INTERNATIONL MATHEMATICS AND SCIENCE OLYMPIAD FOR PRIMARY SCHOOLS (IMSO) 2006

Mathematics Contest in Taiwan, Exploration Problems

Name:________ School:_______ Grade:______ ID number:_________

Answer the following 5 questions, and show your detailed solution in the answer sheet. Write down the question number in each paper. Each question is worth 8 points. Time limit: 60 minutes.

1. The solution to each clue of this crossnumber is a two-digit number. None of these numbers begins with zero. Complete the crossnumber, stating the order in which you solved the clues and explaining why there is only one solution.

Clues Across

1. A square number
3. A multiple of 11

Clues Down

1. A multiple of 7
2. A cube number

1.	2.
3.	

2. Notice that $2^2 + 2^2 = 2^3$, so two squares can sum to give a cube; however, the two squares here are equal (to 4).
   (a) Find two unequal squares whose sum is a cube.
   (b) Show that there are infinitely many pairs of unequal squares whose sum is equal to a cube.

3. Is it possible to find a number 11…11 that is divisible by 19?

4. The menu in the school cafeteria never changes. It consists of 10 different dishes. Peter decides to make his school lunch different everyday (at least 1 dish). For each lunch, he may eat any number of dished, but no two are identical.

(a) What is the maximum numbers of days Peter can do so?

(b) What is the total number of dishes Peter has consumed during this period?

5. A sequence of shapes is made as follows.

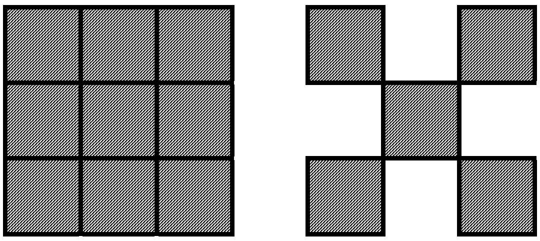
(1) Shape S_1 is a shaded square of side 1 unit.

(2) Shape S_2 is made by dividing S_1 into 9 equal squares and removing four of these, so that only the central and corner squares remain.

(3) Shape S_3 is made by applying the process in (2) to each of the squares of S_2.

(4) Shape S_4 is made by applying the process in (2) to each of the squares of S_3.

And so on.



S_1 S_2

(a) Find the area and perimeter of shape S_3, giving your answers as fractions.

(b) Find the least value of k for which the shape S_k has the area less than $\frac{1}{30}$ and also has perimeter greater than 30.