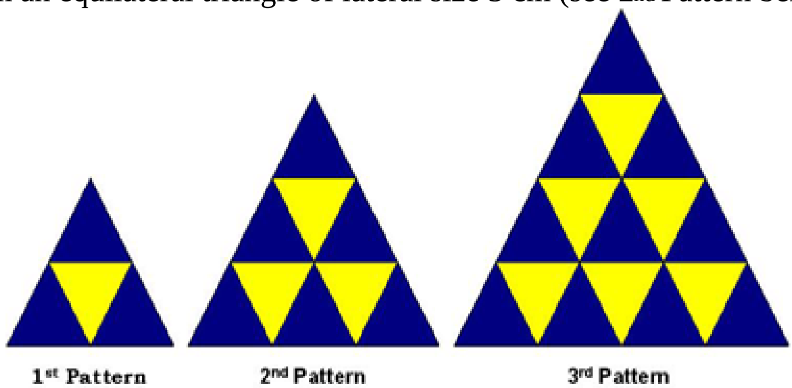
Mathematics Exploration Problems

International Mathematics and Science Olympiad (IMSO) for Primary School 2004

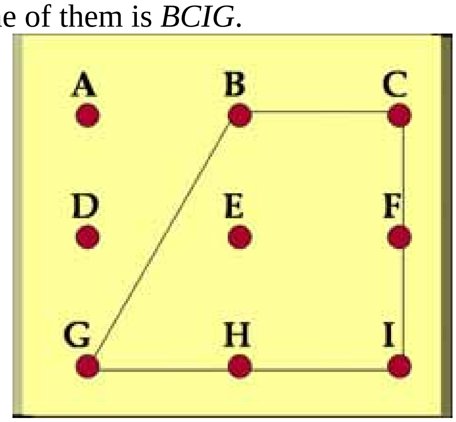
Jakarta, November 29 - December 3, 2004

1. We are given a number of equilateral triangles with lateral length 1 cm. They come in two colors, yellow and blue. Three blue and one yellow triangles can be arranged to make an equilateral triangle of lateral size 2 cm (see 1st Pattern below). Six blue and three yellow triangles are arranged to form an equilateral triangle of lateral size 3 cm (see 2nd Pattern below).



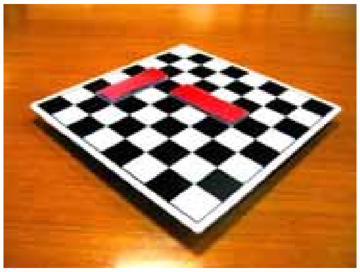
a. How many blue triangles and yellow triangles are required in the arrangement with lateral length 6 cm?
b. If you would like to make a similar arrangement to form an equilateral triangle of lateral size 10 cm, how many blue triangles and yellow triangles are needed?
c. If you would like to make equilateral triangle of lateral size 20 cm, how many blue triangles and yellow triangles are needed?

2. We define a trapezoid as a quadrilateral, which has a pair of parallel laterals; another pair of laterals are not parallel. In the rectangular arrangement below, there are exactly three noncongruent trapezoids. One of them is *BCIG*.

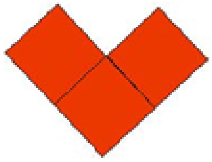


a. Find the other two non-congruent trapezoids.
b. Find the other 7 trapezoids which are congruent to *BCIG*.
c. What is the total number of trapezoids that can be made on the arrangement (both congruent and non-congruent, including *BCIG*).

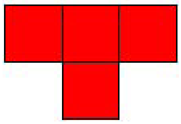
3. There are 96 distinct ways an I-tromino (1 × 3 rectangular tile) can be positioned on squares of an 8 × 8 chessboard, along the lines of the chessboard. There are 48 vertical positions and 48 horizontal positions. (see picture).



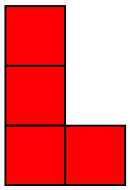
a. In how many distinct ways can an V-tromino (see picture) be positioned on squares of the chessboard?



b. In how many distinct ways can a T-tetramino (see picture) be positioned on squares of the chessboard?



c. In how many distinct ways can an L-tetramino (see picture) be positioned on squares of the chessboard?

