# INTERNATIONL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2005

## Mathematics Contest in Taiwan, Mathematics Short Answer Problems

**Name:________ School:_______ Grade:______ ID number:_________**

**Short Answer: there are 20 questions, fill in the correct answers in the answer sheet. Each correct answer is worth 2 points. Time limit: 60 minutes.**

1. A foundation has allocated a certain amount of money for 1$^{st}$, 2$^{nd}$ and 3$^{rd}$ prizes in a competition. The money is divided in the ratio of 3∶2 where the larger amount is for the 1$^{st}$ prize and the smaller amount is divided again in the ratio of 3∶2 for the 2$^{nd}$ and 3$^{rd}$ prizes respectively. It becomes known that the 3$^{rd}$ prize is \$3300 less than the first prize. How much is the 2$^{nd}$ prize?

Solution:

Let the total money be $x$. Then the 1$^{st}$ prize is $\frac{3}{5}x$ and the 3$^{rd}$ prize is $\frac{4}{25}x$.

So $\frac{3}{5}x-\frac{4}{25}x=\$3300$ $\qquad$ $\frac{11}{25}x=\$3300$ $\qquad$ $x=\$7500$.

Hence the 2$^{nd}$ prize is $\$7500\times\frac{2}{5}\times\frac{3}{5}=\$1800$.

2. Three man and three children arrive at the river where there is a small boat that will hold one adult or two children. What is the minimum number of trips across the river in either direction to get the family across?

Solution:

Clearly, we must start with two children crossing and one coming back, as starting with an adult crossing means that the adult must come back and two trips are wasted without changing the initial situation. So the sequence is as following:

| | | | |
|---|---|---|---|
| 1. | AAAC | CC → | 0 |
| 2. | AAAC | ← C | C |
| 3. | AACC | A → | C |
| 4. | AACC | ← C | A |
| 5. | AAC | CC → | A |
| 6. | AAC | ← C | AC |
| 7. | ACC | A → | AC |
| 8. | ACC | ← C | AA |
| 9. | AC | CC → | AA |
| 10. | AC | ← C | AAC |
| 11. | CC | A → | AAC |
| 12. | CC | ← C | AAA |
| 13. | C | CC → | AAA |
| 14. | C | ← C | AAAC |
| 15. | 0 | CC → | AAAC |

3. Mr. Sun has a broken calculator. When just turned on, it displays 0. If the + key is pressed, it adds 35. If the − key is pressed, it subtracts 35. If the × key is pressed, it adds 91. If the ÷ key is pressed, it subtracts 91. The other keys do not function. Mr. Sun turns the calculator on. What is the number closest to 2005 that he can get using this calculator?

Solution:

The numbers that can be obtained using the calculator are of the form $35a+91b$ where $a$ and $b$ are integers (possibly negative). Since $35=5\times7$ and $91=7\times13$, every number of the form $35a+91b$ is divisible by 7. The closest multiple of 7 to 2005 is $2002=7\times286$. Since $2002=35\times52+91\times2=2002$, the number closest to 2005 that is possible to get using this calculator is 2002.

4. There are 500 unit cubes. As many of these cubes as needed are glued together to form the largest possible cube which looks solid from any point on the outside but is hollow inside. What is the side length of the largest cube?

Solution:

For the hollow cube, each of the faces would be a square close to $\frac{500}{6}\approx 83$.

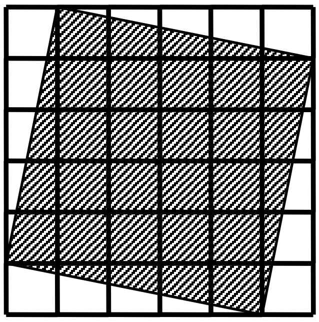
The possibilities around 83 are $9^2=81$ and $10^2=100$.

A $11\times11\times11$ hollow cube would contain $11^3-9^3=1331-729=602$ unit cubes, so it is not possible.

A $10\times10\times10$ hollow cube would contain $10^3-8^3=1000-512=488$ unit cubes.

Thus the side length of the largest cube is 10.

