INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2009

Mathematics Contest in Taiwan, Exploration Problems

Name:_____ School:_____ Grade:_____ ID number:_____

Answer the following 5 questions. Write down your answer in the space provided after each question. Each question is worth 8 points. Time limit: 60 minutes.

1. At a recent athletics meeting, five old acquaintances: Fred, Greta, Hans, Iolo and Jan met together for the first time since leaving college, so they had a lot of news to catch up on.
   It seemed they all lived in different towns: Acton, Buswick, Coalford, Derby and Eccles; and that they all had different jobs which were, in no particular order: an engineer, a lawyer, a teacher, a doctor and a shopkeeper.
   Also, each one was the winner in just one event at the meeting. These events were: 100 m, 400 m, 1500 m, High Jump and Javelin.
   The following facts were also known:
   (1) Hans the shopkeeper from Derby won the High Jump.
   (2) The lawyer was from Eccles and said he was not a runner.
   (3) Greta was a P.E. teacher from Buswick and won the 1500 m.
   (4) The doctor, who came from Acton, did not win the 100 m.
   (5) The person from Derby was not an engineer.
   (6) Iolo was an engineer from Coalford and did not win the 400 m.
   (7) Jan was not a lawyer, but did win the 400 m.
   (8) Fred did not come from Acton and was not a runner.

   (a) Which event did the person from Coalford win? (1 point)
   (b) Which town did Jan come from? (1 point)
   (c) What was the name of the lawyer? (2 point)
   (d) Which event did the engineer win? (2 point)
   (e) Which event did Fred win? (2 point)

【*Solution*】

From (1), we know:

Name	*Town*	*Job*	*Event*
Hans	Derby	Shopkeeper	High Jump

From (2) and (1), we know:

Name	*Town*	*Job*	*Event*
-	Eccles	Lawyer	Javelin

From (3), we know:

Name	*Town*	*Job*	*Event*
Greta	Buswick	Teacher	1500 m

From (4) and (3), we know:

Name	*Town*	*Job*	*Event*
-	Acton	Doctor	400 m

(5) can be known from (1).
From (6) and (3), (4), (7), we know:

Name	*Town*	*Job*	*Event*
Iolo	Coalford	Engineer	100 m

From (7), we know:

Name	*Town*	*Job*	*Event*
Jan	Acton	Doctor	400 m

From (8), we know:

Name	*Town*	*Job*	*Event*
Fred	Eccles	Lawyer	Javelin

(a) The person from Coalford won the 100 m race.
(b) Jan came from Acton.
(c) The name of the lawyer is Fred.
(d) The engineer won the 100 m race.
(e) Fred won the Javelin contest

ANS:(a)100 m (b)Acton (c)Fred (d)100 m (e)Javelin

2. In 2008, a school had 450 new pupils, making a total of 1600 pupils. In 2009, there were 504 new pupils, 8% of the number of the previous year's pupils left the school.
   (1) Find the percentage increase in the number of new pupils from 2008 to 2009. (2 point)
   (2) How many pupils left the school? (3 point)
   (3) What was the total number of pupils in the school in 2009? (3 point)

【***Solution***】

(a) In 2008, the school had 450 new pupils. In 2009, the school had 504 new pupils. So the percentage increase in the number of new pupils from 2008 to 2009 is $(504-450)\div 450\times 100\%=12\%$.
(b) Since we know 8% of the number of 2008's pupils left the school, there are $1600\times 8\%=128$ pupils left the school.
(c) Since there were 504 new pupils and 128 pupils left the school, the total number of pupils in 2009 is $1600-128+504=1976$.

ANS:(a) 12% (b) 128 pupils (c) 1976

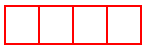
3. The diagram represents a small sheet of 12 postage stamps, as they are usually sold, all perforated at the edges and all of the same value. (The letters are only there to identify the separate stamps).
   You need 4 of the stamps in order to post a letter but would like all 4 to be properly joined together at their edges (not at their corners). For example: ABCD, EFGH, JKLM, FGHL would all do, but NOT EFLM. In how many different ways can you get such a group of 4? Write down this number.