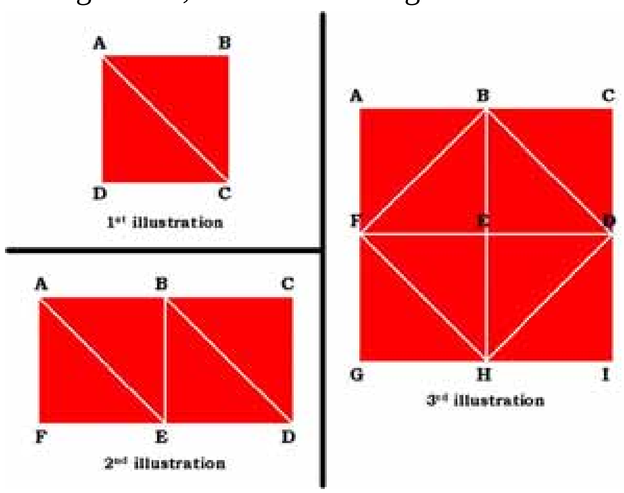


4. Right-isosceles triangles are used to make various arrangements, so that the arrangements contain squares, as in the following illustrations:

1st illustration: Using two triangles, we can make an arrangement, which contains one square:*ABCD*.

2nd illustration: Using four triangles, we can make an arrangement, which contains two squares:*ABEF* and *BCDE*.

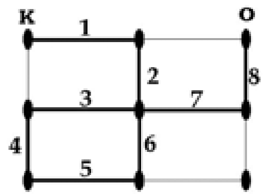
3rd illustration: Using eight triangles, we have six squares:*ABEF*,*BCDE*,*EDIH*, *FEHG*,*BDHF* and *ACIG*.



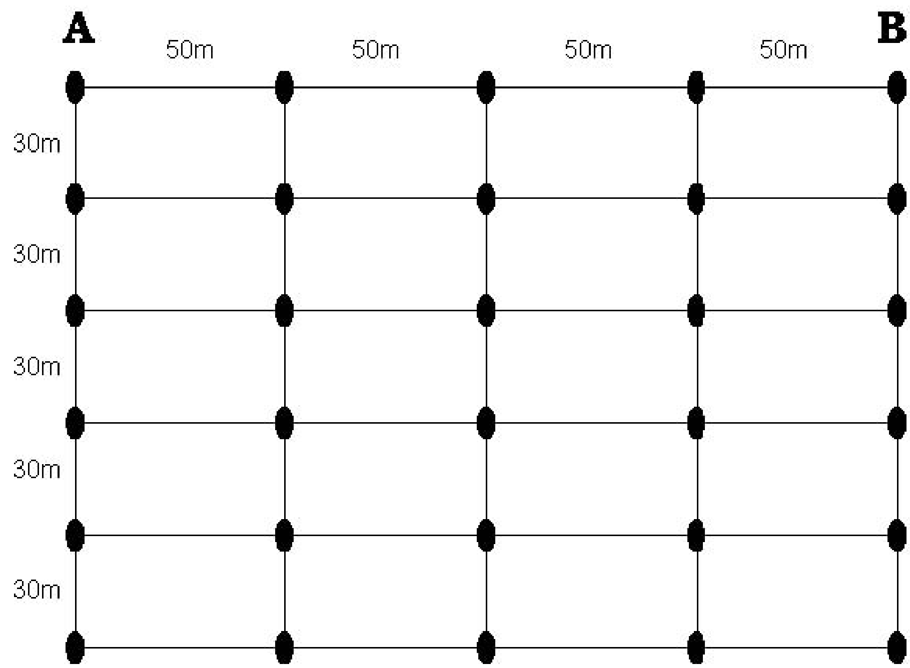
a. Using 10 of such triangles, how many squares at most can we find?
b. How about using 12 triangles?
c. How about using 18 triangles?
d. How about using 24 triangles?

5. Popon is to deliver newspapers along the streets in his neighborhood. He is paid by the distance he makes, and thus the farther he makes, the higher the pay is. While he can cross any intersection as many times as he likes, he cannot pass any street more than once (a street is one segment between 2 adjacent points).

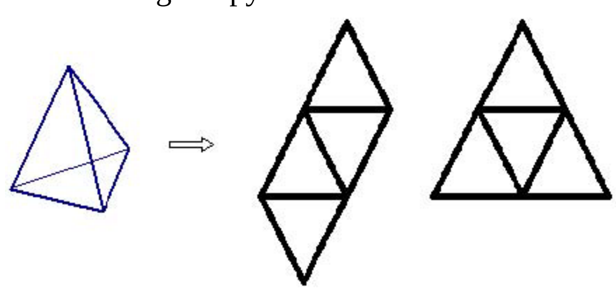
For example, in a neighborhood which looks like this, Popon is to start from *K* and to finish at *O*. To get the highest payment, he takes the longest possible route. One possibility is indicated by 1-2-3-4-5-6-7-8 in the figure below.



If the neighborhood looks like the following picture, what is the longest possible route from *A* to *B*? Trace and indicate the route by writing numbers 1,2,3,... on the streets of the route.



6. The following figures show a triangular pyramid and some of its nets.



You are given 3 sheets of graph paper to work on (you may not use them all), and one sheet of pink paper. You are asked to **draw the largest cube net on the graph paper, such that the cube net fits the pink paper**.