INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2009

Mathematics Contest (Second Round) in Taiwan, Essay Problems

Name:_______ School:_______ Grade:______ ID number:________

Answer the following 10 questions, and show your detailed solution in the space provided after each question. Each question is worth 4 points.
Time limit: 60 minutes.

1. There are two clocks. One of them gains 6 seconds in every hour, while the other loses 9 seconds in every hour. If they are both set to show the same time, and then set going, how long will it be before the time displayed on them is exactly 1 hour apart?

【***Solution***】

In 1 hour, the difference in time between them is 6+9=15 seconds. Since 1 hour is equal to 3600 seconds, they need 3600÷15=240 hours.

ANS:240 hours

2. Replace the asterisks in 86**** with the digits 1, 2, 3 and 4. Using each of them once so that the six-digit number obtained is the largest possible number divisible by 132.

【***Solution***】

Assume the six-digit number is $\overline{86abcd}$.

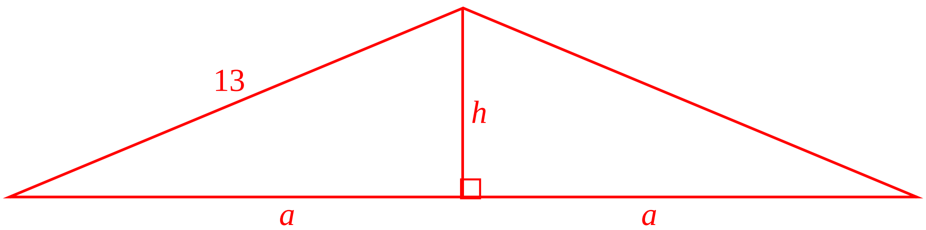
Since 132=3×4×11, (8+a+c)－(6+b+d)=2+(a+c)－(b+d) must be a multiple of 11.

Since the four digits are 1, 2, 3, and 4, 2+(a+c)－(b+d)=0, i.e. a and c are 1 and 3 and b and d are 2 and 4. Since the six-digit number is divisible by 4, the last two digits can only be 12 or 32, hence the largest number is 863412.

ANS:863412

3. There are two isosceles triangles. They are equal in area. In both triangles all edges measure an exact number of cm, and the two edges of equal length are 13 cm. In one of them the third edge measures 10 cm. What is the length of the third edge of the other?

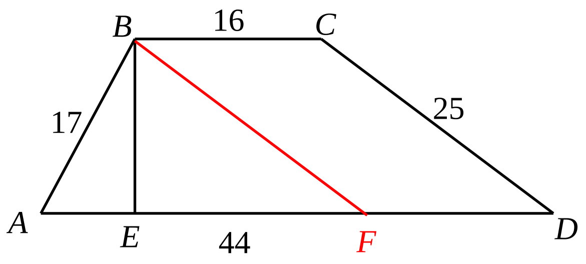
【***Solution***】



Let the above triangle be the other triangle. From the conditions, we know that the area is $\frac{1}{2}\times 10\times\sqrt{13^2-(\frac{1}{2}\times 10)^2}=\frac{1}{2}\times 10\times 12$=60 cm^2. And since $13^2=a^2+h^2$, we know 13>h and 13>a. Thus (h, a)=(5, 12) or (12, 5) because 60=ha. If (h, a)=(12, 5), then the triangle is the known triangle. So (h, a)=(5, 12) and the answer is 12×2=24 cm.

ANS:24 cm.

4. In a quadrilateral *ABCD*, *BC* is parallel to *AD*. *E* is the foot of the perpendicular from *B* to *AD*. Find *BE* if *AB*=17, *BC*=16, *CD*=25 and *AD*=44.



【Solution】

Let point *F* lie on *AD* such that *BF*//*CD*, thus quadrilateral *FBCD* is a parallelogram and hence *BF*=25 and *AF*=44－16=28. Since △*ABE* and △*FBE* are right triangles, $BE=\sqrt{AB^2-AE^2}=\sqrt{BF^2-EF^2}$, i.e. $17^2-AE^2=25^2-(28-AE)^2$. Solve the equation to get *AE*=8. So *BE*=$\sqrt{17^2-8^2}$=15.