A	B	C	D
E	F	G	H
J	K	L	M

【*Solution*】

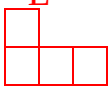
It is necessary to find the number of tetrominoes in the figure. There are five kinds of tetromino:

(i) I



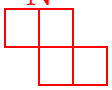
ABCD, EFGH, JKLM
There are in total 3 I-tetrominoes.

(ii) L



ABCG, EABC, BCDH, FBCD, EFGC, EFGL, AEFG, JEFG, FGHD, FGHM, BFGH, KFGH, JKLG, EJKL, KLMH, FKLM, AEJK, BAEJ, CBFK, ABFK, BFKL, BFKJ, BCGL, DCGL, CGLM, CGLK, DHML, CDHM
There are in total 28 L-tetrominoes.

(iii) N

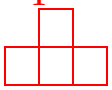


ABFG, BCGH, EFKL, FGLM, EFBC, FGCD, JKFG, KLGH, AEFK, BFGL, CGHM, BFEJ, CGFK, DHGL
There are in total 14 N-tetrominoes.

(iv) O

ABFE, BCGF, CDHG, EFKJ, FGLK, GHLM
There are in total 6 O-tetrominoes.

(v) T



ABCF, BCDG, EFGK, FGHL, EFGB, FGHC, JKLF, KLMG, AEJF, BFKG, CGLH, BFKE, CGLF, DHMG
There are in total 14 T-tetrominoes.

Hence there are 3+28+14+6+14=65 tetrominoes.

ANS:65

4. A domino consists of two unit squares joined edge to edge, each with a number on it. Fifteen dominoes, numbered 11, 12, 13, 14, 15, 22, 23, 24, 25, 33, 34, 35, 44, 45, and 55, are assembled into the 6 by 5 rectangle shown in the diagram below. However, the boundary of the individual dominoes has been erased. Reconstruct the dominoes by drawing in the boundary lines.

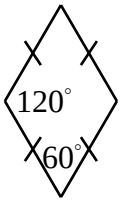
5	3	5	2	2	3
2	3	4	4	4	5
3	1	3	4	1	3
2	1	5	2	4	5
4	2	1	1	1	5

【*Solution*】

5	3	5	2	2	3
2	3	4	4	4	5
3	1	3	4	1	3
2	1	5	2	4	5
4	2	1	1	1	5

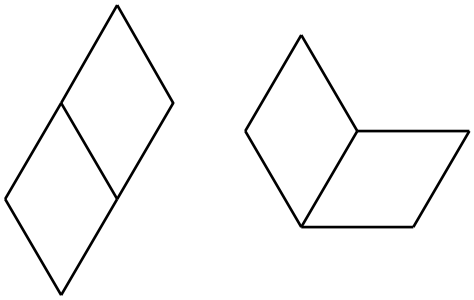
5. We can make shapes by joining diamonds together edge to edge. There are exactly two different shapes that can be made in this way from two diamonds. We call them *bimonds*.

Diamon



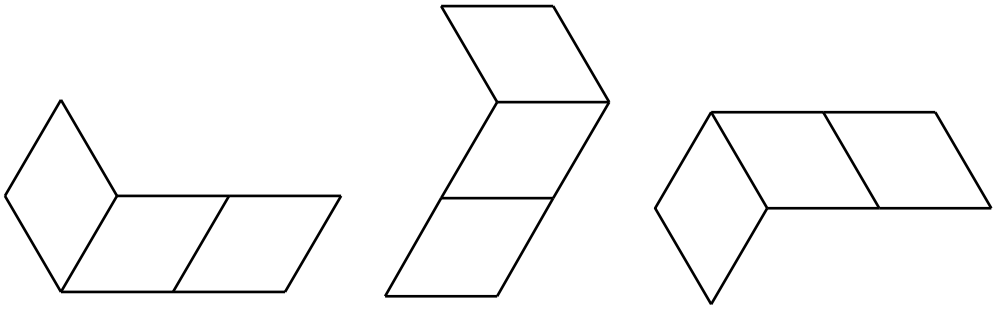
Sides equal in length, one angle 60°, adjacent angle 120°; area 1 unit^2.

Bimond



The two different bimonds, each with area 2 units^2.

