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INTERNATIONL MATHENATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2005

Mathematics Contest in Taiwan, Mathematics Short Answer Problems

Name: _____ School: _____ Grade: _____ ID number: _____

Short Answer: there are 20 questions, fill in the correct answers in the answer sheet. Each correct answer is worth 2 points. Time limit: 60 minutes.

1. A foundation has allocated a certain amount of money for 1st, 2nd and 3rd prizes in a competition. The money is divided in the ratio of 3 : 2 where the larger amount is for the 1st prize and the smaller amount is divided again in the ratio of 3 : 2 for the 2nd and 3rd prizes respectively. It becomes known that the 3rd prize is \$3300 less than the first prize. How much is the 2nd prize?

Solution:

Let the total money be x . Then the 1st prize is $\frac{3}{5}x$ and the 3rd prize is $\frac{4}{25}x$.

$$\text{So } \frac{3}{5}x - \frac{4}{25}x = \$3300 \qquad \frac{11}{25}x = \$3300 \qquad x = \$7500.$$

Hence the 2nd prize is $\$7500 \times \frac{2}{5} \times \frac{3}{5} = \text{\textcolor{red}{\$1800}}.$

2. Three man and three children arrive at the river where there is a small boat that will hold one adult or two children. What is the minimum number of trips across the river in either direction to get the family across?

Solution:

Clearly, we must start with two children crossing and one coming back, as starting with an adult crossing means that the adult must come back and two trips are wasted without changing the initial situation. So the sequence is as following:

1. AAAC	CC →	0
2. AAAC	← C	C
3. AACC	A →	C
4. AACC	← C	A
5. AAC	CC →	A
6. AAC	← C	AC
7. ACC	A →	AC
8. ACC	← C	AA
9. AC	CC →	AA
10. AC	← C	AAC
11. CC	A →	AAC
12. CC	← C	AAA
13. C	CC →	AAA
14. C	← C	AAAC
15. 0	CC →	AAAC

3. Mr. Sun has a broken calculator. When just turned on, it displays 0. If the + key is pressed, it adds 35. If the - key is pressed, it subtracts 35. If the \times key is pressed, it adds 91. If the \div key is pressed, it subtracts 91. The other keys do not function. Mr. Sun turns the calculator on. What is the number closest to 2005 that he can get using this calculator?

Solution:

The numbers that can be obtained using the calculator are of the form $35a+91b$ where a and b are integers (possibly negative). Since $35=5\times 7$ and $91=7\times 13$, every number of the form $35a+91b$ is divisible by 7. The closest multiple of 7 to 2005 is $2002=7\times 286$. Since $2002=35\times 52+91\times 2=2002$, the number closest to 2005 that is possible to get using this calculator is **2002**.

4. There are 500 unit cubes. As many of these cubes as needed are glued together to form the largest possible cube which looks solid from any point on the outside but is hollow inside. What is the side length of the largest cube?

Solution:

For the hollow cube, each of the faces would be a square close to $\frac{500}{6} \approx 83$.

The possibilities around 83 are $9^2=81$ and $10^2=100$.

A $11\times 11\times 11$ hollow cube would contain $11^3-9^3=1331-729=602$ unit cubes, so it is not possible.

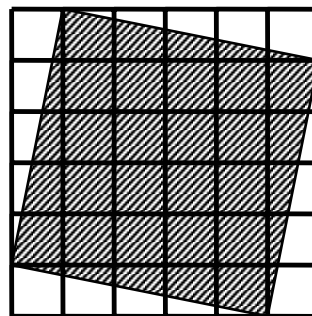
A $10\times 10\times 10$ hollow cube would contain $10^3-8^3=1000-512=488$ unit cubes.

Thus the side length of the largest cube is **10**.

