Junior Division

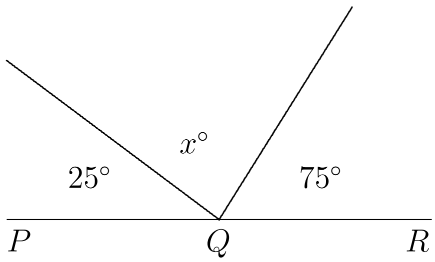
1. $37 - 16 = 21$

hence (D).

2. $\dfrac{6 \times 7}{3} = 2 \times 7 = 14$,

hence (B).

3. (Also I2)
PQR is a straight line, so $x + 25 + 75 = 180$.



So $x = 180 - 100 = 80$.

hence (C).

4. Five hours after 11 am is 4 pm,

hence (C).

5. The largest and smallest of the five numbers 3.1, 0.6, 3, 6, 0.5 are 6 and 0.5, with sum 6.5,

hence (E).

6. His result is $100 + 300$ while the correct answer is $97 + 298$. 100 is 3 more than 97 and 300 is 2 more than 298. Hence his answer is too big by $3 + 2$ and so he must subtract 5,

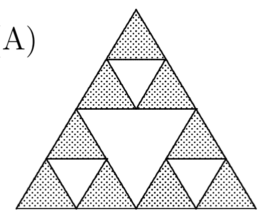
hence (C).

7. The cost of five rows of mangoes with six in each row is \$46. So 30 mangoes will cost \$46 and 3 mangoes will cost $\$45 \div 10 = \$4.60$,

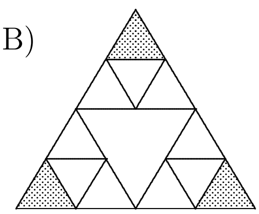
hence (D).

8. 16 of the smallest triangles make up the largest triangle.

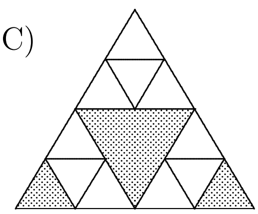
(A)

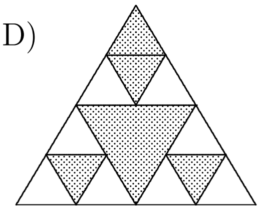
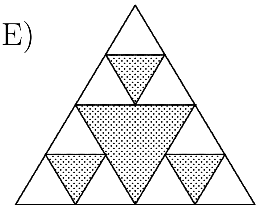


(B)



(C)



(D)  (E) 

The middle sized triangle is made up of 4 of the smallest triangles. So looking at the areas shaded, (A) has $\frac{9}{16}$ shaded, (B) has $\frac{3}{16}$ shaded, (C) has $\frac{6}{16}=\frac{3}{8}$ shaded, (D) has $\frac{8}{16}=\frac{1}{2}$ shaded, and (E) has $\frac{7}{16}$ shaded,

hence (C).

9. If $97+a=100+b$, then $a=100-97+b=3+b$

hence (A).

10. (Also I5 & S4)
The fractions $\frac{7}{15}$, $\frac{3}{7}$ and $\frac{4}{9}$ are each less than $\frac{1}{2}$. The fraction $\frac{6}{11}>\frac{1}{2}$ and is thus the largest,

hence (C).

11. Shorty catches 2 out of every 5 they catch. Since they together catch 60 mice, Shorty catches $\frac{2}{5}\times 60=24$ mice,

hence (B).

12. The class had 30 students, with 17 getting 100% on Monday, and 18 getting 100% on Tuesday, so there are 35 100% scores. Since there are 30 students in the class, at least 5 students must have obtained 100% in both tests,

hence (B).

13. (Also S12 & I12)
Using the rules that each row, column and samll square contains 1, 2, 3 and 4, we fill in the squares one at a time, then

	2		
			1
	1	3	
4			

$\longrightarrow$

1	2		
			1
	1	3	
4	3		

$\longrightarrow$

1	2		
	4		1
2	1	3	
4	3		

$\longrightarrow$

1	2		3
3	4		1
2	1	3	4
4	3		

$\longrightarrow$

1	2	4	3
3	4	2	1
2	1	3	4
4	3	1	2

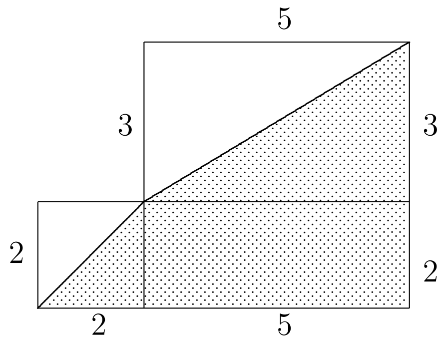
The sum of the numbers in the four corners is $1+4+2+3=10$,

hence (E).

