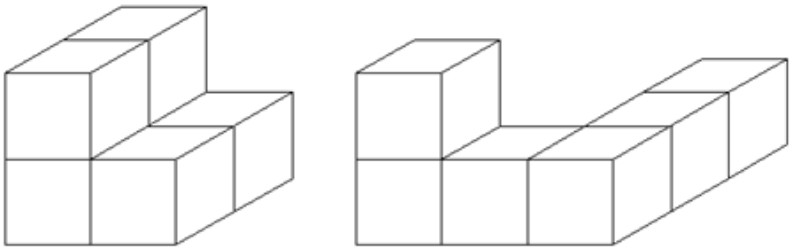
EXPLORATION PROBLEMS

1. Faces of identical dice are labeled by digits 1, 2, 3, 4, 5 and 6. The digit 1 is opposite the digit 6, the digit 2 is opposite the digit 5, and the digit 3 is opposite digit 4. A three-dimensional shape can be formed by putting together a number of these dice in such a way that only two faces with the same digits are glued. Two dice can be put together either horizontally or vertically. After putting the dice together, the digits on the glued faces cannot be seen anymore. If we put the shape on the floor, the digits on the bottom faces cannot be seen as well. In the figure below we see two examples of the shapes.

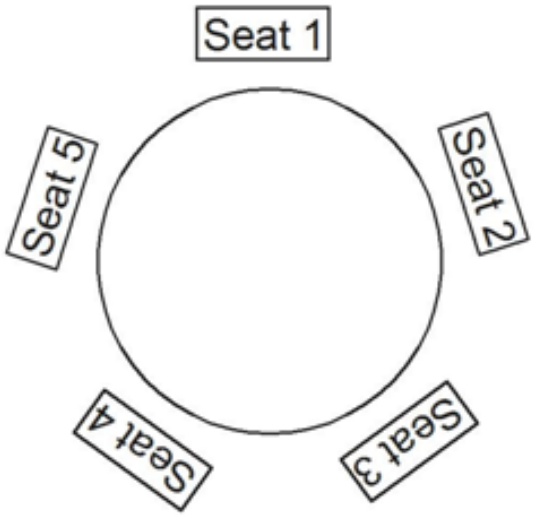


Questions:

(a) Construct a shape by putting together three dice, then place it on the floor so that the sum of the visible digits on the side and the top faces of the shape is minimum.

(b) Construct a shape by putting together four dice, then place it on the floor so that the sum of the visible digits on the side and the top faces of the shape is minimum.

2. Five scientists, Ms. Anna, Mr. Bill, Ms. Chelsea, Mr. Danny, and Mr. Ewing sit at a round table. Determine where each person sits and what his or her profession is, if:

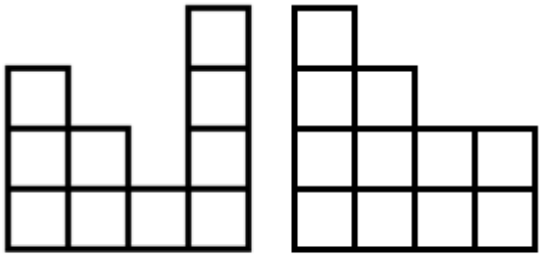


- Ms. Anna sits in Seat 1.
- Ms. Chelsea, who is not the mathematician, sits next to at least one man.
- Mr. Danny is not the physician.
- The biologist and the physician are both men and sit next to each other.
- The astronomer sits to immediate right of a woman.
- The mathematician sits in Seat 3.
- Mr. Bill, who is not the physician, sits two seats away to the right of the chemist.

Seat Number	Name of the Scientist	Profession
1		
2		
3		
4		
5		

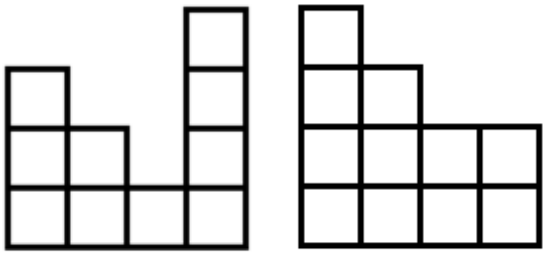
3. A three-dimensional shape is formed from a number of cubes.
Questions:

(a) What is the maximum number of cubes that may be used to form a shape so that the front view and the side view of the shape follow the figures below?