5. What is the ratio of the shaded square to that of the largest square shown in the diagram?

Solution:

The area of the larger square is $6^2=36$ square units. The shaded area is the area of the larger square less the area of 4 triangles with base 5 and height 1. So the shaded area is



$$36 - 4 \times \left(\frac{1}{2} \times 1 \times 5 \right) = 36 - 10 = 26$$

Hence the ratio is $26 : 36 = 13 : 18$.

6. A three-digit number N leaves remainder 3 when divided by 7, remainder 5 when divided by 11 and remainder 8 when divided by 17. What is the number N ?

Solution:

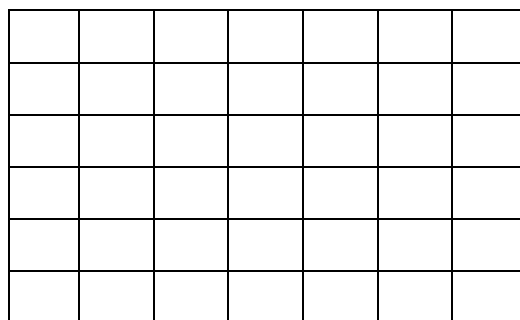
Consider $K=2N+1$. Since N leaves remainder 3 when divided by 7, K is divisible by 7. Similarly, K is divisible by 11 and 17. Hence K is divisible by $7 \times 11 \times 17 = 1309$ as 7, 11 and 17 are relatively prime. Since N is a three-digit number, $K=1309$.

$$\text{Hence } N = \frac{K-1}{2} = 654.$$

7. How many rectangles are there in this grid, where vertices are points of the grid and the edges are lines of the grid?

Solution:

Any rectangle is uniquely determined by pairing a horizontal segment with an orthogonographical segment to get the four edges of a rectangle. We can choose $8 \times 7 \div 2 = 28$ ways of horizontal segments and we can choose $7 \times 6 \div 2 = 21$ ways of orthogonographical segments. This gives a total of $21 \times 28 = 588$ rectangles.



8. A six digit number is represented by \overline{abcdef} , where a, b, c, d, e and f are its

digits. If this number is multiplied by 6, the result is \overline{defabc} . What is this six digit number?

Solution:

Given $\frac{\overline{abcdef}}{\overline{defabc}} \times 6$, we get $a=1$. $\overline{abcdef} \times 6 = \overline{defabc}$, this means

$$(\overline{abc} \times 1000 + \overline{def}) \times 6 = \overline{def} \times 1000 + \overline{abc}. \text{ Thus } 5999 \times \overline{abc} = 994 \times \overline{def} \text{ and}$$

this implies $7 \times 857 \times \overline{abc} = 2 \times 7 \times 71 \times \overline{def}$. So we have

$857 \times \overline{abc} = 142 \times \overline{def}$. Since $a=1$, $\overline{abc}=142$ and hence $\overline{def}=857$. Thus the six digit number is **142857**.

9. If a, b, c and d are positive integers such that

$$a + \frac{1}{b + \frac{d}{c}} = \frac{2005}{101}.$$

What is the value of $a+b+c+d$?

Solution:

$$\frac{2005}{101} = 19 + \frac{86}{101} = 19 + \frac{1}{\frac{101}{86}} = 19 + \frac{1}{1 + \frac{15}{86}}.$$

Hence $a+b+c+d=19+1+86+15=\mathbf{121}$.

10. Alan has a stride 75 cm . If he travels by walking 5 steps forward and one step back, what is the least number of steps he needs to reach a spot 24 metres away?

Solution:

After 6 steps, Alan will finish at 3 metres, after 12 steps at 6 metres, and so on.

So he will take $6 \times 7 = 42$ steps to reach 21 metres, and will only take the four forward steps from there to reach the 24 metres, hence will take $42+4=\mathbf{46}$ steps.

