INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2007

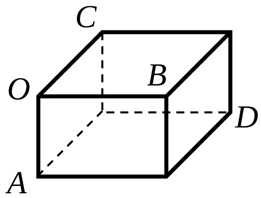
Mathematics Contest (Second Round) in Taiwan

Short Answer Problems

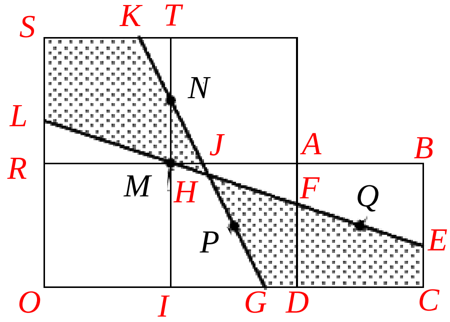
Name:________ School:________ Grade:______ ID number:__________

Short Answer: there are 20 questions, fill in the correct answers in the answer sheet. Each correct answer is worth 2 points. Time limit: 60 minutes.

1. Amongst the children in a family each boy has many sisters as brothers, but each girl has only half as many sisters as brothers. How many children are there in the family?
Since each boy has many sisters as brothers, there are 1 more boys than girls. And because each girl has only half as many sisters as brothers, half of the number of boys is 1+1=2. Hence there are 2×2=4 boys and 4－1=3 girls, i.e. there are 4+3=7 children in the family.
2. A litre of orange fruit juice drink contains 20% orange juice. How many milliliters of orange juice must be added to produce a mixture containing 50% orange juice?
Since there are 1000×20%=200 millilitres of orange juice and 1000×80%=800 millilitres of other liquid, 800－200=600 millilitres of orange juice must be added to produce a mixture containing 50% orange juice.
3. The sum of seven consecutive odd numbers is 539. What is the smallest of the seven numbers?
Set the seven consecutive odd numbers be $a-6$, $a-4$, $a-2$, a, $a+3$, $a+4$, $a+6$. Thus the sum is $7a$. Since 539=7×77, a=77 and hence the smallest of the seven is $77-6=71$.
4. The diagram represents a rectangular box in which the lengths of ages OA, OB and OC are respectively 3, 4 and 5 units. What is the length of OD in the same units?

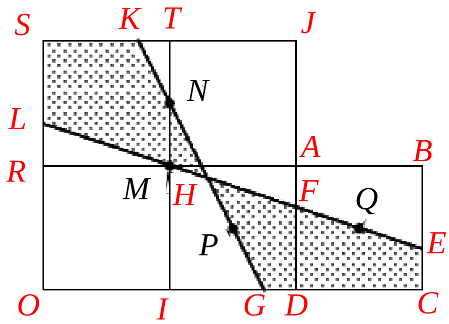


$OD^2 = OA^2 + AD^2 = OA^2 + BC^2 = OA^2 + OB^2 + OC^2 = 9+16+25 = 50$ by Pythagoras' theorem, so the length of OD is $\sqrt{50}$.
5. The diagram below shows five unit squares joined edge to edge. M is a corner, N is the midpoint of a side and P and Q are the centres (intersection point of the two diagonal of a square) of two squares. What is the non-negative difference in the areas of the two shaded regions between PN and QM?
<Alternative 1>
Set the area of a unit square be 1.
Since P and Q are centers of square $ADIM$ and $ABCD$, respectively, the area of quadrilateral $JGIM$ and the area of quadrilateral $FECD$ are equal to $\frac{1}{2}$.
So the area of quadrilateral $HECG$ is equal to 1－(the area of quadrilateral $AFHJ$).
Since N is the midpoint of a side, the area of quadrilateral $SKJR$ is equal to 1. Because triangle LMR and triangle AMF are congruent triangles, the area of



triangle MJH is equal to (the area of triangle LMR) − (the area of quadrilateral $AFHJ$).
Since the area of quadrilateral $KHLS$ is equal to (the area of quadrilateral $SKJR$) − (the area of triangle LMR) + (the area of triangle MJH) = 1 − (the area of quadrilateral $AFHJ$), the area of quadrilateral $HECG$ and the area of quadrilateral $KHLS$ are equal. So the non-negative difference in the areas of the two shaded regions between PN and QM is 0.