Each of the following shapes is a trimond made from three diamonds joined edge to edge; each has area 3 units^2.



Note: The three trimonds above are all the same — rotations or reflections of one will produce the others.

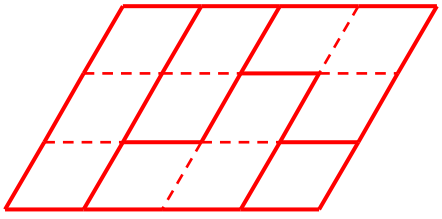
(1) From a set of one diamond, two different bimonds and your collection of different trimonds, fit some of these together to make a parallelogram with area 12 units^2. Draw this parallelogram, showing the pieces you have used. (2 point)

(2) From the same set of pieces, make another parallelogram with area 12 units^2, but with a different perimeter. Draw this parallelogram. (2 point)

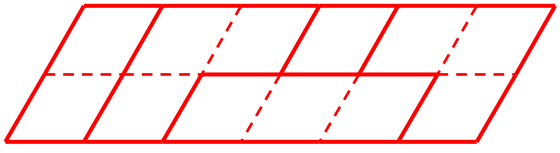
(3) How many different shapes can be made from four diamonds joined edge to edge each having an area of 4 units^2. (4 point)

【*Solution*】

(1)

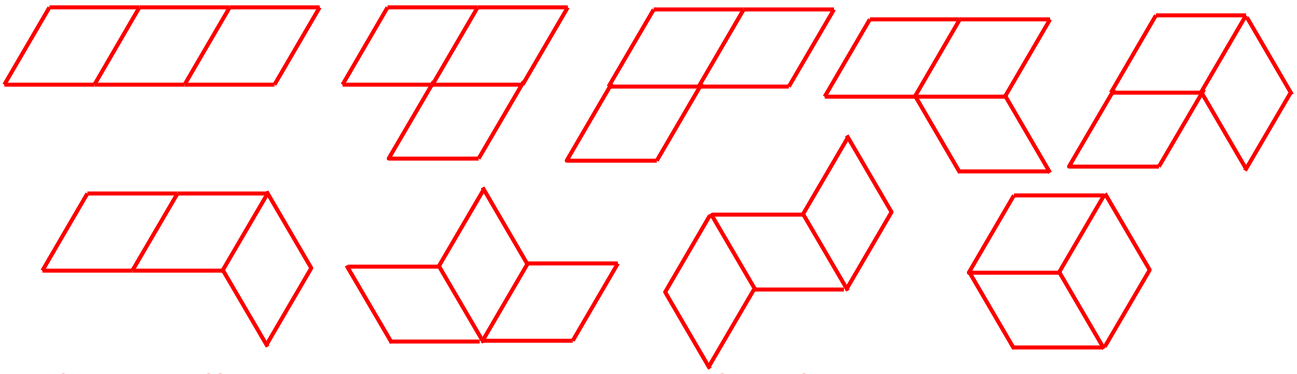


(2)



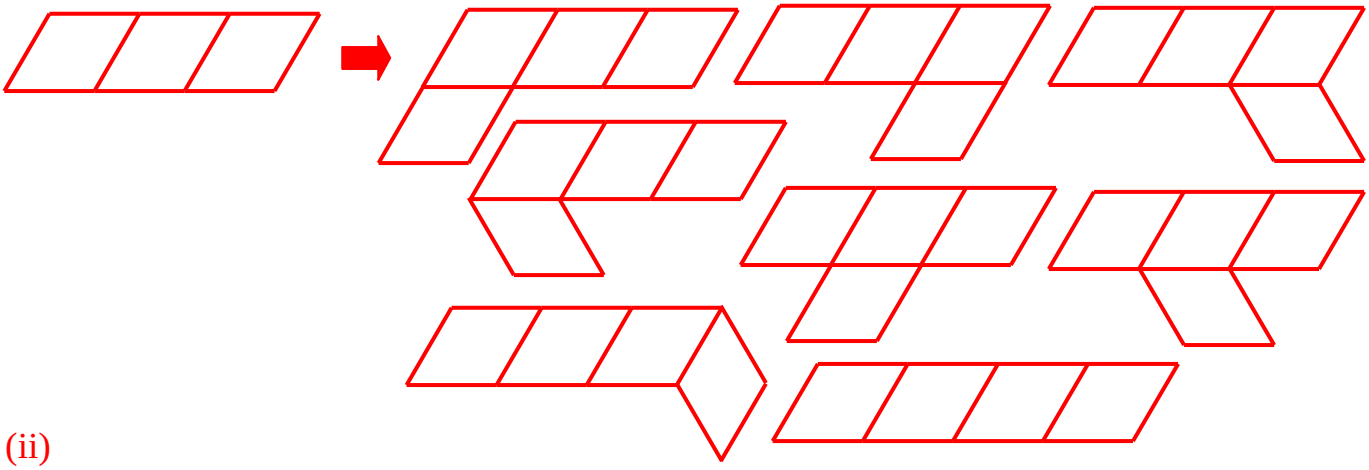
In (1) and (2), other arrangements exit.

