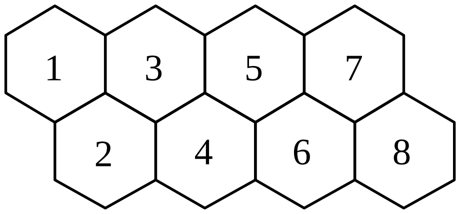
INTERNATIONAL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2007

Mathematics Contest in Taiwan, Exploration Problems

Name:_______ School:_______ Grade:______ ID number:________

Answer the following 5 questions. Write down your answer in the space provided after each question. Each question is worth 8 points. Time limit: 60 minutes.

1. How many different routes from cell 1 to cell n can you find if you always move to an adjacent cell labeled with a higher number?



   (a) 1 to 3 (1 point)
   (b) 1 to 6 (3 point)
   (c) 1 to 12 (4 point)

Let $F(n)$ mean the different routes from cell 1 to cell n. Observe that there are two different ways to enter cell n: (1) through cell $n-1$; (2) through cell $n-2$ without through cell $n-1$. So we have $F(n)=F(n-1)+F(n-2)$. Since $F(1)=1$ and $F(2)=1$, the sequence $\{F(n)\}$ is the Fibonacci sequence.

(a) $F(3)=1+1=2$.
(b) $F(4)=1+2=3$; $F(5)=2+3=5$; $F(6)=5+3=8$, the 6^{th} Fibonacci number.
(c) $F(12)$=the 12^{th} Fibonacci number=144.

2. The number 21 can be expressed as a sum of two or more consecutive positive integers in 3 different ways, namely

$$10+11$$
$$6+7+8$$
$$1+2+3+4+5+6.$$

How many different ways can (a)100 (3 point) ; (b)210 (5 point) be expressed as such sum in?

Observe that (i) if a number is expressed as a sum of $2n+1$ consecutive inters, then the number must be the product of $2n+1$ and the midterm of the series;
(ii) if a number is expressed as a sum of $2n$ consecutive inters, then the number must be the product of n and the sum of the mid-two terms. Note that the sum of the mid-two terms must be an odd number.

(a) $100=2^2\times5^2$ and $1+2+3+\dots+12+13=91<100<1+2+3+\dots+13+14=105$, so the number of terms must be less than 13 and hence the possible numbers are 5 and 8.
100 can be expressed as a sum of 5 consecutive positive integers since the midterm is $100\div5=20$ and $100=18+19+20+21+22$.
100 can be expressed as a sum of 8 consecutive positive integers since the midterm is $100\div4=25$ and $100=9+10+11+12+13+14+15+16$.
So 100 can be expressed as a sum of two or more consecutive positive integers in 2 different ways.

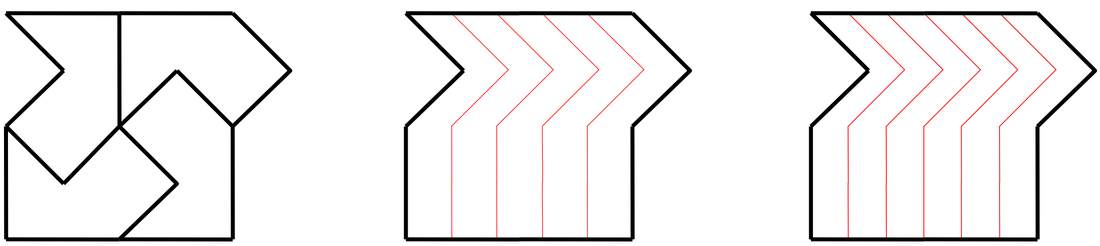
(b) $210=2\times3\times5\times7$ and $1+2+3+\dots+19+20=210$, so the number of terms must be less than 20 and hence the possible number of terms are 3, 5, 7, 15, 4, 12 and 20.
210 can be expressed as a sum of 3 consecutive positive integers since the midterm is $210\div3=70$ and $210=69+70+71$.
210 can be expressed as a sum of 5 consecutive positive integers since the midterm is

