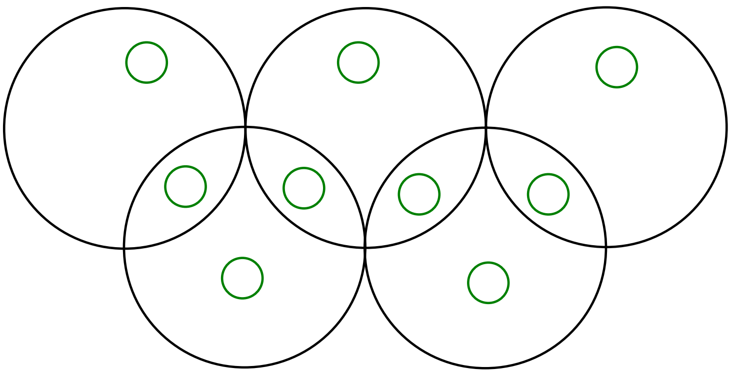
2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

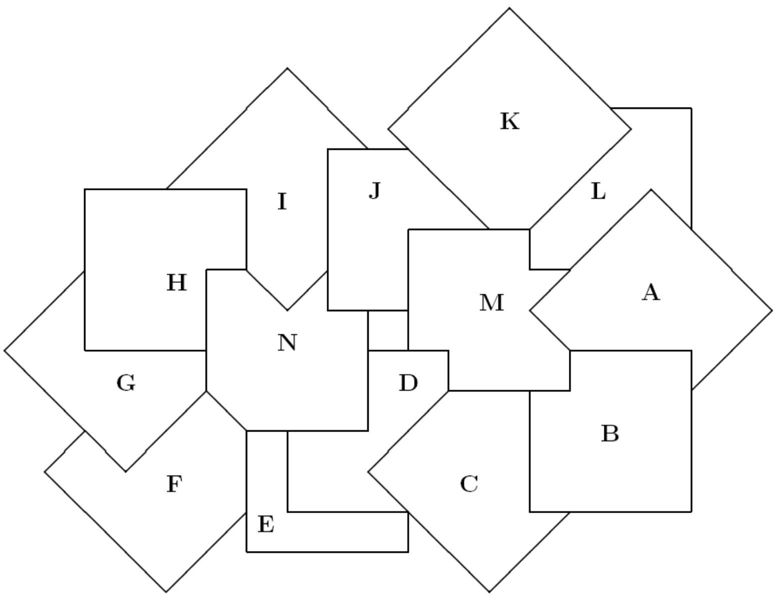
2007/7/23 Changchun, China

Team: ________________________ **Score**: ________________

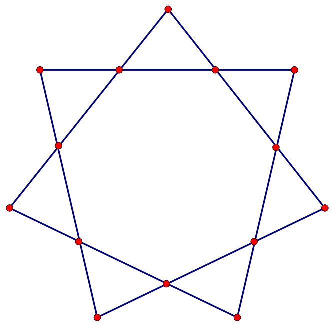
1. Use each of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 exactly once to fill in the nine small circles in the Olympic symbol below, so that the sum of all the numbers inside each large circle is 14. Write down the correct number in each small circle.



2. The diagram below shows fourteen pieces of paper stacked on top of one another. Beginning on the pieces marked B, move from piece to adjacent piece in order to finish at the piece marked F. The path must alternately climb up to a piece of paper stacked higher and come down to a piece of paper stacked lower. The same piece may be visited more than once, and it is not necessary to visit every piece. List the pieces of paper in the order visited.



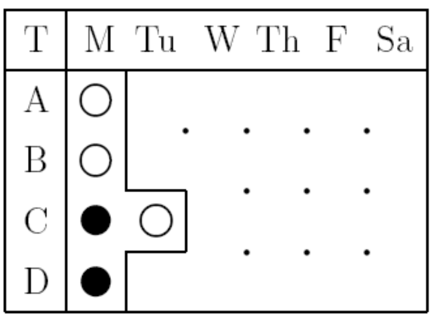
3. There are 14 points of intersection in the seven-pointed star in the diagram on the below. Label these points with the numbers 1, 2, 3, …, 14 such that the sum of the labels of the four points on each line is the same. Give one set of solution, no explanation needed.

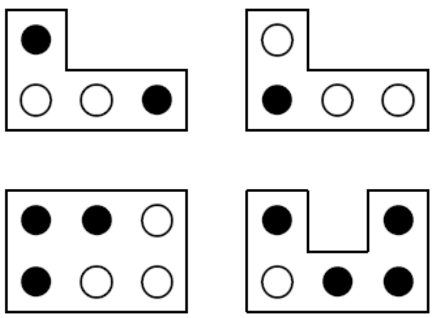


4. Mary found a 3-digit number that, when multiplied by itself, produced a number which ended in her 3-digit number. What is the sum of all the distinct 3-digit numbers which have this property?

5. Determine all positive integers m and n such that m^2+1 is a prime number and $10(m^2+1)=n^2+1$.

6. Four teams take part in a week-long tournament in which every team plays every other team twice, and each team plays one game per day. The diagram below on the left shows the final scoreboard, part of which has broken off into four pieces, as shown on the diagram below on the right. These pieces are printed only on one side. A black circle indicates a victory and a white circle indicates a defeat. Which team wins the tournament?





7. Let A be a 3 by 3 array consisting of the numbers 1, 2, 3, …, 9. Compute the sum of the three numbers on the i-th row of A and the sum of the three numbers on the j-th column of A. The number at the intersection of the i-th row and the j-th column of another 3 by 3 array B is equal to the absolute difference of the two sums of array A. For Example,

$$b_{12} = \left|(a_{11} + a_{12} + a_{13}) - (a_{12} + a_{22} + a_{32})\right|.$$

Is it possible to arrange the numbers in array A so that the numbers 1, 2, 3, …, 9 will also appear in array B?

a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}

A

b_{11}	b_{12}	b_{13}
b_{21}	b_{22}	b_{23}
b_{31}	b_{32}	b_{33}

B

8. The diagonals AC and BD of a convex quadrilateral are perpendicular to each other. Draw a line that passes through point M, the midpoint of AB and perpendicular to CD; draw another line through point N, the midpoint of AD and perpendicular to CB. Prove that the point of intersection of these two lines lies on the line AC.

9. The positive integers from 1 to n (where n>1) are arranged in a line such that the sum of any two adjacent numbers is a square. What is the minimum value of n?

10. Use one of the five colours (R represent red, Y represent yellow, B represent blue, G represent green and W represent white) to paint each square of an 8×8 chessboard, as shown in the diagram below. Then paint the rest of the squares so that all the squares of the same colour are connected to one another edge to edge. What is the largest number of squares of the same colour as compare to the other colours?

R							
						Y	
		B					
G							G
			R				
	W					W	
		B	Y				