INTERNATIONL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2007

Mathematics Contest in Taiwan

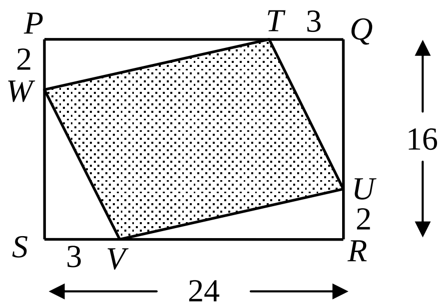
Name:________ School:___________ Grade:____ number:_____

Short Answer: there are 12 questions, fill in the correct answers in the answer sheet. Each correct answer is worth 10 points. Time limit: 90 minutes.

1. The fraction $\frac{2007}{7000}$ is written as a decimal. What digit is in the 2007^{th} place? (In the decimal 0.23456 the digit 4 is in the 3^{rd} place.)

   Since $\frac{2007}{7000}=0.286\overline{714285}$ and $2007=6\times334+3$, hence the digit in the 2007^{th} place is 5.

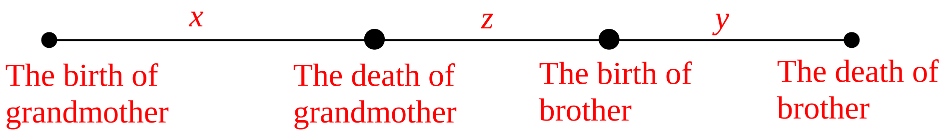
2. The rectangle *PQRS* measures 24cm by 16cm. Points *T*, *U*, *V* and *W* are on the sides with measurements, in centimeters, as shown. Find the area, in square centimeters, of shaded portion.



   $24\times16-2\times(\frac{1}{2}\times2\times21+\frac{1}{2}\times3\times14)=300\ \text{cm}^2$

3. Mary's brother and grandmother both died young. The sum of their lifespans equaled 78 years. Mary's brother died 99 years after their grandmother was born. How many years after their grandmother died was Mary's brother born?

   Let x be the age of Mary's grandmother when she died, y be the age of Mary's brother when he died and z be the number of years between the death of Mary's grandmother and the birth of Mary's brother. Then we have the following figure:



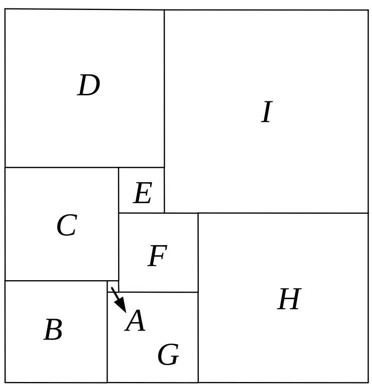
   From the given conditions, we have $x+y+z=99$ and $x+y=78$. So $z=99-78=21$.

4. What would be the third number from the left of the 75^{th} row of the accompanying triangular number pattern?

   1
   2 3 4
   5 6 7 8 9
   10 11 12 13 14 15 16
   ⋮

   Observe that the last element from the left of i^{th} row is i^2, so the last element of 74^{th} row is $74^2=5476$. Hence the third number of the 75^{th} row is $5476+3=5479$.

5. Nine squares are arranged as shown. If square *A* has area 4cm^2 and square *B* has area 324cm^2, then what is the area of square *I*, in square centimeters?



   Let a, b, c, d, e, f, g, h and i be the length of edge of A, B, C, D, E, F, G, H and I, respectively.

   Since A has area 4cm^2 and B has area 324cm^2, $a=2$ and $b=18$.

   Then we know: $c=a+b=20$, $g=b-a=16$, $f=g-a=14$, $h=g+f=30$, $e=c+a-f=8$, $i=h+f-e=36$.

   Hence the area if I is $36^2=1296\ \text{cm}^2$.

6. Suzanne has 20 coins in her purse. They are \$10, \$20 and \$50 coins and the total value of the coins is \$500. If she has more \$50 than \$10 coins, how many \$10 coins she has?
Suppose Suzanne has x \$50 coins and y \$10 coins. Since the total value of the coins is \$500 and she has more \$50 than \$10 coins, we know $10>x>y$. Thus she has $20-x-y$ \$20 coins. So we have $50x+20(20-x-y)+10y=500$, $30x-10y=100$, i.e. $3x-y=10$. Hence $x=4$ and $y=2$.

7. Each of faces of a regular octahedron is numbered with a different integer. Each vertex is assigned a “vertex number” which is the sum of the numbers on the faces which intersect in that vertex and then the sum of the vertex numbers is calculated. What is the highest number which must divide this sum, for every possible numbering of the faces?
Let the numbers on the faces be a, b, c, d, e, f, g and h. There are six vertices, each vertex number being a sum of these face numbers, i.e. 24 numbers altogether. The 8 face numbers must occur equally often, in the sum of the vertex numbers. The sum of the vertex numbers will be $3(a+b+c+d+e+f+g+h)$, i.e. a number divisible by at least 3.

8. The number 119 is very curious:

When divided by 2 it leaves a remainder of 1.
When divided by 3 it leaves a remainder of 2.
When divided by 4 it leaves a remainder of 3.
When divided by 5 it leaves a remainder of 4.
When divided by 6 it leaves a remainder of 5.

How many 4-digit numbers have this property?
Since the least common multiple of 2, 3, 4, 5 and 6 is 60 and $2-1=3-2=4-3=5-4=6-5=1$, the numbers satisfy the conditions must be of the form $60k-1$.

$$999<60k-1<10000$$
$$1000<60k<10001$$
$$16<k<167$$

Since there are $167-16-1=150$ numbers between 16 and 167, the answer is 150.

