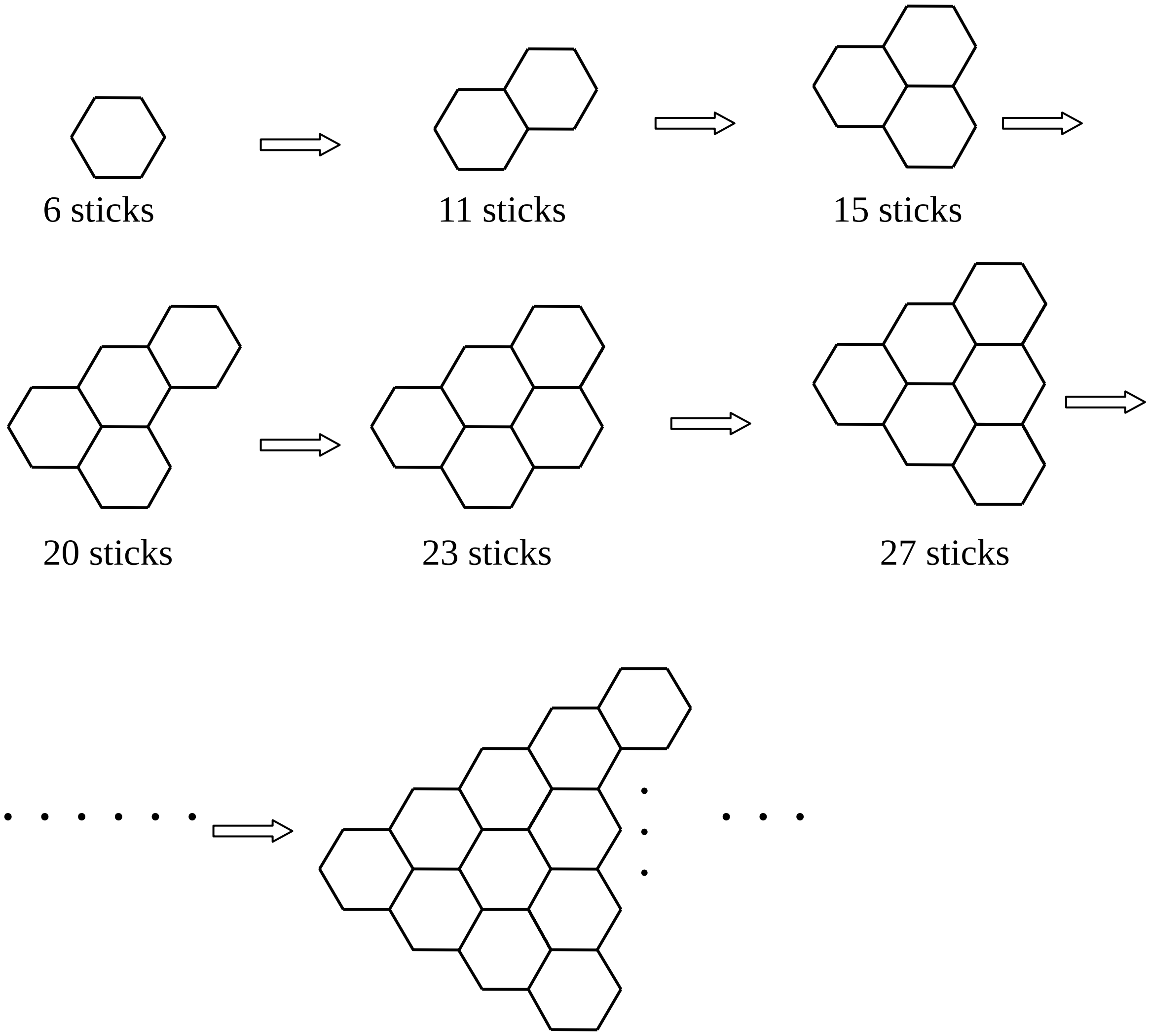
INTERNATIONL MATHENATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2005

Mathematics Contest in Taiwan, Exploration Problems

Name:________ School:_______ Grade:______ ID number:_________

Answer the following 5 questions, and show your detailed solution in the answer sheet. Write down the question number in each paper. Each question is worth 8 points. Time limit: 60 minutes.