11. N is a positive integer such that N and $N + 97$ are both perfect squares. What is the positive integer N ?

Solution:

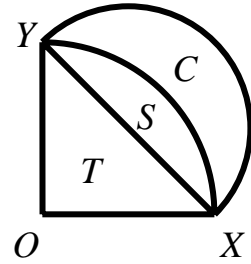
Let $N = p^2$ and $N + 97 = q^2$. Then $97 = q^2 - p^2 = (q - p)(q + p) = 1 \times 97$. Hence $q - p = 1$ and $q + p = 97$, that is $q = 49$ and $p = 48$. Thus $N = 48^2 = \mathbf{2304}$.

12. Five students sit for an exam which has a maximum score of 100. The average of the five scores achieved by the students in the exam was 89. What could the minimum score be gained?

Solution:

The minimum score can be gained by one student when each of the other four students achieve the maximum score. Hence the minimum score is $89 \times 5 - 400 = 45$.

13. In Figure, $OX=OY=10$ are radii of a circular quadrant. A semi-circle is drawn on XY as shown. T , S and C denote the resulting triangles, segment and crescent. What is the area of C ?



Solution:

Since $OX=OY=10$, from Pythagoras Theorem in $\triangle OXY$,

$XY = 10\sqrt{2}$. $T+S$ is a quadrant of a circle radius 10, so

$T + S = \frac{1}{4} \times 10 \times 10 \times \pi = 25\pi$. $S+C$ is a semi-circle with a diameter of $10\sqrt{2}$, so

$S + C = \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} \times \pi = 25\pi$. This means $T+S=S+C$, i.e. $T=C$. Because the

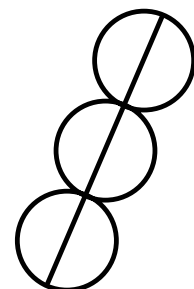
area of T is $\frac{1}{2} \times 10 \times 10 = 50$, the area of C is **50**.

14. A large watermelon weighs 12 kg, with 97% of its weight being water. It is left to stand in the sun, and some of the water evaporates so that now only 90% of its weight is water. What does it now weigh?

Solution:

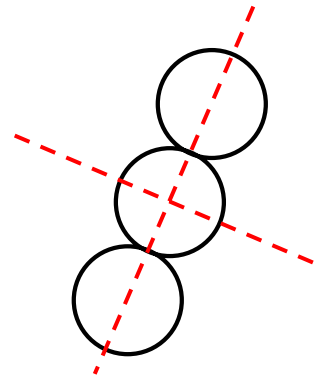
Originally, 97% of the watermelon is water, so the solids are 3% or $12 \times 0.03 = 0.36$ kg. When the water becomes 90% of its weight, these solids become 10% of its weight. Thus its weight then is $0.36 \div 10\% = 3.6$ kg.

15. What is the number of lines of symmetry in the plane of the diagram?

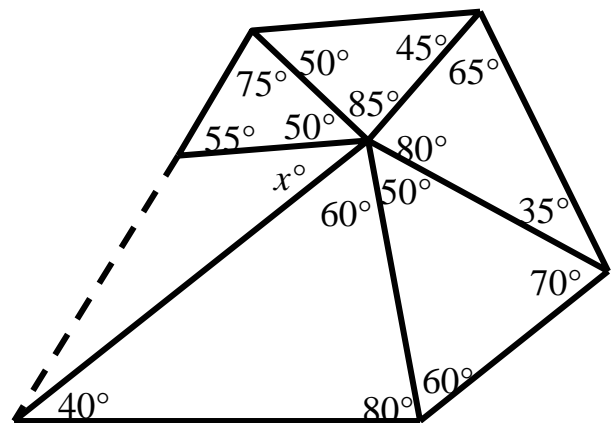
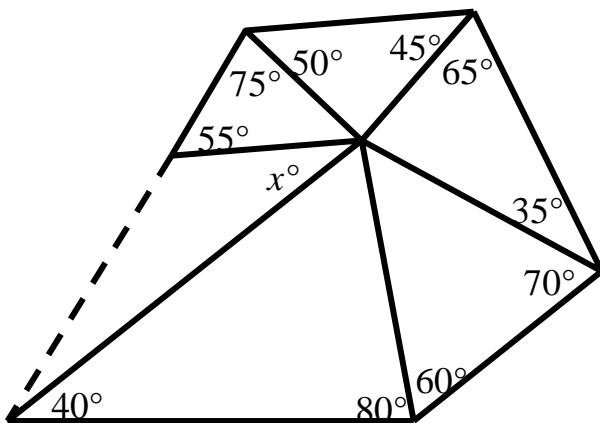


Solution:

In the plane of the diagram, there are **two** axes of symmetry, the two dotted lines as shown in the diagram.