ANS:15

5. Three different numbers from 1 to 10 were written on three cards. The cards were shuffled and dealt to three players. Each player got one card and wrote down the number of his card. Then the cards were collected and dealt again. After several deals the three players reported the totals of their written numbers, which were 13, 15, and 23. What numbers were written down on the cards at the beginning?

【Solution】

Assume the numbers written on the cards are x, y and z, where $x<y<z$. Since x, y and z are different numbers and 13+15+23=51=3×17, $x+y+z=17$ and hence there are 3 deals.
Suppose A got 13, B got 15, C got 23.
Since C got 23 and 23>17= $x+y+z$, C got one x and two z's or one y and two z's.
When C got one x and two z's, then there are two cases:

(i) If B got one x and two y's, i.e. B got in total $x+2y$, then A got one x, one y, one z and hence A got in total $x+y+z=17$, this is impossible.

(ii) If B got two x's and one y, i.e. B got in total $2x+y$, then A got two y's, one z and hence A got in total $13=2y+z>2x+y=15$, this is impossible.

When C got one y and two z's, then there are two cases:

(i) If B got two x's and one y, i.e. B got in total $2x+y$, then A got one x, one y, one z and hence A got in total $x+y+z=17$, this is impossible.

If B got one x and two y's, i.e. B got in total $x+2y$, then A got two x's, one z and hence A got in total $2x+y$. Thus we have $x=3$, $y=5$ and $z=9$, i.e. the numbers written on cards are 3, 5 and 9.

ANS: 3, 5 and 9

6. Five students A, B, C, D, and E competed in solving a math problem. The complete solution to the problem was awarded 10 points and a partial solution – an integer between 2 and 9. Each student scored some number of points so that : A, B, and C were awarded 15 points together; and B, C and D were awarded 12

points together. All students got different scores. The student A had the highest score and student E who scored 6 points, was placed third. What was the score of student D?

【***Solution***】

Let a, b, c, d and e be the scores of A, B, C, D and E, respectively. Thus we know $2 \le a, b, c, d, e \le 10$, $a+b+c=15$ and $b+c+d=12$.

If $a>b>e=6$, then a and b are two of {7, 8, 9, 10} and hence $a+b \ge 15$, which contradicts with $a+b+c=15$. So $e>b$.

If $a>c>e=6$, then a and c are two of {7, 8, 9, 10} and hence $a+c \ge 15$, which contradicts with $a+b+c=15$. So $e>c$.

Hence we know $a>d>e=6$. Because $a+b+c=15$ and $b+c+d=12$, $a-d=3$ and the only possible solution is $a=10$, $d=7$.

ANS:7

7. A grandmother has two grandsons. Her age is a two-digit number. The first digit is equal to the age of the first grandson, and the second digit is equal to the age of the second grandson. If the sum of their ages is 69, how old is the grandmother?

【***Solution***】

Assume the age of the grandmother is $\overline{ab}$, where a and b are digits, then we get $10a+b+a+b=69$, i.e. $11a+2b=69$. Thus we know that $0 < a \le 6$ and a must be an odd number, hence $a=1$, 3 or 5. If $a=1$, then $b=29$. If $a=3$, then $b=18$. If $a=5$, then $b=7$. Only $(a, b)=(5, 7)$ satisfies the conditions.

ANS:57

8. A 'Lucky number' has been defined as a number which can be divided exactly by the sum of its digits. For example: 1729 is a Lucky number since 1 + 7 + 2 + 9 = 19 and 1729 can be divided exactly by 19. Find the smallest Lucky number which is divisible by 13

【***Solution***】

The multiples of 13 are 13, 26, 39, 52, 65, 78, 91, 104, 117, …. So the smallest Lucky number which is divisible by 13 is 117.