14. The first multiple of 6 over 100 is $6 \times 17 = 102$. The last multiple of 6 under 1000 is $6 \times 166 = 996$.
So, the number of multiples of 6 between 100 and 1000 is
$166 - 17 + 1 = 150$,

hence (B).

15. (Also I10)
The shaded area is half the 2×2 square, plus the 2×5 rectangle, plus half the 3×5 rectangle.



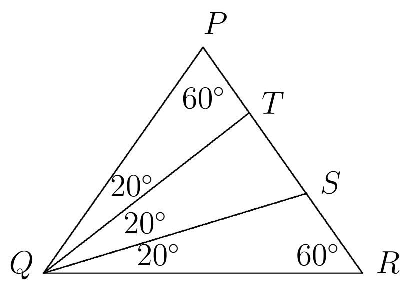
So the shaded area is $\frac{1}{2} \times 4 + 10 + \frac{1}{2} \times 5 \times 3 = 19.5\,\text{cm}^2$,

hence (D).

16. If Bea is 2, then Dee is 6.
Now Eve and Cec are 1 and 5 respectively, or 3 and 7 respectively. In each case, the highest of the remaining three ages is equal to the sum of the other two.
Hence Bea is 1.
It follows that Dee is 3 while Eve and Cec are 2 and 6 respectively. Now Fie is 7, Geo is 4 and Ace is 5.

hence (D).

17. (Also S7)
As QT and QS trisect $\angle PQR$ we get the angles as shown in the diagram.



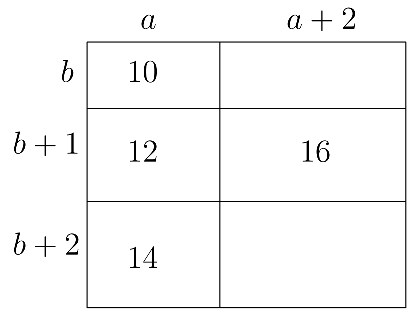
$\angle QTS = 60° + 20° = 80°$ (exterior angle $\triangle QPT$),

hence (C).

18. Jim's average score in his first six matches was 8.5, so his total score for 6 matches was $6 \times 8.5 = 51$. $5 + 9 + 9 + 9 + 9 + 9 = 50 < 51$, so his highest score is at least 10. Now, $5 + 9 + 9 + 9 + 9 + 10 = 51$, so his highest score is 10,

hence (B).

19. let the dimesnsions of the top left rectangle be a and b units, and the dimensions of the top right rectangle be a and c units. Then the other dimensions are as shown on the diagram.



Since the dimensions are all whole numbers, and the perimeter of the top left rectangle is 10 units, then $a + b = 5$.

The perimeter of the largest rectangle is $2(b+b+1+b+2+a+a+2) = 6b+4a+10$

Now $b = 5 - a$, so the perimeter is $6(5 - a) + 4a + 10 = 40 - 2a$

Since $a + b = 5$ and a is a whole number, the largest a can be is 4, so the smallest perimeter is $40 - 8 = 32$ units,

hence (B).

20. (Also S8)
The possibilities are

(A) 13 & 14. $14 = 2 \times 7$ which has 4 factors.
(B) 19 & 20. $20 = 2^2 \times 5$ which has 6 factors.
(C) 37 & 38. $38 = 19 \times 2$ which 4 factors.
(D) 43 & 44. $44 = 2^2 \times 11$ which has 6 factors.
(E) 53 & 54. $54 = 2 \times 3^3$ which has 8 factors.

Andy's age has 8 factors,

hence (E).

21. (Also I21)
Suppose Rachel was born last century in $19ab$. Then

$$\begin{aligned} 2007 - (1900 + 10a + b) &= 2(1 + 9 + a + b) \\ 107 - 10a - b &= 20 + 2a + 2b \\ 87 &= 12a + 3b \\ 29 &= 4a + b. \end{aligned}$$

Testing possible digits for a gives $a = 7, b = 1$ or $a = 6, b = 5$ or $a = 5, b = 9$. This gives the years 1971, 1965 and 1959 which satisfy the condition.

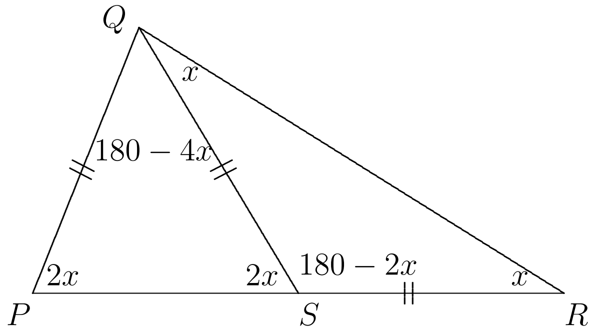
If Rachel was born this century, assume she was born in 200a. Then

$$\begin{aligned} 2007 - 200a &= 2(2 + 0 + 0 + a) \\ 7 - a &= 4 + 2a \\ 3a &= 3 \quad \text{and} \quad a \;=\; 1. \end{aligned}$$

So 2001 also satisfies as a birthdate, so there are 4 possibles years in which she could have been born,

hence (D).

