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# 2009 Durban Invitational World Youth Mathematics Intercity Competition



## World Youth Mathematics Intercity Competition

### Team Contest

#### Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on the first page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must solve at least one problem by themselves. Each will then have 35 minutes to write the solutions of their allotted problems independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together. Each problem is worth 40 points and complete solutions of problem 1, 2, 6, 7, 8, 9 and 10 are required for full credits. Partial credits may be awarded.
- No calculator, calculating device or electronic devices are allowed.
- Answer in pencil or in blue or black ball point pen.
- All materials will be collected at the end of the competition.

English Version

# 2009 Durban Invitational World Youth Mathematics Intercity Competition



## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

1. The cards 1 to 15 are arranged in a deck, not in numerical order. The top card is placed on the table and the next card is transferred to the bottom of the deck. Now the new top card is placed on top of the card on the table and the next card is transferred to the bottom of the remaining deck. This process is repeated until all 15 cards are on the table. If the cards on the table are now in their natural order, 1 to 15, from top to bottom, what was the fourth card from the bottom in the original deck?

ANSWER: \_\_\_\_\_

# 2009 Durban Invitational World Youth Mathematics Intercity Competition



## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

2. Find the smallest positive integer with at least one factor ending in each of the digits 0 to 9 i.e. at least one factor ends in 0, at least one factor ends in 1, ..., at least one factor ends in 9.

**ANSWER:** \_\_\_\_\_

# 2009 Durban Invitational World Youth Mathematics Intercity Competition



## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

3. Place the digits 1 to 6 in each of the rows and columns as well as the two diagonals such that no digit is repeated in a row, column or diagonal.

2			1		
					4
	2				
				6	
		5			1
3					

2			1		
					4
	2				
				6	
		5			1
3					

**ANSWER:** \_\_\_\_\_

# 2009 Durban Invitational World Youth Mathematics Intercity Competition



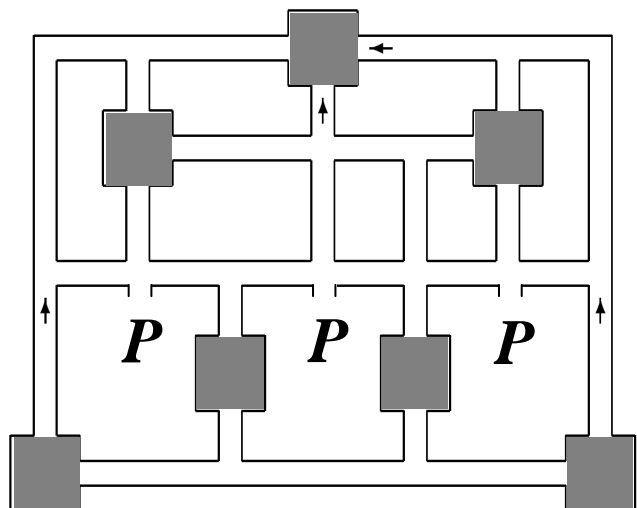
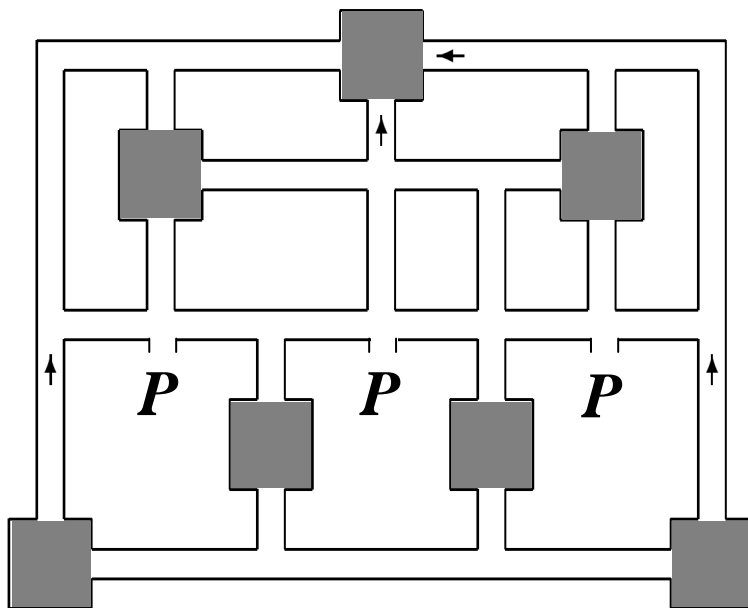
## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

4. We have indicated the positions of three parking areas (indicated by the letter *P*) and seven squares (the shaded areas) on the map of this small town centre. Some of the streets only allow one-way traffic. This is shown by arrows which indicate the direction of traffic up to the first side street. Can you find a route that begins at one of the parking areas, passes through all the squares and ends at another parking area? Make sure that you do not visit any point, including intersection areas, on your route more than once.



