INTERNATIONL MATHENATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2004

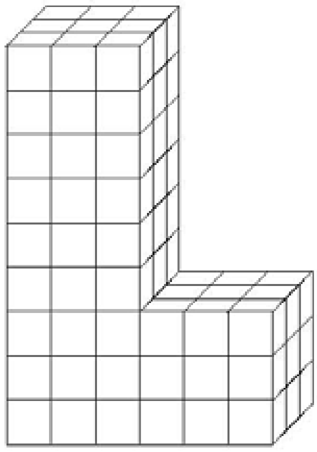
Mathematics Contest in Taiwan
The Second Round
Exploration Examination

Solutions

Name:_______ School:_______ Grade:_______ ID number:________

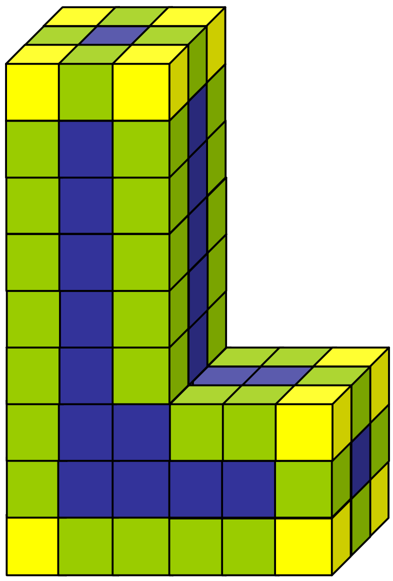
Show your detailed solution in the space provided after each question. Each question is worth 10 points.

1. This is a tower made from cubes (see figure). The outside part of this tower is painted. How many cubes are painted on:
   (a) three sides
   (b) two sides
   (c) one side



Sol:

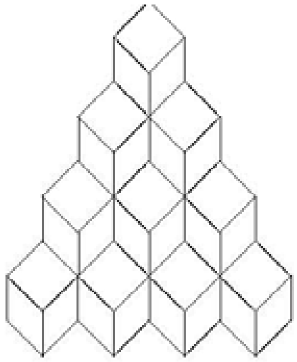
(a) (3 points)
The cubes painted on three sides should be located at a corner (the yellow ones in the figure). There are four at bottom, four at the top of the higher tower and two at the top of the lower tower. Thus there are 4+4+2=10 cubes painted on three sides.



(b) (3 points)
The cubes painted on two sides should be located at each edge, but not at a corner (the green ones in the figure). (7+4+1+2+5+1)╳2+2+2+1＝45. So there are 45 cubes painted on two sides.

(c) (4 points)
The cubes painted on one sides are just those at the surface excluding (a) and (b) (the blue ones in the figure). (3╳9+3╳3)╳2+(3+3)╳2+5+1+7－10－45＝42 or (7+2+2)╳2+1+7+1+4+7＝42. So there are 42 cubes painted on one sides.

2. Mark wants to make a building, 4 cubical tall, from cubes. (see figure)



   (a) How many cubes does Mark need?

   (b) If Mark wants to build a similar building but has 8 cubical height, how many cubes does Mark need?

Sol:

(a) (4 points)

Mark needs 1 cube for the top layer;

$1+2=3$ cubes for the 2nd layer;

$1+2+3=6$ cubes for the 3rd layer;

$1+2+3+4=10$ cubes for the bottom (4th) layer.

Therefore Mark needs $1+3+6+10=20$ cubes to make a 4-cubical-tall building.

(b) (6 points)

Similarly, Mark needs

$1+2+3+4+5=15$ cubes for the 5th layer;

……

$1+2+3+4+5+\cdots+8=36$ cubes for the 8th layer.

$$1+(1+2)+(1+2+3)+\cdots+(1+2+3+4+5+\cdots+8)$$
$$=1\times8+2\times7+\cdots+4\times5+5\times4+\cdots+8\times1=(1\times8+2\times7+\cdots+4\times5)\times2=120$$

Therefore Mark needs 120 cubes to make an 8-cubical-tall building.

3. Two people want to play a game with some marbles in a box. The game rule is that each person can take one or two marbles at a time. After the first person takes marble(s), the other person will have the same opportunity. The loser is the person who takes the last marble.

   (a) If it is your turn to take marble(s) and there are only 6 marbles left in the box, how many marble(s) will you take to be sure you are going to win?

   (b) If it is your turn to take marble(s) and there are only 8 marbles left in the box, how many marble(s) will you take to be sure you are going to win?

   (c) If it is your turn to take marble(s) and there are only 20 marbles left in the box, how many marble(s) will you take to be sure you are going to win?

   (d) Now we change the rule. There are 100 marbles and each player can take 1, 2, 3 or 4 marble(s). If you get the first turn to play, how many marble(s) will you take to be sure you are going to win?

Sol:

(a) (2 points)

I will take 2 marbles and then can be sure I will win.

Let ◯ and □ denotes the number of marbles I and the other person take respectively and ( ) denotes the number of marbles left after one's turn. The following chat shows all the possibilities:

2(4)→	$\boxed{2}$(2) →	1(1) →	I win.	
	$\boxed{1}$(3) →	2(1) →	I win.	
1(5)→	$\boxed{2}$(3) →	2(1) →	I win.	
	$\boxed{1}$(4) →	2(2) →	$\boxed{1}$(1) →	I lose.
		1(3) →	$\boxed{2}$(1) →	I lose.