(3) There are 9 different trimonds altogether.

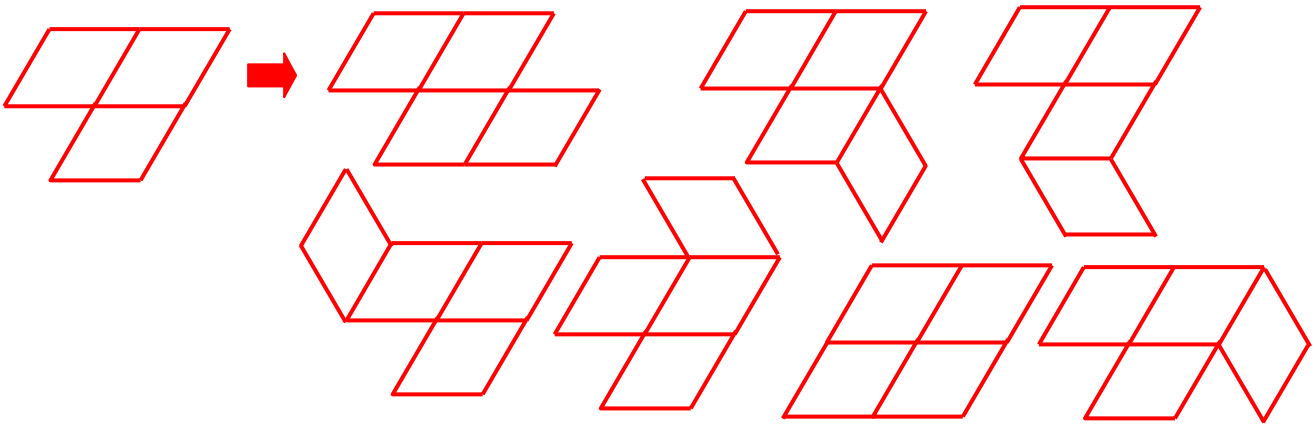


To find all different shapes which can be made from four diamonds joined edge to edge, each having an area of 4 units^2, we can add one diamond to each trimond.

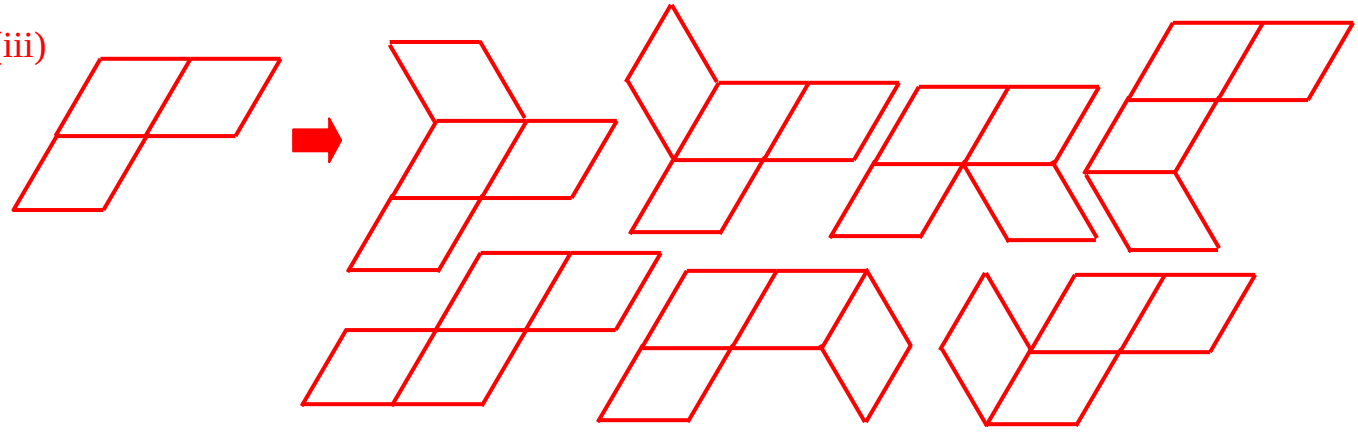
(i)



