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2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

World Youth Mathematics Intercity Competition

Individual Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and ID number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All materials will be collected at the end of the competition.

English Version

2009 Durban Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes | 8th July 2009 | Durban, South Africa

Team: ____________ Name: ____________________________ ID No.: ______________

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. If a, b and c are three consecutive odd numbers in increasing order, find the value of $a^2 - 2b^2 + c^2$.

Answer **:__________**

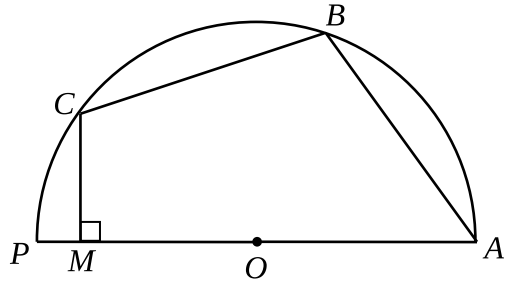
2. When the positive integer n is put into a machine, the positive integer $\dfrac{n(n+1)}{2}$ is produced. If we put 5 into this machine, and then put the produced number into the machine, what number will be produced?

Answer **:__________**

3. Children A, B and C collect mangos.
   A and B together collect 6 mangos less than C.
   B and C together collect 16 mangos more than A.
   C and A together collect 8 mangos more than B.
   What is the product of the number of mangos that A, B and C collect individually?

Answer **:__________**

4. The diagram shows a semicircle with centre O. A beam of light leaves the point M in a direction perpendicular to the diameter PA, bounces off the semicircle at C in such a way that *angle* MCO = *angle* OCB and then bounces off the semicircle at B in a similar way, hitting A. Determine *angle* COM, in degrees.



Answer **:__________**

5. Nineteen children, aged 1 to 19 respectively, are standing in a circle. The difference between the ages of each pair of adjacent children is recorded. What is the maximum value of the sum of these 19 positive integers?

***Answer* :**__________

6. Simplify as a fraction in lowest terms

$$\frac{(2^4+2^2+1)(4^4+4^2+1)(6^4+6^2+1)(8^4+8^2+1)(10^4+10^2+1)}{(3^4+3^2+1)(5^4+5^2+1)(7^4+7^2+1)(9^4+9^2+1)(11^4+11^2+1)}.$$

***Answer* :**__________

7. Given a quadrilateral *ABCD* not inscribed in a circle with *E*, *F*, *G* and *H* the circumcentres of triangles *ABD*, *ADC*, *BCD* and *ABC* respectively. If *I* is the intersection of *EG* and *FH*, and $AI = 4$ and $BI = 3$, find *CI*.

Answer : __________

8. To pass a certain test, 65 out of 100 is needed. The class average is 66. The average score of the students who pass the test is 71, and the average score of the students who fail the test is 56. It is decided to add 5 to every score, so that a few more students pass the test. Now the average score of the students who pass the test is 75, and the average score of the students who fail the test is 59. How many students are in this class, given that the number of students is between 15 and 30?

Answer :__________

9. How many right angled triangles are there, all the sides of which are integers, having 2009^{12} as one of its shorter sides?
Note that a triangle with sides *a*, *b*, *c* is the same as a triangle with sides *b*, *a*, c; where *c* is the hypotenuse.

***Answer* :**__________

10. Find the smallest six-digit number such that the sum of its digits is divisible by 26, and the sum of the digits of the next positive number is also divisible by 26.

Answer :__________

11. On a circle, there are 2009 blue points and 1 red point. Jordan counts the number of convex polygons that can be drawn by joining only blue vertices. Kiril counts the number of convex polygons which include the red point among its vertices. What is the difference between Jordan's number and Kiril's number?

***Answer* :**__________

12. Musa sold drinks at a sports match. He sold bottles of spring water at R4 each, and bottles of cold drinks at R7 each. He started with a total of 350 bottles. Not all were sold and his total income was R2009. What was the minimum number of bottles of cold drink that Musa could have sold?

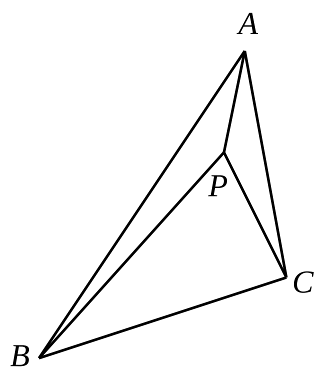
***Answer* :**__________

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. In a chess tournament, each of the 10 players plays each other player exactly once. After some games have been played, it is noticed that among any three players, there are at least two of them who have not yet played each other. What is the maximum number of games played so far?

2. P is a point inside triangle ABC such that *angle PBC* = 30°, *angle PBA* = 8° and *angle PAB* = *angle PAC* = 22°. Find *angle APC*, in degrees.



3. Find the smallest positive integer which can be expressed as the sum of four Positive squares, not necessarily different, and divides $2^n + 15$ for some positive integer n.