2005 Kaohsiung Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2005/8/3 Kaohsiung

Section I:

In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

1. The sum of a four-digit number and its four digits is 2005. What is this four-digit number ?

Answer:______________

2. In triangle *ABC*, *AB*=10 and *AC*=18. *M* is the midpoint of of *BC*, and the line through *M* parallel to the bisector of $\angle CAB$ cuts *AC* at *D*. Find the length of *AD*.

Answer:______________

3. Let *x*, *y* and *z* be positive numbers such that $\begin{cases} x+y+xy=8, \\ y+z+yz=15, \\ z+x+zx=35. \end{cases}$ Find the value of $x+y+z+xy$.

Answer:________

4. The total number of mushroom gathered by 11 boys and *n* girls is n^2+9n-2, with each gathering exactly the same number. Determine the positive integer *n*.

Answer:___________

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5. The positive integer x is such that both x and $x + 99$ are squares of integers. Find the total value of all such integers x.

Answer:__________

6. The lengths of all sides of a right triangle are positive integers, and the length of one of the legs is at most 20. The ratio of the circumradius to the inradius of this triangle is 5:2. Determine the maximum value of the perimenter of this triangle.

Answer: ____________

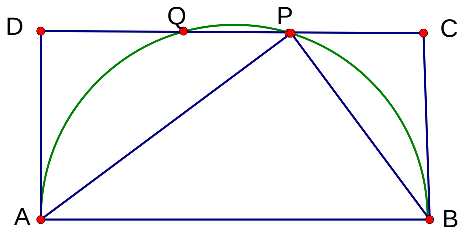
7. Let α be the larger root of $(2004x)^2 - 2003\cdot 2005x - 1 = 0$ and β be the smaller root of $x^2 + 2003x - 2004 = 0$. Determine the value of $\alpha - \beta$.

Answer: ______________

8.Let a be a positive number such that $a^2 + \frac{1}{a^2} = 5$, Determine the value of $a^3 + \frac{1}{a^3}$.

Answer:_____________

9.In the figure, $ABCD$ is a rectangle with AB=5 such that the semicircle on AB as diameter cuts CD at two points. If the distance from one of them to A is 4, find the area of $ABCD$.



Answer:____________

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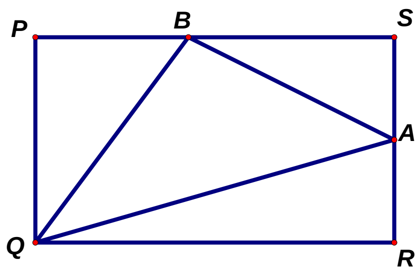
10.Let $a=9\left[n\left(\frac{10}{9}\right)^{n}-1-\left(\frac{10}{9}\right)-\left(\frac{10}{9}\right)^{2}-\cdots-\left(\frac{10}{9}\right)^{n-1}\right]$ where n is a positive integer. If a is an integer, determine the maximum value of a.

Answer: ____________

11. In a two-digit number, the tens digit is greater than the units digit, and the units digit is nonzero. The product of these two digits is divisible by their sum. What is this two-digit number?

Answer: ____________

12. In Figure, $PQRS$ is a rectangle of area 10. A is a point on RS and B is a point on PS such that the area of triangle QAB is 4. Determine the smallest possible value of $PB+AR$.



Answer:____________

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Section II:
Answer the following 3 questions, and show your detailed solution in the space provided after each question. Write down the question number in each paper. Each question is worth 20 points.

1. Let a, b and c be real numbers such that $a+bc=b+ca=c+ab=501$. If M is the maximum value of $a+b+c$ and m is the minimum value of $a+b+c$. Determine the value of $M+2m$.

2. The distance from a point inside a quadrilateral to the four vertices are 1, 2, 3 and 4. Determine the maximum value of the area of such a quadrilateral.

3. We have an open-ended table with two rows. Initially, the numbers 1, 2, …, 2005 are written in the first 2005 squares of the first row. In each move, we write down the sum of the first two numbers of the first row as a new number which is then added to the end of this row, and drop the two numbers used in the addition to the corresponding squares in the second row. We continue until there is only one number left in the first row, and drop it to the corresponding square in the second row. Determine the sum of all numbers in the second row. (For example, if 1, 2, 3, 4 and 5 are written in the first row, at the end, we have 1, 2, 3, 4, 5, 3, 7, 8 and 15 in the second row. Hence its sum is 48.)