International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper **Fall 2008.**

1. Each of ten boxes contains a different number of pencils. No two pencils in the same box are of the same colour. Prove that one can choose one pencil from each box so that no two are of the same colour.
2. Twenty-five of the numbers 1, 2, …, 50 are chosen. Twenty-five of the numbers 51, 52, …, 100 are also chosen. No two chosen numbers differ by 0 or 50. Find the sum of all 50 chosen numbers.
3. *ACE* is inscribed in a circle of radius 2. Prove that one can choose a point *D* on the arc *CE*, a point *F* on the arc *EA* and a point *B* on the arc *AC*, such that the numerical value of the area of the hexagon *ABCDEF* is equal to the numerical value of the perimeter of triangle *ACE*.
4. The average of two distinct positive integers are calculated. Can the product of all three numbers be equal to a^{2008} for some positive integer a?
5. On a straight track are several runners, each running at a different constant speed. They start at one end of the track at the same time. When they reach the end of the track, they turn around and continue to run indefinitely. Some time after the start, all runners meet at the same point. Prove that this will happen again.

Note: The problems are worth 3, 3, 4, 4 and 4 points respectively.