1. A pattern of hexagons is made from sticks, as shown below.



If there are 200 sticks used, how many hexagons have been formed?

Solution:

From above observation, we can get the following fact: if there are k column of hexagons be formed, than there has $1+2+3+\cdots+(k-1)+k=\frac{k(k+1)}{2}$ hexagons and thus we need $\frac{3k(k+3)}{2}$ sticks.

When k=10, we know that $\frac{3k(k+3)}{2}=195$ and there 55 hexagons can be formed. Since $200-195=5$, there one more hexagon can be formed. Hence, there 56 hexagons can be formed by using 200 sticks.

2. If you have \$1, \$2, \$5, \$10 coins, in how many ways can you make up \$50?

Solution:

Consider the different ways using \$10, then for each of those the number of ways we can combine with \$5 and \$2 and this will then fix the number of \$1 coins to make \$50.

\$10	\$5	\$2	\$1	ways
5	0	0	0	1
4	2	0	0	1
4	1	2,1,0	remainder	3
4	0	5,4,3,2,1,0	remainder	6
3	4	0	0	1
3	3	2,1,0	remainder	3
3	2	5,4,3,2,1,0	remainder	6
3	1	7,6,5,4,3,2,1,0	remainder	8
3	0	10,9,8,7,6,5,4,3,2,1,0	remainder	11
2	6	0	0	1
2	5	2,1,0	remainder	3
2	4	5,4,3,2,1,0	remainder	6
2	3	7,6,5,4,3,2,1,0	remainder	8
2	2	10,9,8,7,6,5,4,3,2,1,0	remainder	11
2	1	12,11,10,9,8,7, 6,5,4,3,2,1,0	remainder	13
2	0	15,14,13,12,11,10,9, 8,7,6,5,4,3,2,1,0	remainder	16
1	8	0	0	1
1	7	2,1,0	remainder	3
1	6	5,4,3,2,1,0	remainder	6
1	5	7,6,5,4,3,2,1,0	remainder	8
1	4	10,9,8,7,6,5,4,3,2,1,0	remainder	11
1	3	12,11,10,9,8,7, 6,5,4,3,2,1,0	remainder	13
1	2	15,14,13,12,11,10,9, 8,7,6,5,4,3,2,1,0	remainder	16
1	1	17,16,15,14,13,12,11,10, 9,8,7,6,5,4,3,2,1,0	remainder	18
1	0	20,19,1817,16,15,14,13,12, 11,10,9,8,7,6,5,4,3,2,1,0	remainder	21
0	10	0	0	1

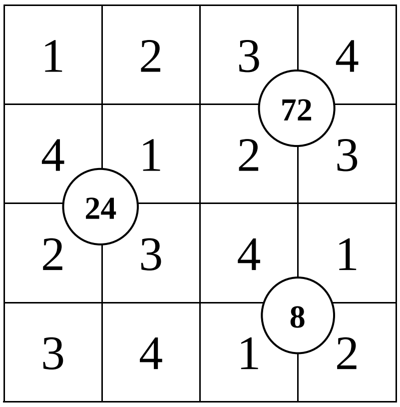
0	9	2,1,0	remainder	3
0	8	5,4,3,2,1,0	remainder	6
0	7	7,6,5,4,3,2,1,0	remainder	8
0	6	10,9,8,7,6,5,4,3,2,1,0	remainder	11
0	5	12,11,10,9,8,7, 6,5,4,3,2,1,0	remainder	13
0	4	15,14,13,12,11,10,9, 8,7,6,5,4,3,2,1,0	remainder	16
0	3	17,16,15,14,13,12,11,10, 9,8,7,6,5,4,3,2,1,0	remainder	18
0	2	20,19,1817,16,15,14,13,12, 11,10,9,8,7,6,5,4,3,2,1,0	remainder	21
0	1	22,21,20,19,1817,16,15, 14,13,12,11,10,9,8, 7,6,5,4,3,2,1,0	remainder	23
0	0	25,24,23,22,20,19,1817, 16,15,14,13,12,11,10,9, 8,7,6,5,4,3,2,1,0	remainder	26