(b) (2 points)

I will take 1 marble and then can be sure I will win.

2(6)→	Here comes the same situation as (a) but it is the other person's turn, so he can be sure to win; i.e. I lose.		
1(7)→	$\boxed{2}$(5)→	1(4)→	The following will be the same as the first two condition in (a) and I can be sure to win.
	$\boxed{1}$(6)→	Here comes the same situation as (a), so I can be sure to win.	

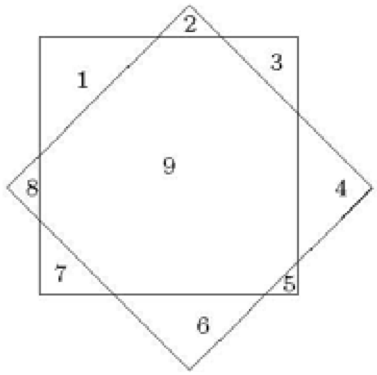
(c) (2 points)

Observe the number of the marbles left in (a) and (b). I can be sure to win when I make the number of the marbles left be 4, 7 … They are in the form of $3k+1$ where k is an integer. $20-2=18=3\times6$; $20-1=19=3\times6+1$. Hence I will take 1 marble and then can be sure I will win.

(d) (4 points)

Similarly, I need make the number of the marbles left be in the form of $5k+1$ where k is an integer since each player can take 4 marbles at most. Hence I will take 4 marbles and then can be sure I will win since the number of the marbles left is $100-4=96=5\times19+1$.

4. Two squares that are put on top of each other can make 9 regions (see figure).



   (a) How many regions at most can be constructed from 3 squares?

   (b) Using 4 squares, construct as many regions as possible by using the grid transparent paper, scissors and glue. Put the result in the envelope.

   (c) How many regions at most can be constructed from 4 squares?

Sol:

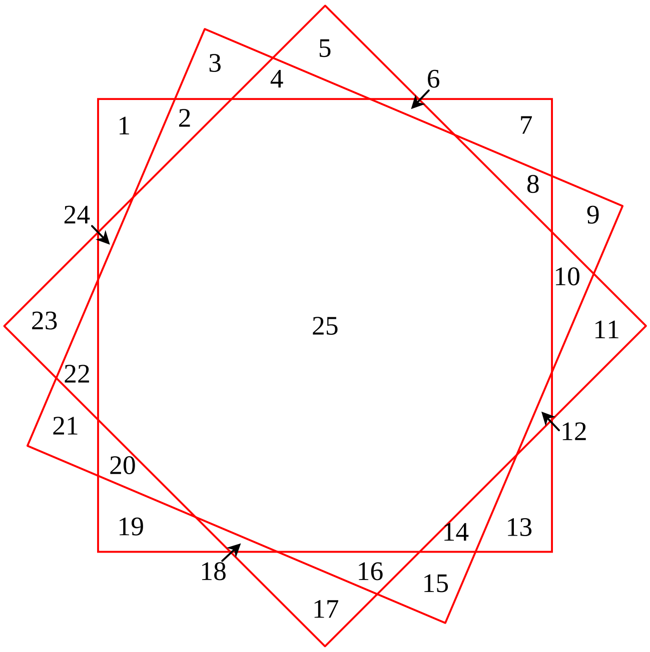
(a) (3 points)

3 squares can construct 25 regions at most. (See figure (a))

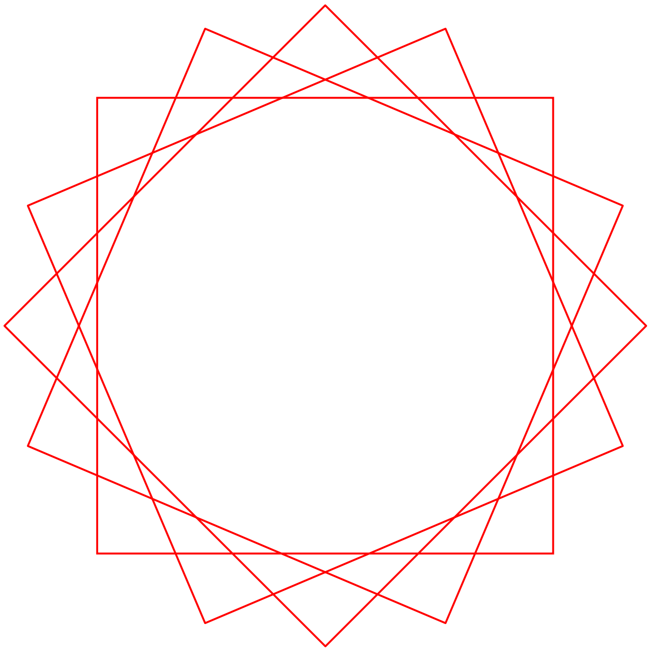
(b) (3 points)

(c) (4 points)

4 squares can construct 49 regions at most. (See figure (c))



(a)



(c)