INTERNATIONL MATHENATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2004
Mathematics Contest in Taiwan
The Second Round
Theoretical Examination

Solutions

1. This year my father is 41－5＝36 years old [1 pt]. So my young brother is $\frac{3}{36}$•36 ＝3 years old [1 pt]. Therefore he was born in the year of 2004－3＝**2001** [1 pt].

2. Every end of the week, Tina reduces the difference by $3＋$2.4＝$5.4[1 pt]. To have $6 less savings than Laila, Tina has to cover $100－$40－$6＝$54[1 pt]. Since $\frac{54}{5.4}$＝10, she can do that after **10 weeks**[1 pt].

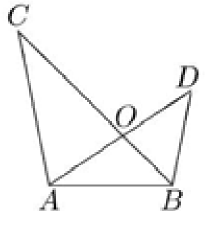
3. If the product of two integers is even, one of the two must be even [1 pt]. If both are even, then their product is divisible by 4, which is not the case. Thus the other one is odd [1 pt], and from this we conclude that their sum is **odd** [1 pt].

4. Deni must take $5 [1 pt]. If Deni does not take a $50 coin, he can take at most 6 more $10coins, and then 3 $5coins. But the total number of coins is less than 9, so Deni must take $50 [1 pt]. Deni then must take 5 $5 to get 6 coins. So Deni can not take any $10 coin [1 pt].

5. The weight of a large box is $15 - 10 = 5$ kg [1 pt]. The weight of a small box and two medium boxes is $10 - 5 = 5$ kg [1 pt]. So, two small boxes and four medium boxes weighs $2 \times 5 =$ **10kg** [1 pt].

6. Since 12 divides $A579B$, then 4 divides $A579B$. So 4 divides $9B$, hence either $B = 2$ or $B = 6$. Also, 3 is a factor of $A579B$. Since $5 + 7 + 9 = 21$ is divisible by 3, $A + B$ is also divisible by 3 [1 pt]. If $B = 2$, either $A = 1$ or $A = 7$. If $B = 6$, then $A = 3$. All possible numbers are $15792, 75792, 35796$ [2 pts].
   [Notes:
   (a) Successful use of at least one divisibility test worths 1 pt.
   (b) Obtain complete solution for a particular B worths 1 pt.]

7. Factor 2032 into $2 \cdot 2 \cdot 2 \cdot 2 \cdot 127$, and 762 into $2 \cdot 3 \cdot 127$. The 3-digit number must be a multiple of 127 that divides both 2032 and 762 [1 pt]. The possibility is 127 or $2 \times 127 = 254$. But $2032 \div 127 = 16$, more than 9 [1 pt]. So the 3-digit number must be 254 and so the 2-digit number is 38 [1 pt].

8. 

The area $\triangle ABC$ is $\frac{1}{2} \times 8 \times (6+7+5) = 72$. The area of $\triangle ABD$ is $\frac{1}{2} \times 8 \times (6+7) = 52$. The area of $\triangle ABO$ is $\frac{1}{2} \times 8 \times 6 = 24$ [obtaining at least two out of three areas worths 1 pt]. The area of the shaded regions is $A(\triangle ABC) + A(\triangle ABD) - 2 \times A(\triangle ABO) = 76$ [2 pts].

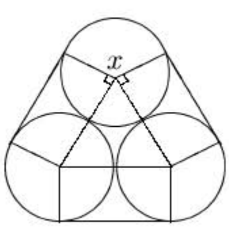
9. Let the side length of the squares be 8 units. Then the radius of the circle in the first figure is 4 units, a circle in the second figure is 2 units, circle is in the third figure is 1 unit [1 pt]. The area of the circle in the first figure is $\pi \times 4^2 = 16 \times \pi$, the area of the circle in the second figure is $\pi \times 2^2 = 4 \times \pi$, the area of the circle in the third figure is $\pi \times 1^2 = \pi$ [1 pt]. So the area of the shaded region on the first, second, and third figure are $16 \times \pi, 4 \times (4 \times \pi) = 16 \times \pi$, and $16 \times \pi$. Therefore the area of shaded region in the three squares are equal [1 pt].

10. 

3-rd Picture

The third figure is as shown. Divide the largest square into 64 smaller squares by dividing its sides equally into 8 segments [1 pt]. The shaded area cover 8 full small squares and $16 + 8 + 4 = 28$ half-squares. So it is equals $8 + \frac{1}{2} \times 28 = 22$ small squares [1 pt]. The percentage of the shaded area is $\frac{22}{64} \times 100\% = 34.375\%$ [1 pt]. [Note: 1 pt for obtaining the area of one shaded triangle.]

11. Larger wheel turn slower than smaller wheel. The wheel B turn at a speed of $\frac{12}{36} \times 450 =$ 150rpm [1 pt]. The wheel C turn at the same speed as wheel B, i.e., 150rpm [1 pt]. The wheel D turn at a speed of $\frac{9}{27} \times 150 = 50$rpm [1 pt].

12. 

The angle x is $360^\circ - (60 + 90 + 90)^\circ = 120^\circ$. So part of the belt that is tight to a disk is $\frac{120}{360} \times \frac{22}{7} \times 2 \times 7 = \frac{44}{3}$cm [1 pt]. Part of the belt that connect two disk is $2 \times 7 = 14$cm [1 pt]. Therefore the total length of the belt is $3 \times (\frac{44}{3} + 14) = 86$cm [1 pt]. [Note: 1 pt for obtaining the 120° angle.]

13. The difference in volume between the two balloons is 3 liters [1 pt]. Each second this difference decreases by 0.42 liters [1 pt]. So the two balloons will have the same volume after $\frac{3}{0.42} = 7.1$ seconds [1 pt].

14. When the 60kph train arrives at the destination, the 50kph train needs 5 minutes more. This means that the 50kph train is about $\frac{5}{60} \times 50 = \frac{25}{6}$km from the destination [1 pt]. This is the amount the faster train leaves the low train behind during the scheduled travel time between the two stations. Each hour the faster train is ahead by 10km of the slower train. So the scheduled travel time is $\frac{25}{6} \div 10 = \frac{25}{60}$ hours = 25 minutes [1 pt]. The 60kph train takes this time to travel between the two stations. So the distance between the two stations is $\frac{25}{60} \times 60 = 25$ km [1 pt].
Alternatively, this problem can be solved using algebra. Obtaining at least one side of the equation worth 1 pt. Stating the equation worth 1 pt. Solving the equation and the problem worth 1 pt.

15. We have $2A < B$, $A + C < B$, $D > B$, $C = 6$, and $D = 9$. So $B - A > 6$ and $B < 9$. So $A + 6 < 9$, hence $A < 3$ [1 pt]. If $A = 2$, then $8 < B < 9$, impossible [1 pt]. So $A = 1$, hence $7 < B < 9$, so $B = 8$ [1 pt].