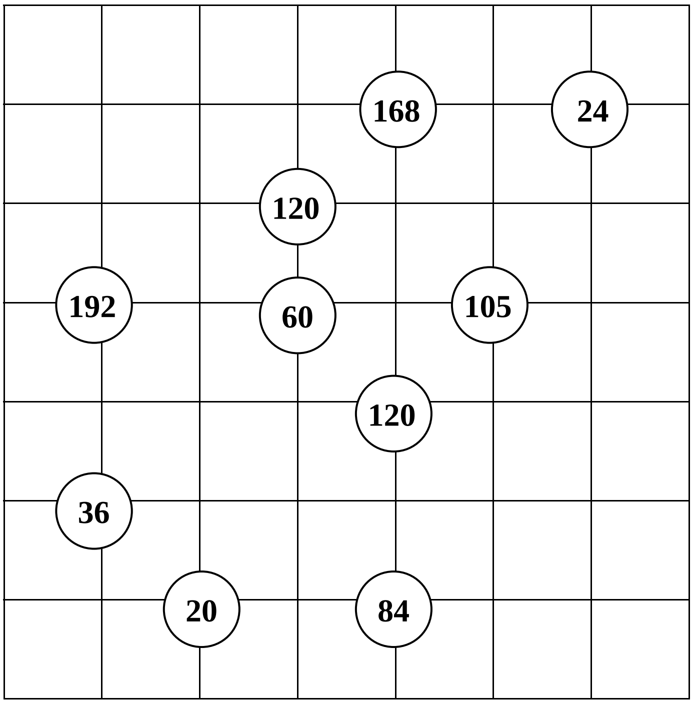
Hence the total ways is

$1\times6+3\times5+6\times5+8\times4+11\times4+13\times3+16\times3+18\times2+21\times2+23\times1+26\times1 = 341$.

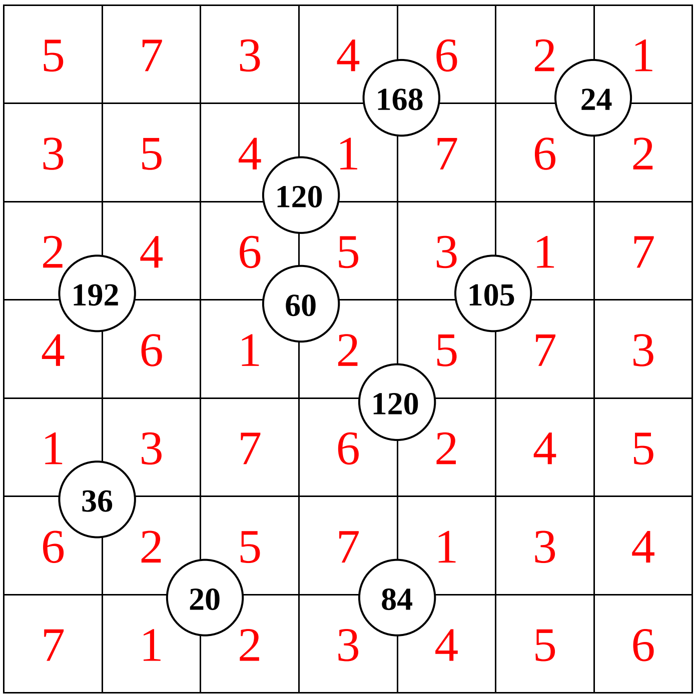
3. Locate digits from 1 to 7 into each row and each column of the grid once. Numbers on the circles tell the product of the four digits around them.

Example:





Solution:



4. The Fibonacci numbers are

$$F_1=1,\ F_2=1,\ F_3=2,\ F_4=3,\ F_5=5,\ F_6=8,\ F_7=13,\cdots$$

where the first two are both equal to 1, and from then on, each one is the sum of the two preceding it. What is the last digit of the sum of the first 2005 Fibonacci numbers?