<Alternative 2>

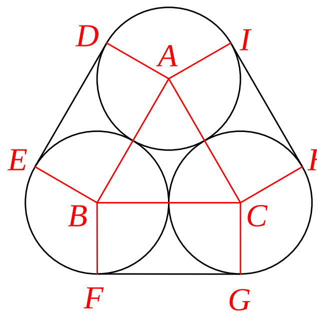


Since Q is the center of square $ABCD$, any line through Q must bisect $ABCD$ and hence line MQ bisect $ABCD$. We can observe that M is the center of square $SJDO$, so line MQ bisects $SJDO$. From above two results, we know line MQ bisect the diagram and hence the area of quadrilateral $LECO$ is equal to the half of the area of the diagram.
Since P is the center of square $ADIM$, line PN bisect $ADIM$. Because M is the midpoint of segment TM, so line PN bisects $SJAR$. So line PN bisect the polygon $SJDIMR$, i.e. line PN bisects the diagram. Hence the area of quadrilateral $SKGO$ is also equal to the half of the area of the diagram. Thus we know the area of quadrilateral $LECO$ is equal to the area of quadrilateral $SKGO$, i.e. the areas of quadrilateral $SKHL$ and quadrilateral $HECG$ are the same, so the difference in the areas of the two shaded regions is 0.

6. What is the least positive integer by which 1512 should be multiplied so that the product is a perfect square?
Since $1512=2^3\times3^3\times7$ and the exponent of each prime factor of a perfect square must be even, the least positive integer by which 1512 should be multiplied so that the product is a perfect square is $2\times3\times7=42$.

7. In his latest game of bowling Tom scored 189 and this raised his average over a number of games from 178 to 179. To raise his average to 180 with the next game, how many points does he have to score?
Since $189-178=11$ and Ken raised his average over 1 point, he has played 11 games. To raise his average to 180 with the next game, he have to score $179+12=191$.

8. Three pipes of diameter 2 m are held together by a taut metal band as shown. What is the length (in metres) of the metal band?



Let A, B and C be the centers of the three circles and D, E, F, G, H and I be the points of tangency. Thus triangle ABC is an equilateral triangle and quadrilateral $ABED$, $BCGF$ and $IHCA$ are rectangles. So $\angle EBF=\angle GCH=\angle IAD=120^\circ$ and hence the length is equal to

$$3\times(1+1+\frac{1}{3}\times2\times3.14)=12.28\text{ m}.$$

9. The 14 digits in a credit card number are to be written in the boxes below. If the sum of any three consecutive digit is 22, what is the value of x?

a	b	c	9				x				7		d

Let the numbers which are written in the first three boxes from left be a, b and c and the least one be d. Since the sum of any three consecutive digit is 22, $a+b+c=22=b+c+9$ and hence $a=9$.

So we can get the boxes below by the same progress.

9	*b*	*c*	9			9	*x*		9		7	9	*d*

Thus we can get d=22－9－7=6 and get boxes below.

9	6	*c*	9	6		9	6		9	6	7	9	6

So the value of x is 6.

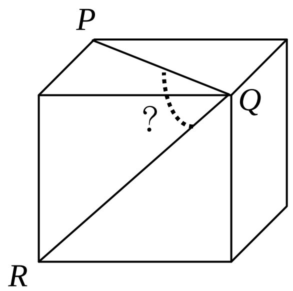
10. 1152 digits are used to number the pages of a book consecutively from page 1. How many pages are there in the book?
Because 9×1=9 digits are used by one-digit numbers and 90×2=180 digits are used by two-digit numbers, 1152－180－9＝963 digits are used by three-digit number and hence there are 963÷3=321 three-digit numbers. So there are 9+90+321=420 pages in the book.