210÷5=42 and 210=40+41+42+43+44.
210 can be expressed as a sum of 7 consecutive positive integers since the midterm is 210÷7=30 and 210=27+28+29+30+31+32+33.
210 can be expressed as a sum of 15 consecutive positive integers since the midterm is 210÷15=14 and 210=7+8+9+10+11+12+13+14+15+16+17+18+19+20+21.
210 can be expressed as a sum of 4 consecutive positive integers since the midterm is 210÷2=105 and 210=51+52+53+54.
210 can be expressed as a sum of 12 consecutive positive integers since the midterm is 210÷6=35 and 210=12+13+14+15+16+17+18+19+20+21+22+23.
210 can be expressed as a sum of 20 consecutive positive integers since the midterm is 210÷10=21 and 210=1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+17+18+19+20.
So 210 can be expressed as a sum of two or more consecutive positive integers in 7 different ways.

3. There are at least 89 things in the right picture with names that begin with the letter B. How many can you find? Write down 16 things.
(0.5 point each correct answer;
－0.5 point each incorrect answer.)

Baby	Basket	Blacksmith	Breeches
Back	Bat	Blade	Bricks
Background	Bellows	Blanket	Bridle
Bacon	Belt	Blinders	Brig
Badge	Bench	Blinds	Bridge
Bag	Bicycle	Blindman	Brook
Baggage	Billows	Board	Broom
Baker	Biplane	Boat	Brush
Bakery	Bay	Body	Bubbles
Balcony	Bay window	Bonnet	Bucket
bale	Beach	Book	Buckle
ball	Beacon	Boot	Buggy
Balustrade	Beak	Bough	Buildings
Band	Beard	Bow	Buns
Bananas	Beast	Bowl	Bundle
Bandage	Bed	Bowsprit	Buoy
Banjo	Bedroom	Box	Bureau
Banner	Beef	Boy	Bush
Barber	Beets	Braces	Butcher
Bark	Beggar	Bracket	Buttons
Barn	Belfry	Braid	
Barrel	Bell	Branch	
Basin	Bird	Bread	

4. The figure on the left is divided into four congruent parts—four units of identical size and shape that can be laid flush on top of one another. Divide the figure on the right into five congruent parts (4 point), and then six (4 point).



5. In how many different ways can these nine barrels be arranged in three tiers of three so that :

(a) no barrel shall have a smaller number than its own below it? (3 point)

(b) no barrel shall have a smaller number than its own below it or to the right of it? (5 point)

(a) As you pick 3 numbers, there is 1 way to arrange in one tier. So there are totally $C_3^9 \times C_3^6 \times C_3^3 = \frac{9!}{3!(9-3)!} \times \frac{6!}{3!(6-3)!} \times \frac{3!}{3!(3-3)!} = 84 \times 20 \times 1 = 1680$ different ways to satisfy the condition.

(b) Since 1 is the smallest number and 9 is the largest number, the locations of the two numbers are fixed and hence there are two possible positions of 2.

(i) 2 is under 1.
When 3 is under 2, 4 must be on the right of 1. If 5 is on the right of 4, then 6 must be under 4 and hence there are two ways to arrange 7 and 8. If 5 is under 4, then there is one way to arrange 7 and 8 as 6 is under 5 and there are two ways to arrange 7 and 8 as 6 is on the right of 4. So there are totally 2+1+2=5 ways in this case.
When 3 is on the right of 1, there are three possible positions for 4. If 4 is under 2, then there are five ways to arrange 5, 6, 7 and 8 since it is also a similar situation with above discussion. We can also get another five ways as 4 is on the right of 3 by the similar progress. If 4 is under 3, then there are 2 ways to arrange 5. As 5 is on the right of 3, there is one way to arrange 7 and 8 when 6 is under 5 and there are two ways to arrange 7 and 8 when 6 is under 2. So there are totally three ways. As 5 is under 2, there are also three ways to arrange 6, 7 and 8 since it is a similar situation.
Hence there are 5+5+5+3+3=21 ways as 2 is under 1.

(ii) 2 is on the right of 1.
We can get another 21 ways since the discussion of this situation is the same with (i).

So there are 21+21=42 different ways to satisfy the conditions.