Solution:

The last digits of the first 60 Fibonacci numbers are:

1, 1, 2, 3, 5, 8, 3, 1, 4, 5,
9, 4, 3, 7, 0, 7, 7, 4, 1, 5,
6, 1, 7, 8, 5, 3, 8, 1, 9, 0,
9, 9, 8, 7, 5, 2, 7, 9, 6, 5,
1, 6, 7, 3, 0, 3, 3, 6, 9, 5,
4, 9, 3, 2, 5, 7, 2, 9, 1, 0.

After which the whole sequence repeats.

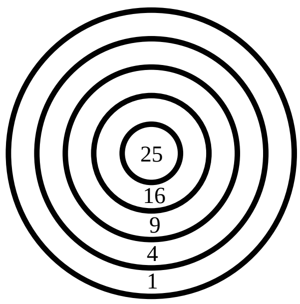
Counting the sum of those last digit, we get 0 as its last digit in the complete cycle of 60.

Now, 2005=33x60+25, so the last digit of the sum of the last 25 Fibonacci numbers is

$$1+1+2+3+5+8+3+1+4+5+9+4+3+7+0+7+7+4+1+5+6+1+7+8+5.$$

So the solution is 7.

5. Four darts are thrown at the dartboard illustrated on the right. The four scores are added together, a miss counted as zero. What is the smallest positive total score which is impossible to obtain?



Solution:

Every positive score up to 52 can be obtained:

1=0+0+0+1,	2=0+0+1+1,	3=0+1+1+1,	4=0+0+0+4,
5=0+0+1+4,	6=0+1+1+4,	7=1+1+1+4,	8=0+0+4+4,
9=0+0+0+9,	10=0+0+1+9,	11=0+1+1+9	12=0+4+4+4,
13=1+4+4+4,	14=0+1+4+9,	15=1+1+4+9,	16=0+0+0+16,
17=0+0+1+16,	18=0+1+1+16,	19=1+1+1+16,	20=0+0+4+16,
21=0+1+4+16,	22=1+1+4+16,	23=1+4+9+9,	24=0+4+4+16,
25=0+0+0+25	26=0+0+1+25,	27=0+1+1+25,	28=1+1+1+25,
29=0+0+4+25,	30=0+1+4+25,	31=1+1+4+25,	32=0+0+16+16,
33=0+1+16+16,	34=1+1+16+16,	35=0+1+9+25,	36=9+9+9+9,
37=4+4+4+25,	38=0+4+9+25,	39=1+4+9+25,	40=4+4+16+16,
41=0+9+16+16,	42=0+1+16+25,	43=1+1+16+25,	44=1+9+9+25,
45=0+4+16+25,	46=1+4+16+25,	47=4+9+9+35,	48=0+16+16+16,
49=1+16+16+16	50=0+0+25+25,	51=0+1+25+25,	52=1+1+25+25.

But 53 cannot be obtained. The reason is as following:

Assume $53=25a+16b+9c+4d+e$, where a, b, c, d and e are non-negative integers and $a+b+c+d+e=4$. Thus the possible values of a are 0, 1 or 2.

First, when $a=0$, $b\neq 4$ since $16\times 4=64>53$. If b=3, then $53-3\times 16=5$ and there is no 5. If $b\leqq 2$, then the maximal score we can get is $16\times 2+9\times 2=50$ which is less than 53. So it is impossible to get score 53.

Next, when $a=1$, $b\leqq 1$ since $16\times 2=32>28=53-25$. As b=1, we can not get score 12 by choosing two of 9, 4 and 1. If b=0, then the maximal score we can get is $25+9\times 3=52$ which is smaller than 53. So it is also impossible to get score 53.

Finally, if $a=2$, then the maximal score we can get is 52 since $b=c=d=0$ and hence $e=2$. So it is still impossible to get score 53.

From above discussion, we know that 53 is the smallest number.