2005 Kaohsiung Invitational World Youth Mathematics Intercity Competition

Team Contest 2005/8/3 Kaohsiung

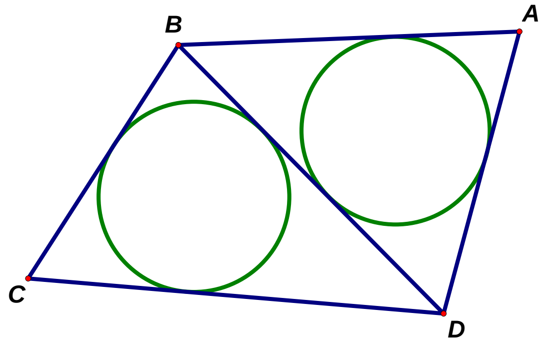
Team: ______________________ ***Score:*** ________________

1. The positive integers a, b and c are such that $a + b + c = 20 = ab + bc - ca - b^2$. Determine all possible values of abc.

2. The sum of 49 positive integers is 624. Prove that three of them are equal to one another.

3. The list 2, 3, 5, 6, 7, 10, … consists of all positive integers which are neither squares nor cubes in increasing order. What is the 2005th number in this list?

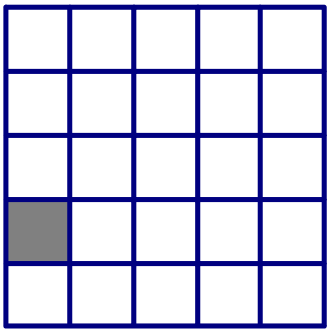
4. *ABCD* is a convex quadrilateral such that the incricles of triangles *BAD* and *BCD* are tangent to each other. Prove that *ABCD* has an incircle.



5. Find a dissection of a triangle into 20 congruent triangles.

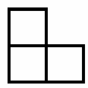
6. You are gambling with the Devil with 3 dollars in your pocket. The Devil will play five games with you. In each game, you give the Devil an integral number of dollars, from 0 up to what you have at the time. If you win, you get back from the Devil double the amount of what you pay. If you lose, the Devil just keeps what you pay. The Devil guarantees that you will only lose once, but the Devil decides which game you will lose, after receiving the amount you pay. What is the highest amount of money you can guarantee to get after the five games?

7. A frog is sitting on a square adjacent to a corner square of a 5×5 board. It hops from square to adjacent square, horizontally or vertically but not diagonally. Prove that it cannot visit each square exactly once.



8. Determine all integers n such that $n^4-4n^3+15n^2-30n+27$ is a prime number.

9. A V-shaped tile consists of a 2×2 square with one corner square missing. Show that no matter which square is omitted from a 7×7 board, the remaining part of the board can be covered by 16 tiles.



V-shaped

10. Let $a_0, a_1, a_2, \dots, a_n$ be positive integers and $a_0 > a_1 > a_2 > \dots > a_n > 1$ such that

$$\left(1-\frac{1}{a_1}\right)+\left(1-\frac{1}{a_2}\right)+\dots+\left(1-\frac{1}{a_n}\right) = 2\left(1-\frac{1}{a_0}\right).$$

Find all possible solutions for $(a_0, a_1, a_2, \dots, a_n)$.