11. By placing a 3 at both ends of a number, its value is increased by 34215. What is the sum of the digits of the original number?
The number is a 3 digits number since its value is increased by a 5 digits number. Hence we can assume the number is abc and get the following:

$$\begin{array}{cccccc} & 3 & a & b & c & 3 \\ - & & & a & b & c \\ \hline & 3 & 4 & 2 & 1 & 5 \end{array}$$

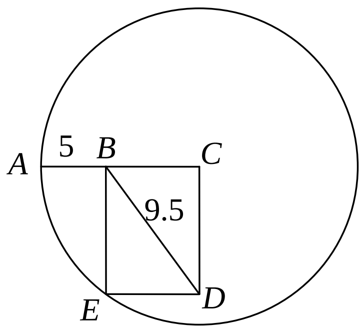
Since the unit digit of 34215 is 5, c=8. Because the tens' digit of 34215 is 1, b=6. Finally, we can get a=4 since the hundreds' digit of 34215 is 2. So the original number is 468 and hence the sum of the digits is 18.

12. PQ and QR are diagonals on two faces of a cube, as shown. What is the angle formed by PQR, in degree?



Since PQ=QR=PR, triangle PQR is an equilateral triangle and hence $\angle PQR=60^\circ$

13. Given this circle, with center C and the rectangle $BCDE$ so that AB=5 and BD=9.5, find the diameter of the circle.



Since $BCDE$ is a rectangle, CE=BD=9.5. So the diameter is 9.5×2=19.

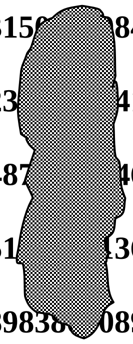
14. An office manager figures out that 40 typists can type 25 complete books in two hours. If he has to cut his work force to two typists, how long would it take them to type ten books?
40 typists can type 25 complete books in two hours, so 40 typists can type 10 complete books

in $2\times\frac{10}{25}=0.8$ hours. Hence 2 typists type 10 complete books in $0.8\times\frac{40}{2}=16$ hours.

15. In an election for school captain, 1320 votes were cast for five candidates. The winner's margins over the other four candidates were 19, 33, 48 and 65. What was the lowest number of votes received by a candidate?

Assume the winner got a votes, then the others got $a-19$, $a-33$, $a-48$ and $a-65$. Hence we have $a+(a-19)+(a-33)+(a-48)+(a-65)=1320$, $a=297$. So the lowest number of votes received by a candidate was $297-65=232$.

16. A teacher asked Garfield to calculate five 10-digit perfect square numbers. After a lot of arithmetic, Garfield turned in a list of the following five numbers (see below), but he made two mistakes. First, he spilled milk on the paper so that the middle six digits of each number were impossible to read. Second, he made an error in calculating one of the five numbers and that number is not a perfect square. Don't cry over spilled milk, but determine which of these five 10-digit numbers is **NOT** a perfect square.



A. 315 84

B. 23 41

C. 487 46

D. 51 36

E. 89838 089

Observe that the least digits of above five numbers are 1, 4, 6 and 9.

(i) If the unit digit of a square number is 1, then the square number must be the square of the number which is of the from 10k+1 or 10k+9. Since $(10k+1)^2=100k+20k+1$ and $(10k+9)^2=100k+180k+1$, the tens' digit of the square number must be even. So ***B*** satisfies the condition.

(ii) If the unit digit of a square number is 4, then the square number must be the square of the number which is of the from 10k+2 or 10k+8. Since $(10k+2)^2=100k+40k+4$ and $(10k+8)^2=100k+160k+64$, the tens' digit of the square number must be even. So ***A*** satisfies the condition.