22. Let $\angle SRQ = x°$. Then $\angle SQR = x$, $\angle QSP = \angle QPS = 2x$.



Now $\angle PQR = 180 - 3x$, so will be its largest when $x = 1$ and this value is 177°, hence (D).

23. (Alternative 1)
Let $10a + b$ be a two-digit number, where a and b are digits.

We are given $10a + b = 3ab$.
Then b must be divisible by a. Hence $b = ka$, where k is an integer.
Then $10a + ka = 3a^2k$. Cancelling by a yields $10 + k = 3ak$.

It follows that k divides 10, so k equals 1, 2 or 5.
If $k = 1$, then $11 = 3a$, which is not possible.
If $k = 2$, then $12 = 6a$, so $a = 2$ and $b = 4$.
If $k = 5$, then $15 = 15a$, so $a = 1$ and $b = 5$.

So there are two such numbers, 15 and 24,

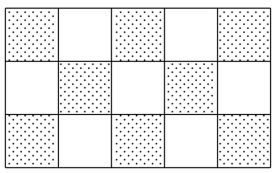
hence (C).

(Alternative 2)
Let $10a + b$ be a two-digit number, where a and b are digits.

We are given $10a + b = 3ab$.
This gives $b = \dfrac{10a}{(3a-1)}$.
Substituting for A, $a = 1$ givesd $b = 5$; $a = 2$ gives $b = 4$, and no other value of a from 3 to 9 gives a whole number for b.
So there are two such numbers, 15 and 24,

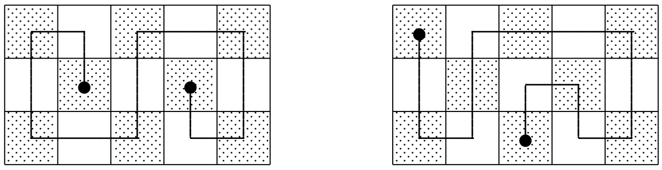
hence (C).

24. (Also S19)
Colour the 15 squares black and white in the usual chessboard fashion with dark squares at all the corners. Then there are 8 black squares and 7 white squares.



Since the squares alternate in colour along the counter's path, the starting square and the end square have to be black. Hence it is impossible for any of the 7 white squares to be starting squares.

There are only three non-equivalent positions of black squares, the corner ones, the interior ones and the ones on the edge but not in a corner. The following two paths show that any black square could have been a starting square.



So there are 8 starting squares,

hence (D).

25. (Also I20)
We are given the following statements:

1. Andrew says that Bill is a liar.
2. Bill says that Clair is a liar.
3. Clair says that Daniel is a liar.
4. Daniel says that Eva is a liar.

Assume that Andrew is a liar.
Then from 1, Bill must tell the truth.
Then, from 2, Clair must be a liar.
Then, from 3, Daniel must tell the truth.
Then, from 4, Eva must be a liar. This gives 3 liars and 2 who tell the truth.

Assume that Andrew tells the truth.
Then from 1, Bill is a liar.
Then from 2, Clair tells the truth.
Then from 3, Daniel is a liar.
Then from 4, Eva tells the truth.
This gives 2 liars and 3 who tell the truth. So, the maximum number of liars is 3,

hence (C).

26. The last digit of the two-digit number must be either 0, 1, 5 or 6.

Any two-digit number ending with 0 when squared ends with 00.

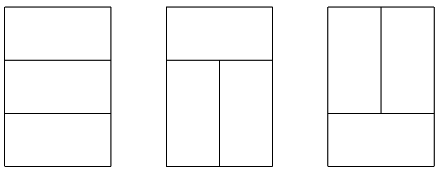
No two-digit number $X1$ when squared ends with $X1$.

There is one two-digit number with units digit 5 when squared ends with that number and that is 25.

There is one two-digit number with units digit 6 when squared ends with that number and that is 76.

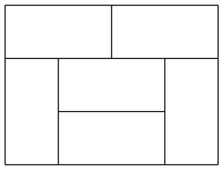
The sum of these two numbers is $25 + 76 = 101$.

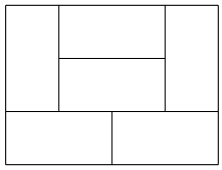
27. A 3 by 4 rectangle can be broken down into two 3 by 2 rectangles. For a 3 by 2 rectangle we have 3 tilings as shown.



For the 3 by 4 rectangle, we can obtain 9 tilings by combining two tilings for the 3 by 2 rectangles, as we can place each of the 3 on the left and each of the 3 on the right to get $3 \times 3 = 9$ different patterns.

