International Mathematics Tournament of Towns

Spring 2001, 11 March, 0-level, Junior.

(Your total score is based on the three problems for which you earn the most points; the scores for the individual parts of a single problem are summed. Points for each problem are shown in brackets [ ].)

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1. [3] The natural number n can be replaced by ab if a+b=n, where a and b are natural numbers. Can the number 2001 be obtained from 22 by such replacements?

2. [4] One of the midlines of a triangle is longer than one of its medians. Prove that the triangle has an obtuse angle.

3. [4] Twenty kilograms of cheese are on sale in a food store, and customers are queued up to buy this cheese. After a while, having sold the demanded portion of cheese to the next customer, the salesgirl correctly calculates the average weight of the portions of cheese already sold and declares the number of customers for whom there is exactly enough cheese if each customer will buy a portion of cheese of weight exactly equal to the average weight of the previous purchases. Could it happen that the salesgirl can declare, after each of the first 10 customers has made his purchase, that there just enough cheese for the next 10 customers? If so, how much cheese will be left in the store after the first 10 customers have made their purchases? (The average weight of a series of purchases is the total weight of the cheese sold divided by the number of purchases.)

4. a) [2] There are 5 identical paper triangles on a table. Each may be moved in any direction parallel to itself (i.e., without rotating it). Is it true that then any one of them can be covered by the 4 others?
b) [3] There are 5 identical equilateral paper triangles on the table. Each may be moved in any direction parallel to itself. Prove that any one of them can be covered by the 4 others in this way.

5. [5] On a square board divided into 15 x 15 little squares there are 15 rooks that do not attack each other. Then each rook makes one move like that of a knight. Prove that after this is done a pair of rooks will necessarily attack each other.

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Spring 2001, 11 March, 0-level, Senior.

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1. [3] A bus that moves along a 100 km route is supplied with a computer, which predicts how must more time is needed to arrive at its final destination. This prediction is made from the assumption that the average speed of the bus in the remaining part of the route is the same as that in the part already covered. Forty minutes after the departure of the bus, the computer predicts that the remaining travelling time will be 1 hour. And this predicted time remains the same for the next 5 hours. Could this possibly occur? If so, how many kilometers did the bus cover after these 5 hours? (The average speed of the bus is the number of kilometers covered divided by the time it took to cover them.)

2. [4] The decimal expression of the natural number a consists of n digits, while that of a^3 consists of m digits. Can n+m equal 2001?

3. [4] In triangle ABC the point X is on side AB, while the point Y is on BC. The segments AY and CX intersect at the point Z. It is known that AY=YC and AB=ZC. Prove that the points B, X, Z, and Y lie on one circle.

4. [5] Two persons play a game on a board divided into 3 x 100 squares. They move in turn: the first places dominoes of size 1 x 2 lengthwise (along the long axis of the board), the second, in the perpendicular direction. The loser is the one who cannot make a move. Which of the payers can always win (no matter how his opponent plays), and how should he play in order to win?

5. [5] Nine points are drawn on the surface of a regular tetrahedron of edge 1cm. Prove that among these points there are two located at a distance (in space) no greater than 0.5cm.