16. What is the value of x in the diagram?



Solution:

Completing the angle sum of the five triangles, we get the angles to be 60° , 50° , 80° , 85° and 50° as shown. Then we have

$$x + 60 + 50 + 80 + 85 + 50 = 360.$$

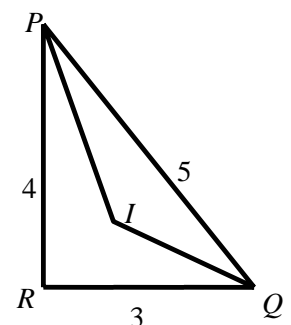
Hence $x = 35$.

17. When $10^{2005} - 2005$ is expressed as a single number, what is the sum of the digits?

Solution:

The number is 999 97995, where the leading 2001 digits are all 9, so the sum of all digits is $9 \times 2001 + 7 + 9 + 9 + 5 = 18039$.

18. The length of the sides of a triangle PQR are $PQ=5$, $QR=3$ and $RP=4$. The bisectors of the angles P and Q meet at the point I . What is the area of the triangle PQI ?

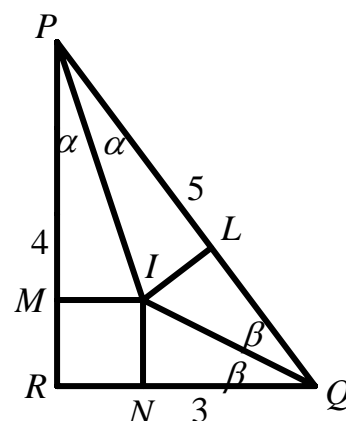


Solution:

Draw the perpendiculars from I to each of the sides PQ , QR and RP and intersect at L , N and M respectively. It follows that the pairs of triangles PIL , PIM and QIL , QIN are congruent, as they have two angles and corresponding side equal. Thus $IL=IN=IM$ and we can get

$$MI=IN=MR=RN=\frac{(4+3-5)}{2}=1.$$

Hence the area of the triangle PQI is $5 \times \frac{1}{2} = 2.5$.



19. A cube with edge of length 10 is painted. The cube is then divided into 1000 unit cubes. Among these small cubes, how many cubes which have one or two painted faces?

Solution:

<method 1>

The numbers of small cubes which have exactly one painted face is $8^2=64$ on each of the 6 faces, i.e. $64 \times 6=384$ cubes.

Also, the number of small cubes which have exactly two painted face is 8 on each of the 12 edges, i.e. $8 \times 12=96$ cubes.

Hence there are $384+96=480$ cubes.

<method 2>

There are $8^3=512$ cubes without painting and 8 cubes painting three faces. So there are $10^3-8^3-8=1000-512-8=480$ cubes having one or two painted faces.

20. In the 5×5 square the numbers 1, 2, 3, 4 and 5 are arranged in such a way that every number occurs precisely once in each column. In the 5×5 square shown, what is the entry in the position marked with ?

1	2			
				1
		4		
2		5		
	5			4

Solution:

The third entry in the first row must be 3, then the third column completes to 3, 2, 4, 5, 1 and the last row to 3, 5, 1, 2, 4. The first column can now be completed to 1, 4, 5, 2, 3. And so on, we get the 5×5 square as shown. Hence the must be 1.

1	2	3	4	5
4	3	2	5	1
5	1	4	3	2
2	4	5	1	3
3	5	1	2	4