5. What is the ratio of the shaded square to that of the largest square shown in the diagram?



Solution:

The area of the larger square is $6^2$=36 square units. The shaded area is the area of the larger square less the area of 4 triangles with base 5 and height 1. So the shaded area is

$$36-4\times\left(\frac{1}{2}\times1\times5\right)=36-10=26$$

Hence the ratio is 26∶36＝13∶18.

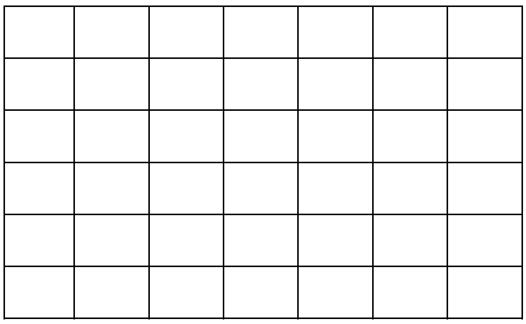
6. A three-digit number $N$ leaves remainder 3 when divided by 7, remainder 5 when divided by 11 and remainder 8 when divided by 17. What is the number $N$ ?

Solution:

Consider $K$=2$N$+1. Since $N$ leaves remainder 3 when divided by 7, $K$ is divisible by 7. Similarly, $K$ is divisible by 11 and 17. Hence $K$ is divisible by 7╳11╳17=1309 as 7, 11 and 17 are relatively prime. Since $N$ is a three-digit number, $K$=1309.

Hence $N=\dfrac{K-1}{2}$=654.

7. How many rectangles are there in this grid, where vertices are points of the grid and the edges are lines of the grid?



Solution:

Any rectangle is uniquely determined by pairing a horizontal segment with an orthographical segment to get the four edges of a rectangle. We can choose 8╳7÷2=28ways of horizontal segments and we can choose 7╳6÷2=21 ways of orthographical segments. This gives a total of 21╳28=588 rectangles.

8. A six digit number is represented by $\overline{abcdef}$ , where $a$, $b$, $c$, $d$, $e$ and $f$ are its digits. If this number is multiplied by 6, the result is $\overline{defabc}$. What is this six digit number?

Solution:

Given $\begin{array}{r} \overline{abcdef} \\ \times \quad 6 \\ \hline \overline{defabc} \end{array}$, we get $a$=1. $\overline{abcdef}\times 6=\overline{defabc}$, this means

$\left(\overline{abc}\times 1000+\overline{def}\right)\times 6=\overline{def}\times 1000+\overline{abc}$. Thus $5999\times\overline{abc}=994\times\overline{def}$ and

this implies $7\times 857\times\overline{abc}=2\times 7\times 71\times\overline{def}$. So we have

$857\times\overline{abc}=142\times\overline{def}$. Since a=1, $\overline{abc}$=142 and hence $\overline{def}$=857. Thus the six digit number is 142857.

9. If $a$, $b$, $c$ and $d$ are positive integers such that

$$a+\frac{1}{b+\frac{d}{c}}=\frac{2005}{101}.$$

What is the value of $a+b+c+d$ ?

Solution:

$$\frac{2005}{101}=19+\frac{86}{101}=19+\frac{1}{\frac{101}{86}}=19+\frac{1}{1+\frac{15}{86}}.$$

Hence $a+b+c+d$=19+1+86+15=121.

10. Alan has a stride 75 *cm*. If he travels by walking 5 steps forward and one step back, what is the least number of steps he needs to reach a spot 24 metres away?

Solution:

After 6 steps, Alan will finish at 3 metres, after 12 steps at 6 metres, and so on. So he will take 6×7=42 steps to reach 21 metres, and will only take the four forward steps from there to reach the 24 metres, hence will take 42+4=46 steps.

11. $N$ is a positive integer such that $N$ and $N$ + 97 are both perfect squares. What is the positive integer $N$ ?