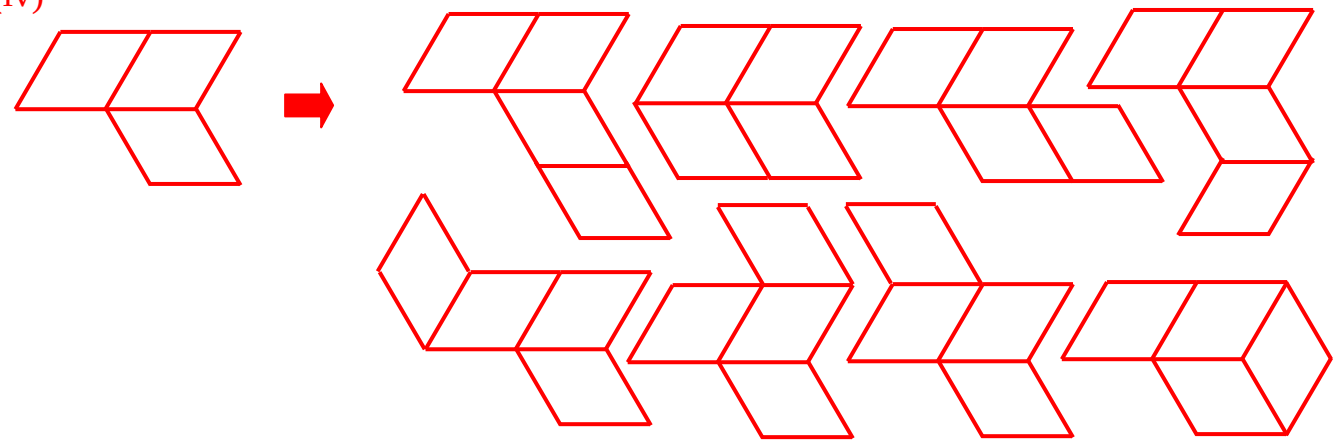
(ii)



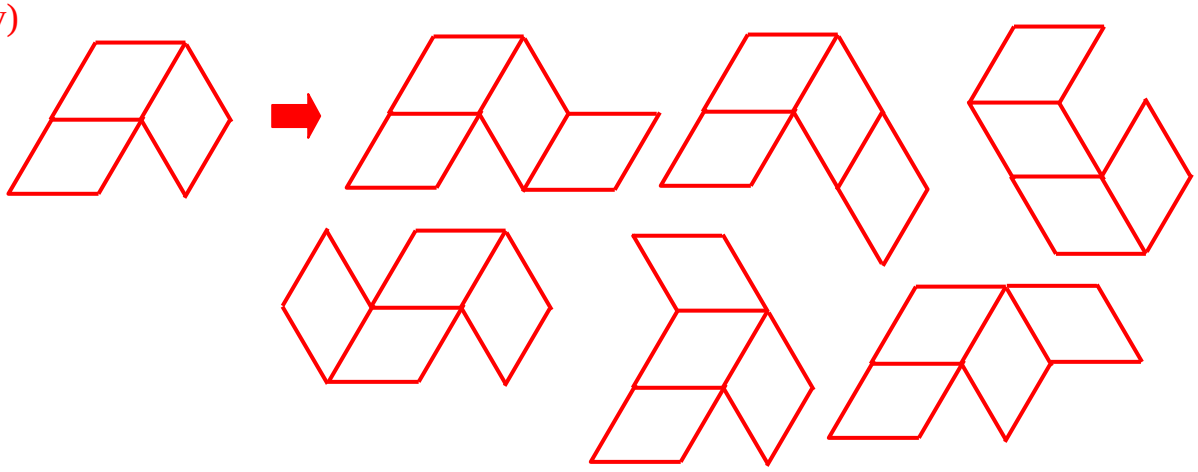
(iii)



