = 3$ 7×7 squares. So there are $63+48+35+24+15+8+3=196$ distinct squares.

ANS:196

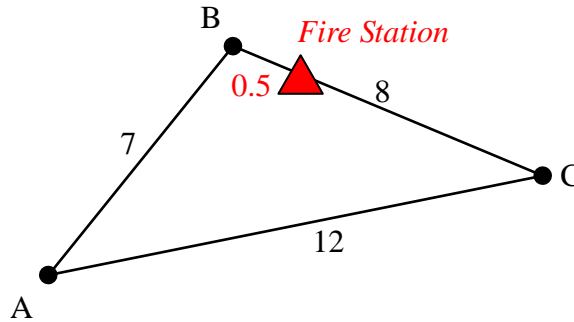
18. A 4-wheeler car has travelled 24,000 km and, in that distance, has worn out 6 tyres. Each tyre travelled the same distance. How far did each separate tyre travel?

【Solution】

If the car didn't change the tyres, then the total distance of the four tyres travelled is $24,000 \times 4 = 96,000$ km. But it has used 6 tyres, so the answer is $96,000 \div 6 = 16,000$ km.

ANS:16,000 km

19. A, B and C are three villages near to each other, shown in the diagram below, where the straight lines represent the only roads joining the villages. The figures give the distances in km between villages.



A new fire station is to be built to serve all three villages. It is to be on a roadside at such a position that the greatest distance that the fire-engine has to travel along the roads in an emergency at one of the villages is as small as it can be. What is this smallest distance?

【Solution】

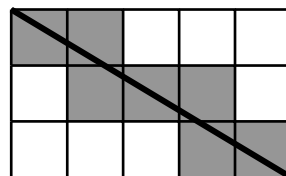
We are told that the greatest distance the fire-engine has to travel to any of the villages is to be as small as it can be. At the position show in the diagram above, 0.5 km from B, the shortest distance to A, B and C are 7.5 km, 0.5 km and 7.5 km respectively. To show that no other position is satisfactory

- At any other position on the road joining B to C, the shortest distance to either A or C will be more than 7.5 km.
- At any other position on the road joining A to B, the shortest distance to C will be more than 8 km.
- At any other position on the road joining A to C, the shortest distance to either B or C will be more than 7.5 km.

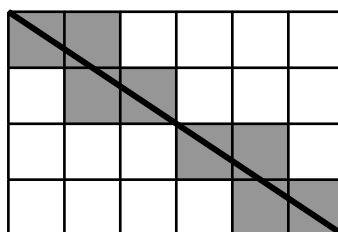
Hence the fire station should be on the road from B to C, 0.5 km from B.

ANS:7.5 km

20. The diagonal of this 5×3 rectangle passes through 7 squares.



The diagonal of this 6×4 rectangle passes through 8 squares.



What is the number of squares passed through by the diagonal of a 360×2009 rectangle?

【Solution】

Observe that in passing from one shaded square to the next, the diagonal passes from one vertical line to the next or from one horizontal line to the next, but not both unless it passes through a 'corner.' Since the greatest common factor of 360 and 2009 is 1, the diagonal doesn't pass corners. Thus the diagonal passes through the top left square and then $360 - 1 = 359$ squares (one for each of the remaining rows) and $2009 - 1 = 2008$ squares (one for each of the remaining columns). This gives a total of $1 + 359 + 2008 = 2368$ squares.

ANS:2368