ANSWER: \_\_\_\_\_

# 2009 Durban Invitational World Youth Mathematics Intercity Competition



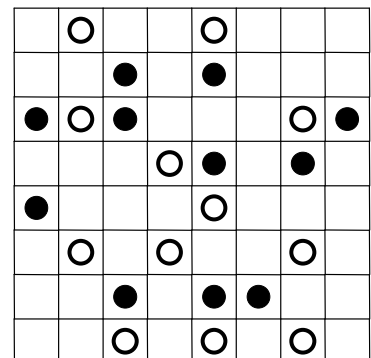
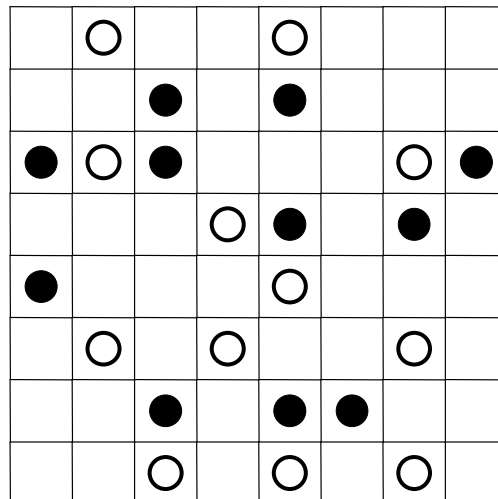
## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

5. In the diagram below, draw a continuous path that begins and ends at the same place and runs through every square exactly once without crossing itself, so that between two consecutive circles on the path, if those circles are the same colour, then they must be joined by one straight line segment and if they are different colours, then they must be joined by two straight line segments which form a right angle. (You may only move horizontally or vertically.)



ANSWER: \_\_\_\_\_

# 2009 Durban Invitational World Youth Mathematics Intercity Competition



## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

6. Let  $a_n = \frac{2^n}{2^{2n+1} - 2^{n+1} - 2^n + 1}$  for all positive integers  $n$ .

Prove that  $a_1 + a_2 + \cdots + a_{2009} < 1$ .

< ***Proof*** >



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## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

7. Find **all the possible** ways of splitting the positive integers into cold numbers and hot numbers such that the sum of a hot number and a cold number is hot and their product is cold.

ANSWER: \_\_\_\_\_

# 2009 Durban Invitational World Youth Mathematics Intercity Competition



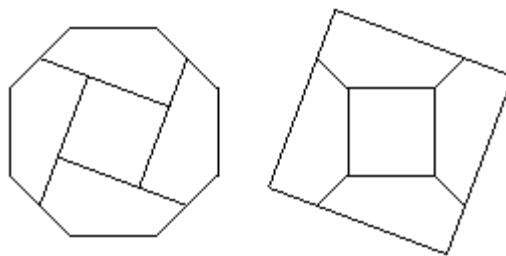
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8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

8. The diagram below shows how a regular octagon may be cut into a  $1 \times 1$  square and four congruent pentagons which may be reassembled to form a square. Determine the perimeter of one of those pentagons.



ANSWER: \_\_\_\_\_

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## Team Contest

8<sup>th</sup> July 2009

Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

9. A game of cards involves 4 players. In a contest, the total number of games played is equal to the total number of players entered in the contest. Every two players are together in at least one game. Determine the maximum number of players that can enter the contest.

**ANSWER:** \_\_\_\_\_

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Durban, South Africa

Team: \_\_\_\_\_ Score: \_\_\_\_\_

10. Which of the numbers 2008, 2009 and 2010 may be expressed in the form

$x^3 + y^3 + z^3 - 3xyz$ , where  $x$ ,  $y$  and  $z$  are positive integers?

ANSWER: \_\_\_\_\_