

Po Leung Kuk
12th Primary Mathematics World Contest
Individual Contest 2008

1. Four couples are eating oranges. Among the four wives, **A** has eaten 3 oranges, **B** has eaten 2 oranges, **C** has eaten 4 oranges and **D** has eaten only 1 orange. Among the husbands, **R** has eaten as many oranges as his wife has, **S** has eaten twice as many as his wife has, **T** has eaten 3 times as many as his wife has, and **U** has eaten 4 times as many as his wife has. If 32 oranges are eaten, who is **T**'s wife?
2. There is a 5-digit number that is divisible by 9 and 11. If the first, the third and the fifth digits are removed, it becomes 35. If the first three digits are removed, it becomes a 2-digit number that is divisible by 9. If the last three digits are removed, it becomes a 2-digit number that is also divisible by 9. What is this number?
3. How many integers from 1 to 100 do not include the digit 1?
4. A man gives $\frac{1}{3}$ of his money to his son, $\frac{1}{5}$ of his money to his daughter and the remaining money to his wife. If his wife gets \$35000, how much money did the man originally have?
5. Calculate
$$\frac{1 \times 2 \times 3 + 2 \times 4 \times 6 + 3 \times 6 \times 9 + \cdots + 2008 \times 4016 \times 6024}{3 \times 4 \times 5 + 6 \times 8 \times 10 + 9 \times 12 \times 15 + \cdots + 6024 \times 8032 \times 10040}.$$
6. Three girls, **A**, **B** and **C** are running a 100 m race. Spectators **D**, **E** and **F** are discussing each girl's chance to win:
D says **A** will be first.
E says **C** will not be last.
F says **B** will not be first.
If only one spectator is right, then who will be first?

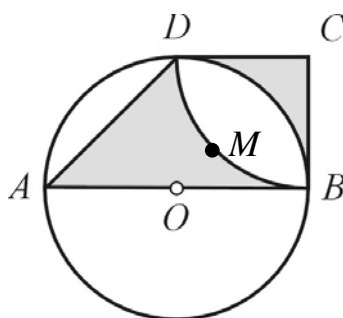
7. In the following figure, AB is the diameter of a circle with centre O . Point D is on the circle. In the trapezoid $ABCD$,

- line segments AB and DC are both perpendicular to BC , and
- $AB = 2CD$.

Arc DMB is part of a circle with centre C .

What is the ratio between the area of the shaded part and the area of the circle?

(Take π as $\frac{22}{7}$)



8. Find the smallest positive integer, divisible by 45 and 4, whose digits are either 0 or 1.

9. Find the greatest value of $a + \frac{1}{b + \frac{1}{c}} + d + \frac{1}{e + \frac{1}{f}} + g + \frac{1}{h + \frac{1}{i}}$, where each letter

represents a different non-zero digit.

10. In the two arithmetic problems below, the four different shapes



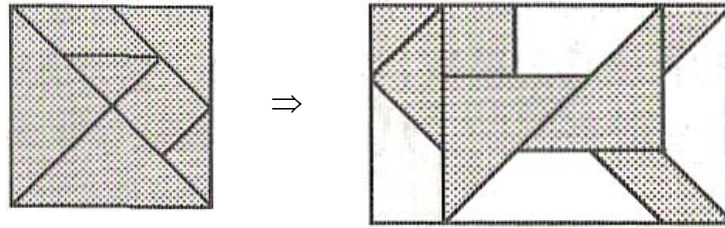
represent exactly one of the numbers 1, 2, 4 or 6 but not necessary in that order.

The symbol \emptyset is zero. What number does each shape represent so that both problems work?

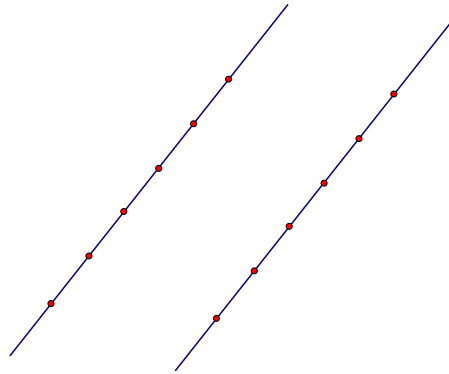
$$\begin{array}{r}
 \text{Hexagon} \quad \text{Triangle} \quad \text{Circle} \\
 - \quad \text{Square} \quad \text{Circle} \quad \text{Circle} \\
 \hline
 \text{Triangle} \quad \text{Circle} \quad \emptyset
 \end{array}$$

$$\begin{array}{r}
 \text{Triangle} \quad \text{Circle} \quad \text{Hexagon} \\
 - \quad \text{Circle} \quad \text{Circle} \quad \text{Triangle} \\
 \hline
 \text{Circle} \quad \emptyset \quad \text{Square}
 \end{array}$$

11. The figure on the right is a rectangle whose shaded area is made up of pieces of a square tangram having an area of 10 cm^2 , as shown on the left. What is the area of the rectangle?



12. Find the remainder of $2^{2008} + 2008^2$ divided by 7.
13. Six different points are marked on each of two parallel lines. How many different triangles may be formed using 3 of the 12 points?



14. There are 12 identical marbles in a bag. Only two, three or four marbles may be removed at a time. How many different ways are there to remove all the marbles from the bag?

For example, here are 3 different ways,

- i) 4 then 3 then 3 and then 2,
- ii) 2 then 3 then 3 and then 4,
- iii) 2 then 2 then 2 then 3 and then 3.

15. John walks from town A to town B. He first walks on flat land, and then uphill. He then returns to town A along the same route. John's walking speed on flat land is 4 km/h. He walks uphill at a speed of 3 km/h and he walks downhill at a speed of 6 km/h. If the entire journey took 6 hours, what is the distance from town A to town B?