Solution:

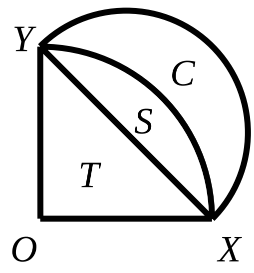
Let $N = p^2$ and $N + 97=q^2$. Then $97= q^2-p^2=(q-p)(q+p)=1\times 97$. Hence $q-p=1$ and $q+p=97$, that is $q$=49 and $p$=48. Thus $N=48^2$=2304.

12. Five students sit for an exam which has a maximum score of 100. The average of the five scores achieved by the students in the exam was 89. What could the minimum score be gained?

Solution:

The minimum score can be gained by one student when each of the other four students achieve the maximum score. Hence the minimum score is 89×5−400=45.

13. In Figure, $OX$=$OY$=10 are radii of a circular quadrant. A semi-circle is drawn on $XY$ as shown. $T$, $S$ and $C$ denote the resulting triangles, segment and crescent. What is the area of $C$ ?




Solution:

Since $OX$=$OY$=10, from Pythagoras Theorem in $\triangle YOX$,

$YX = 10\sqrt{2}$. $T$+$S$ is a quadrant of a circle radius 10, so

$T + S = \frac{1}{4} \times 10 \times 10 \times \pi = 25\pi$. $S$+$C$ is a semi-circle with a diameter of $10\sqrt{2}$, so

$S + C = \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} \times \pi = 25\pi$. This means $T$+$S$=$S$+$C$, i.e. $T$=$C$. Because the

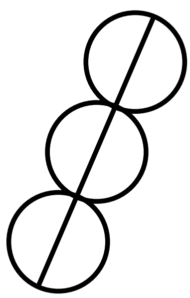
area of $T$ is $\frac{1}{2} \times 10 \times 10 = 50$, the area of $C$ is 50.

14. A large watermelon weighs 12 kg, with 97% of its weight being water. It is left to stand in the sun, and some of the water evaporates so that now only 90% of its weight is water. What does it now weigh?

Solution:

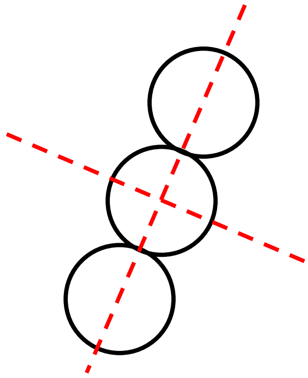
Originally, 97% of the watermelon is water, so the solids are 3% or 12×0.03 = 0.36 kg. When the water becomes 90% of its weight, these solids becomes 10% of its weigh. Thus its weigh then is 0.36÷10%=3.6 kg.

15. What is the number of lines of symmetry in the plane of the diagram?

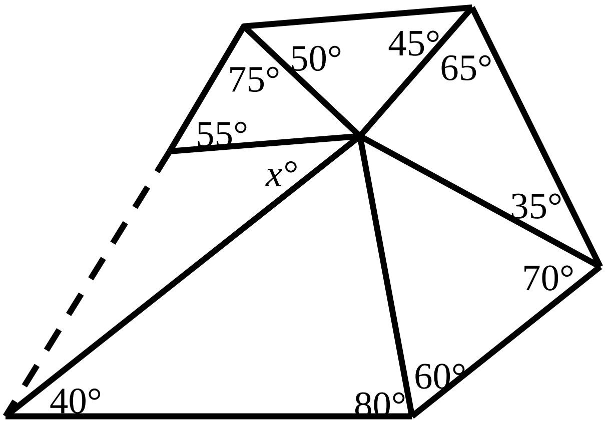


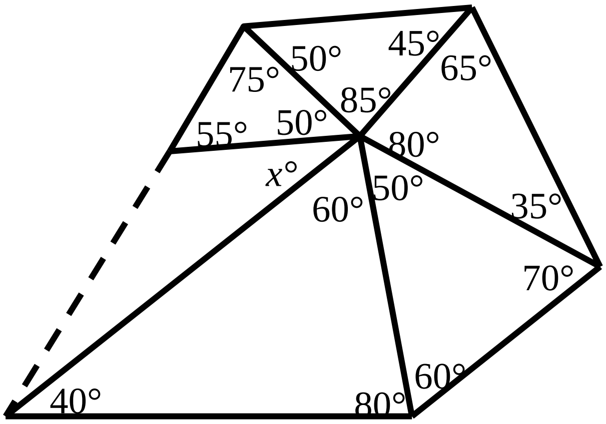
Solution:

In the plane of the diagram, there are two axes of symmetry, the two dotted lines as shown in the diagram.