However, there are also 2 solutions which cannot be obtained this way. These are called irreducible tilings and they are shown below.





This gives a total of $9 + 2 = 11$ different patterns for the tiling.

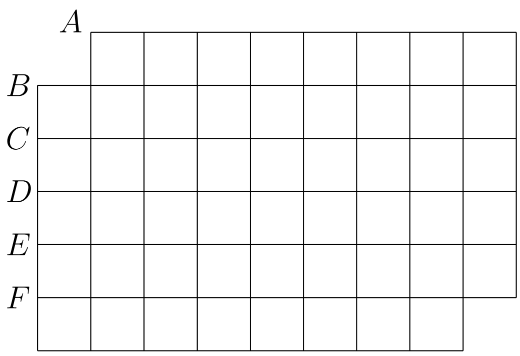
28. (Also I24 & S21)
(Alternative 1)
There are twelve lift stops altogether. Suppose there are six floors. By the Pigeonhole Principle, some floor has at most two lifts stopping there. Each lift connects this floor to two others, so that only four of the other five floors are connected to this floor. If there are seven or more floors, some floor has at most one lift stopping there, and the situation is worse. Hence there are at most five floors.
This is possible if the first lift stops on floors 1, 4 and 5, the second on 2, 4 and 5, the third in 3, 4 and 5, and the fourth on 1, 2 and 3.

So, the maximum number of floors is 5.

(Alternative 2)
For a building with n floors, there are T_{n-1} pairs which have to be connected, where T_{n-1} is the nth triangular number.

Each lift connects 3 pairs (assuming no repeats), so $T_{n=1} \leq 12$.
Now $T_4 = 10$ and $T_5 = 15$, so the largest possible value of n is 5.

29. (Also I29)
(Alternative 1)
Label the points A, B, C, D, E and F as shown.



Consider the 1×1 squares;
Across from A there are 8; from B there are 9, from C there are 9, from D there are 9, from E there are 9, from F there are 8; giving 52 1×1 squares.

Consider the 2×2 squares:
Across from A there are 7, from B there are 8, from C there are 8, from D there are 8 and from E there 7, giving 38 2×2 squares.

Consider the 3×3 squares:
Across from A there are 6, from B there are 7, from C there are 7 and from D there are 6, giving 26.

Consider the 4×4 squares:
Across from A there are 5, from B there are 6 and from C there are 5, giving 16 4×4 squares.

Consider the 5×5 squares:
Across from A there are 3, from B there are 4, giving 8 5×5 squares.

Consider the 6×6 squares, there are 2 across from A and no others.

The total number of squares of all sizes is then $52 + 38 + 26 + 16 + 8 + 2 = 142$.

(Alternative 2)
Add in the two missing squares.

Then for the 1×1 squares there are $6 \times 9 = 54$,
for the 2×2 squares there are $5 \times 8 = 40$,
for the 3×3 squares there are $4 \times 7 = 28$.
for the 4×4 squares there are $3 \times 6 = 18$,
for the 5×5 squares there are $2 \times 5 = 10$,
for the 6×6 squares there are $1 \times 4 = 4$.

This gives a total of 154 squares.

Now take away the squares which include either of the two corners, which is 2 in each case so a total of $2 \times 6 = 12$, and the number of sqaures is $154 - 12 = 142$.

30. If N is any positive integer, we let D be the smallest digit in either N or $7N$. For example, if $N = 34$ then $7N = 238$ and $D = 2$.

Consider single digits N.

N	1	2	3	4	5	6	7	8	9
$7N$	7	14	21	28	35	42	49	56	63
D	1	1	1	2	3	2	4	5	3

So $D = 5$ occurs when $N = 8$. Suppose $D \geq 6$ can occur, then the digits of N can only be 6, 7, 8 or 9. Consider such 2-digit numbers.

N	$7N$	N	$7N$	N	$7N$	N	$7N$
66	462	76	532	86	602	96	672
67	469	77	539	87	609	97	679
68	476	78	546	88	616	98	686
69	483	79	553	89	623	99	693

So, $D = 6$ does occur, but only when $N = 97$ or 98.

Suppose $D = 7$ occurs for a 3 digit number M. The last 2 digits of M and $7M$ are listed in the above table, so, if $D = 7$, M must end in 97. But $7 \times 797 = 5579$, $7 \times 897 = 6279$ and $7 \times 997 = 6979$, so $D \leq 6$ for all 3 digit numbers N. Finally, if $N > 1000$ and N is a candidate for $D = 7$, then $7 \times 10^a < N < 10^{a+1}$ for some a and then $49 \times 10^a < 7N < 7 \times 10^{a+1}$ and so the first digit of $7N$ is less than 7.

Hence largest of the smallest digits in N and $7N$ is 6.

* * *