International Mathematics
TOURNAMENT OF THE TOWNS

Senior O-Level Paper **Fall 2008.**

1. Alex distributes some cookies into several boxes and record the number of cookies in each box. If the same number appears more than once, it is recorded only once. Serge takes one cookie from each box and puts them on the first plate. Then he takes one cookies from each box that is still non-empty and puts the cookies on the second plate. He continues until all the boxes are empty. Then Serge record the number of cookies on each plate. As with Alex, if the same number appears more than once, it is recorded only once. Prove that Alex's record contains the same number of numbers as Serge's record.
2. Let n be an integer such that $n>2$. Find positive numbers x_1, x_2, …, x_n such that $x_1 - x_2 = 1$ and $\sqrt{x_k} + \sqrt{S - x_k}$ has the same value for $1 \le k \le n$, where $S = x_1 + x_2 + \cdots + x_n$.
3. A 30-gon $A_1A_2A_3\cdots A_{30}$ is inscribed in a circle of radius 2. Prove that one can choose a point B_k on the arc A_kA_{k+1} for $1 \le k \le 29$ and a point B_{30} on the arc $A_{30}A_1$, such that the numerical value of the area of the 60-gon $A_1B_1A_2B_2A_3B_3\cdots A_{30}B_{30}$ is equal to the numerical value of the perimeter of the original 30-gon.
4. Five distinct positive integers form an arithmetic progression. Can their product be equal to a^{2008} for some positive integer a?
5. On the infinite chessboard are several rectangles whose sides run along the grid lines. They have no interior points in common, and each consists of an odd number of the squares. Prove that these recangles can be painted in four colours such that two rectangles painted in the same colour do not have any boundary points in common.

Note: The problems are worth 3, 3, 4, 4 and 4 points respectively.