16.What is the value of $x$ in the diagram?




Solution:

Completing the angle sum of the five triangles, we get the angles to be 60°, 50°, 80°, 85°and 50°as shown. Then we have

$$x+60+50+80+85+50=360.$$

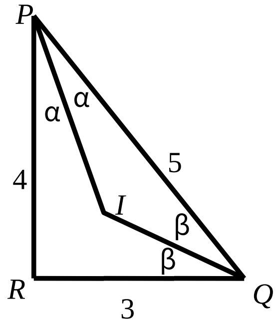
Hence $x$=35.

17.When $10^{2005}-2005$ is express as a single number, what is the sum of the digits?

Solution:

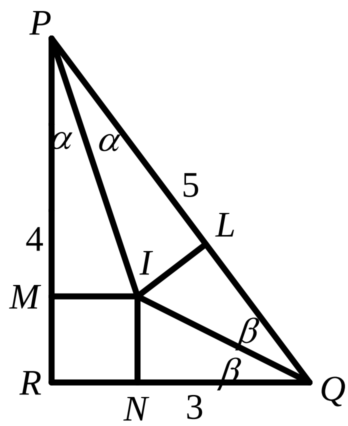
The number is 999⋯97995, where the leading 2001 digits are all 9, so the sum of all digits is $9\times2001+7+9+9+5=$18039.

18.The length of the sides of a triangle $PQR$ are $PQ$=5, $QR$=3 and $RP$=4. The bisectors of the angles $P$ and $Q$ meet at the point $I$. What is the area of the triangle $PQI$?



Solution:

Draw the perpendiculars from $I$ to each of the sides $PQ$, $QR$ and $RP$ and intersect at $L$, $N$ and $M$ respectively. It follows that the pairs of triangles $PIL$, $PIM$ and $QIL$, $QIN$ are congruent, as they have two angles and corresponding side equal. Thus $IL=IN=IM$ and we can get




$$MI=IN=MR=RN=\frac{(4+3-5)}{2}=1.$$

Hence the area of the triangle $PQI$ is $5\times\frac{1}{2}=2.5$.

19.A cube with edge of length 10 is painted. The cube is then divided into 1000 unit cubes. Among these small cubes, how many cubes which have one or two painted faces?

Solution:

<method 1>

The numbers of small cubes which have exactly one painted face is $8^2=64$ on each of the 6 faces, i.e. $64\times6=384$ cubes.

Also, the number of small cubes which have exactly two painted face is 8 on each of the 12 edges, i.e. $8\times12=96$ cubes.

Hence there are $384+96=480$ cubes.

<method 2>

There are $8^3=512$ cubes without painting and 8 cubes painting three faces. So there are $10^3-8^3-8=1000-512-8=480$ cubes having one or two painted faces.

20.In the $5\times5$ square the numbers 1, 2, 3, 4 and 5 are arranged in such a way that every number occurs precisely once in each column. In the $5\times5$ square shown, what is the entry in the position marked with ☆?

| | | | | |
|---|---|---|---|---|
| 1 | 2 | | | |
| | | | | 1 |
| | | 4 | | |
| 2 | | 5 | ☆ | |
| | 5 | | | 4 |

Solution:

The third entry in the first row must be 3, then the third column completes to 3, 2, 4, 5, 1 and the last row to 3, 5, 1, 2, 4. The first column can now be completed to 1, 4, 5, 2, 3. And so on, we get the 5╳5 square as shown. Hence the ☆ must be 1.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 4 | 3 | 2 | 5 | 1 |
| 5 | 1 | 4 | 3 | 2 |
| 2 | 4 | 5 | 1 | 3 |
| 3 | 5 | 1 | 2 | 4 |