ANS:117

9. In the middle of a large field there is a wooden hut on a rectangular base measuring 10 m by 6 m. Outside the hut, and tethered by a chain to one corner is a goat. Over what area can the goat graze if the tether is 15 m long? (Using π = 3.14)

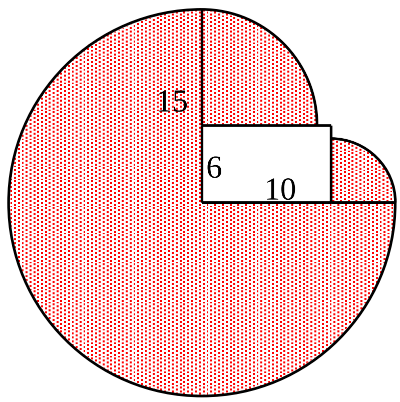
【***Solution***】

In the figure on the right, we need to find the area of the red zone. The area is

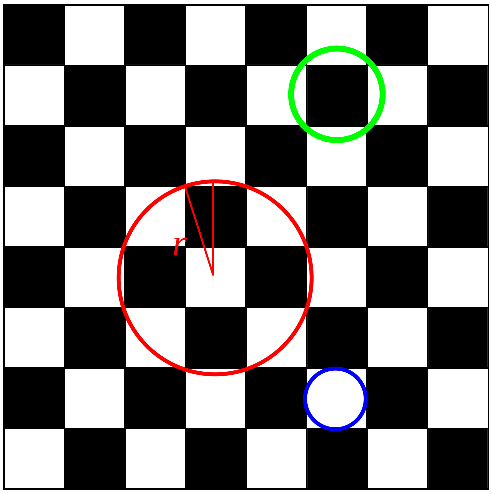
$$\frac{3}{4}\times 3.14\times 15^2+\frac{1}{4}\times 3.14\times(15-6)^2+\frac{1}{4}\times 3.14\times(15-10)^2$$

=613.085.

ANS:613.085



10. A chess-board is made up of 64 black and white squares in the normal way, each having an edge length of 10 cm. On this board the largest possible circle is drawn so that it's circumference does not pass through a black square. What is the radius of the circle?



【*Solution*】

Let O be the center of a circle and r be the radius. We observe that if the circumference of the circle does not pass through a black square, then the circumference can only intersect the black squares at the grid points or touch an edge of a black square.

(i) If O is on a grid point of the chessboard or on an edge of a square, by symmetry, the circumference must pass through a black square

(ii) If O is inside a square, then O should be at the center of the square. Now we'll consider the distances between O and the grid points. By symmetry, we just consider the upper right vertex of a black square.

(1) If O is inside a black square, then there are at least two kinds of distances between the grid points and O, one is of the form $\sqrt{(5+10k)^2+(5+10k)^2}$ which is the distance between O and the grid points on the extension of the diagonal (upper-right to bottom-left) of the black square and the other one is of the form $\sqrt{5^2+(5+20k)^2}$ which is the distance between O and the grid points on the vertical line which is next to O.
They will be the same as k=0 and hence $r=5\sqrt{2}$ cm and we can plot the green circle in the figure.

(2) If O is inside a white square and the circumference of the circle does not pass through the grid points, the r=5 and we can plot the blue circle in the figure.

(3) If O is inside a white square and the circumference of the circle passes some grid points, then there are at least two kinds of distances between the grid points and O, one is of the form $\sqrt{(5+10k)^2+(5+10(k+1))^2}$ which is the distance between O and the grid points on the extend of the diagonal (upper-right to bottom-left) of the black square which is on the right and the other one is of the form $\sqrt{5^2+(15+20k)^2}$ which is the distance between O and the grid points on the vertical line which is next to O.
They will be the same as k=0 and hence $r=5\sqrt{10}$ cm and we can plot the red circle in the figure. So the largest possible radius is $5\sqrt{10}$ cm ANS: $5\sqrt{10}$ cm