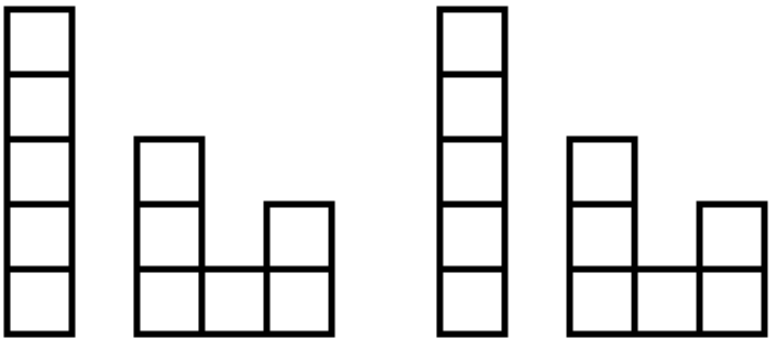
front view side view

(b) What is the minimum number of cubes that may be used to form a shape so that the front view and the side view of the shape follow the figures below?



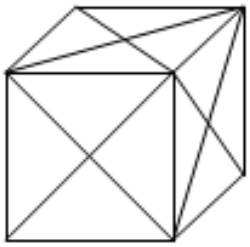
front view side view

(c) What is the minimum number of cubes that may be used to form a shape so that the front view and the side view of the shape follow the figures below?



front view side view

4. The figure shows a cube with sides of 1 unit length on which all 12 face diagonals have been drawn, creating a network with 14 points (the original eight vertices, plus the six points of intersection of diagonals) and 36 segments (the original 12 edges of the cube plus four extra half diagonal segments on each face). Find a shortest path along the segments of the network which passes through all 14 points.
Draw the path that you have found in the given cube net.



5. The digits 1, 3, 4, 5, 6, 7, 8 and 9 are arranged into the following equation

$$2\square \times \square\square\square = \square\square\square\square,$$

such that no digit is repeated more than once.

Find all possible arrangements of the two-digit number, the three-digit number and the four-digit number that satisfy the above equation.

6. An airline has a flight schedule for October 16, 2010, which is shown in the following table.

Flight no	From airport	To airport	Departure time	Arrival time	Flight time (hour)
1	A	B	5:00	7:00	2
2	A	C	7:30	9:00	1.5
3	C	A	9:45	11:15	1.5
4	C	B	10:30	11:30	1
5	A	C	12:00	13:30	1.5
6	B	C	12:15	13:15	1
7	B	A	12:30	14:30	2
8	C	A	14:15	15:45	1.5
9	A	D	16:30	17:15	0.75
10	D	A	18:00	18:45	0.75

A sequence of non-overlapping flights leaving and returning to airport A in which the airport of arrival of a flight is the same as the airport of departure of the flight succeeding it is called a *pairing*. A pairing must satisfy the following labor regulations:

1. The maximum of total flying time for one day is 9 hours.
2. The maximum duty time for one day is 12 hours.
   The duty time is a period of time which consists of flying time of the first flight, transit time 1, flying time of the second flight, transit time 2, flying time of the third flight, ..., flying time of the n^{th} flight ($n \geq 2$).

   The transit time between two non-overlapping consecutive flights x and y is the length of time between the arrival time of the flight x and the departure time of the flight y.

An example of a pairing is given in the table below

Flight	Total flying time (hour)	Duty time (hour)
2-3-5-8	1.5+1.5+1.5+1.5=**6**	1.5+0.75(transit time 1)+1.5+0.75(transit time 2) +1.5+0.75(transit time 3)+1.5=**8.25**

The sequence of flights $1-6$ is not a pairing since it is not returning to airport A. The sequence of flights $6-8$ is not a pairing since it is not starting from airport A. The sequence of flights $1-7-9-10$ is also not a pairing since the duty time is more than 12 hours. Find all possible pairings on October 16, 2010.