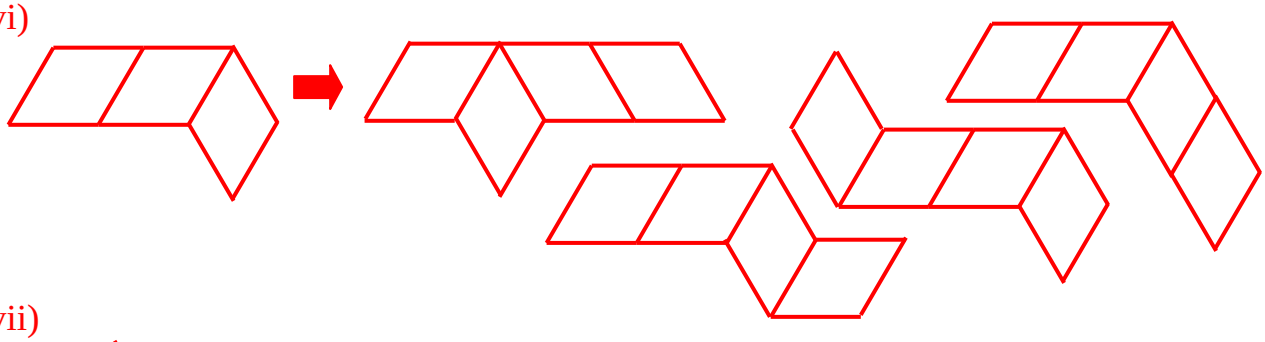
(iv)



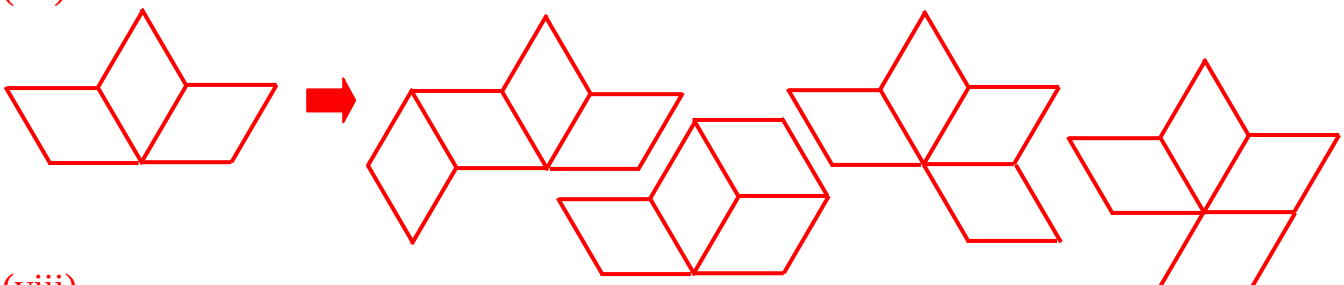
(v)



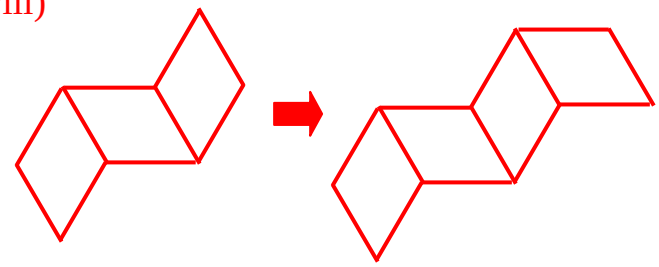
(vi)



(vii)



(viii)



There are in total 8+7+7+8+6+4+4+1=45 ways.

ANS:45