9. In the right multiplication example, all number from 1 through 9 have been used once, and once only. Three of the numbers are given. Can you fill in the rest?

$$\begin{array}{ccccc} & & 2 & \square & \square \\ \times & & & \square & 8 \\ \hline & 5 & \square & \square & \square \end{array}$$

Let the multiplication be

$$\begin{array}{ccccc} & & 2 & a & b \\ \times & & & c & 8 \\ \hline & \square & \square & \square & \square \\ & \square & \square & \square & \\ \hline & 5 & d & e & f \end{array}$$

Then we know $2c<5$, $c=1$ or 2, but 2 is used, so $c=1$. Set

$$\begin{array}{ccccc} & & 2 & a & b \\ \times & & & 1 & 8 \\ \hline & w & x & y & z \\ & 2 & a & b & \\ \hline & 5 & d & e & f \end{array}$$

Since the leading digit of the result is 5, $w=2$ and hence $a=7$ or 9.

And we can observe that $(b, f)=(3, 4)$ or $(7, 6)$.
Case 1: $(b, f)=(3, 4)$

$$\begin{array}{ccccc} & & 2 & a & 3 \\ \times & & & 1 & 8 \\ \hline & w & x & y & 4 \\ & 2 & a & 3 & \\ \hline & 5 & d & e & 4 \end{array}$$

If a=7, then we have 273×18=4919. This is a contradiction.
If a=9, then we have 293×18=5274. This is a contradiction.
Case 2: $(b, f)=(7, 6)$

$$\begin{array}{ccccc} & & 2 & a & 7 \\ \times & & & 1 & 8 \\ \hline & w & x & y & 6 \\ & 2 & a & 7 & \\ \hline & 5 & d & e & 6 \end{array}$$

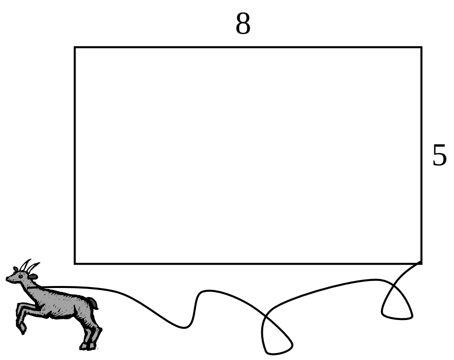
Hence a=9 and we have 297×18=5346.
So the answer is

$$\begin{array}{ccccc} & & 2 & 9 & 7 \\ \times & & & 1 & 8 \\ \hline & 5 & 3 & 4 & 6 \end{array}$$

10. The diagram shows a 5 by 5 table. The top row contains the symbols *A*, *B*, *C*, *D* and *E*. The fourth row contains the symbols *A*, *B* and *C* at the centre. The remaining squares can be filled with *A*'s, *B*'s, *C*'s, *D*'s and *E*'s such that no row, column or diagonal contains the same symbol more than once. What is the symbol that must go into the shaded square?
The right diagram is a Latin Square which satisfies the conditions. So the answer is *A*.

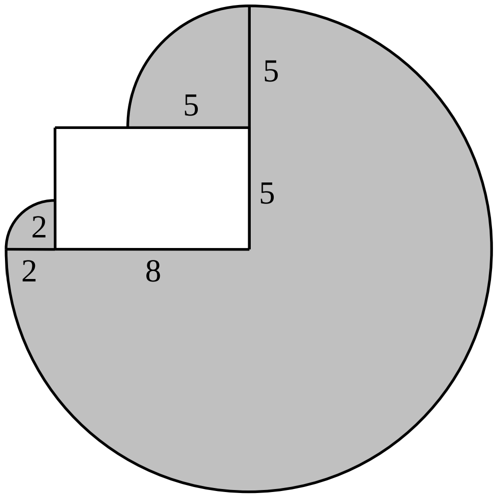
A	*B*	*C*	*D*	*E*
D	*E*	*A*	*B*	*C*
B	*C*	*D*	*E*	*A*
E	*A*	*B*	*C*	*D*
C	*D*	*E*	*A*	*B*

11. A goat is tied by a rope to a corner of a rectangular shed as shown. The shed is 8 metres long and 5 metres wide and the rope is 10 metres long. The shed is surrounded by grass. What is the area, in square metres, that the goat can graze upon? (The ratio of the circumference of the circle to the diameter is π=3.14.)

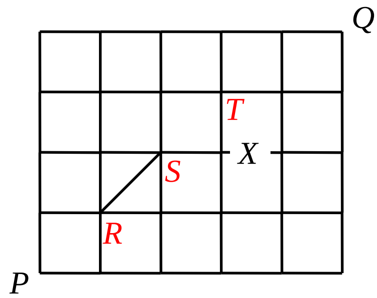


The goat can graze upon the shaded region. Hence the area is

$$\frac{3}{4}\times 10^2\times 3.14+\frac{1}{4}\times 5^2\times 3.14+\frac{1}{4}\times 2^2\times 3.14=(75+\frac{25}{4}+1)\times 3.14=82\frac{1}{4}\times 3.14=258.265\,\mathrm{m}^2$$



12. The accompanying diagram is a road plan of a city. All the roads go east-west or north-south, with the exception of the one short diagonal road shown. Due to repairs one road is impassable at the point X. Of all the possible routes from P to Q, there are several shortest routes. How many such shortest routes are there?



<Alternative 1>

All possible shortest routes must pass R and S. So the numbers are :
(number of the ways from P to R) ×((number of the ways from S to Q via T)+(number of the ways from S to Q without through T))
$=2\times((2\times3)+1)=14$

< Alternative 2>

Observe that the number of ways from A to C is equal to the sum of the number of ways from A to B and the number of ways from A to D.
Thus the numbers on the right diagram means the number of ways from P to that point, so the answer is 14.