(iii) If the unit digit of a square number is 6, then the square number must be the square of the number which is of the from 10k+4 or 10k+6. Since $(10k+4)^2=100k+80k+16$ and $(10k+6)^2=100k+120k+36$, the tens' digit of the square number must be odd. So ***D*** satisfies the condition.

(iv) If the unit digit of a square number is 9, then the square number must be the square of the number which is of the from 10k+3 or 10k+7. Since $(10k+3)^2=100k+60k+9$ and $(10k+7)^2=100k+140k+49$, the tens' digit of the square number must be even. So ***E*** satisfies the condition.

Only ***C*** doesn't satisfy the condition and hence ***C*** is not a perfect square.

17. What is the product of $1001\times\left(1-\frac{1}{1001^2}\right)\times\left(1-\frac{1}{1002^2}\right)\times\left(1-\frac{1}{1003^2}\right)\times\cdots\times\left(1-\frac{1}{2007^2}\right)\times2007$

$$1001\times\left(1-\frac{1}{1001^2}\right)\times\left(1-\frac{1}{1002^2}\right)\times\left(1-\frac{1}{1003^2}\right)\times\cdots\times\left(1-\frac{1}{2007^2}\right)\times 2007$$

$$=\cancel{1001}\times\frac{\cancel{1002}}{\cancel{1001}}\times\frac{1000}{\cancel{1001}}\times\frac{\cancel{1003}}{\cancel{1002}}\times\frac{\cancel{1001}}{\cancel{1002}}\times\frac{\cancel{1004}}{\cancel{1003}}\times\frac{\cancel{1002}}{\cancel{1003}}\times\cdots\times\frac{2008}{\cancel{2007}}\times\frac{\cancel{2006}}{\cancel{2007}}\times\cancel{2007}$$

$$=2008000$$

18. A multiplication magic square has the product of the numbers in each row, column and diagonal the same. If the diagram is filled with positive integers to form a multiplicative magic square, what is the value of X?

20	a	X
16	b	c
d	4	e

(i) $20\times16\times d=4\times d\times e$, $e=80$

(ii) $20\times16\times d=20\times b\times80$, $d=5b$

(iii) $20\times a\times X=80\times c\times X$, $a=4c$

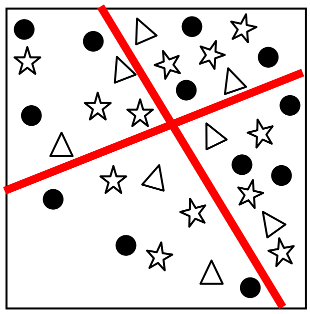
(iv) $20\times16\times5b=16\times b\times c$, $c=100$

(v) $20\times16\times5b=80\times100\times X$, $b=5X$ and hence $d=25X$

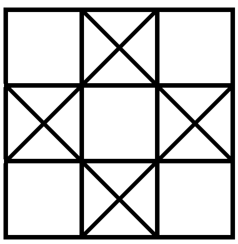
(vi) $25X\times5X\times X=80\times100\times X$, $X^2=64$. Since X is positive, $X=8$ and the multiplication magic square is as follow:

20	400	8
16	40	100
200	4	80

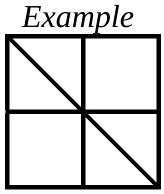
19. Divide the figure at right by drawing two straight lines so that there are three circles, three stars, and two triangles in each section.



20. How many squares and triangles can you count in the figure below? An example is given to show that squares and triangles can be counted more than one.



Example



Ans:
5 squares
6 triangles

There are nine 1×1 squares, four 2×2 squares, one 3×3 square and one $\sqrt{2}\times\sqrt{2}$ square. And there are sixteen right triangles which the length of hypotenuse is 1, sixteen right triangles which the length of hypotenuse is $\sqrt{2}$, eight right triangles which the length of hypotenuse is $2\sqrt{2}$ and four right triangles which the length of hypotenuse is 3. So there are 15 squares and 44 triangles.