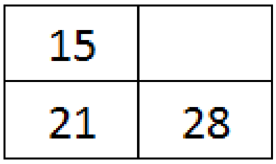
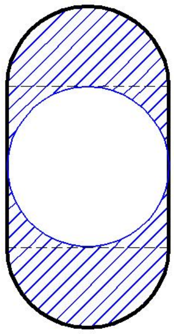
SHORT ANSWER PROBLEMS

1. E is the midpoint of the side $\overline{BC}$ of a triangle ABC. F is on $\overline{AC}$, so that $\overline{AC} = 3\ \overline{FC}$. The ratio of the area of quadrilateral $ABEF$ to the area of triangle ABC is … : … .

2. A rectangle is divided into four rectangles, as shown in the figure below. The areas of three of them are given in cm^2. The area of the original rectangle is … cm^2.

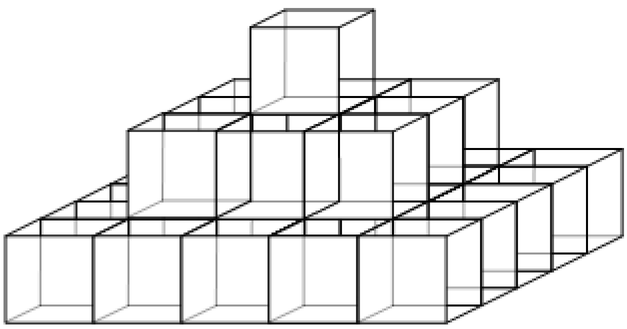


3. A shape on the right is constructed by putting two half circles with diameter 2 cm on the top and on the bottom of a square with 2 cm sides. A circle is inscribed in the square as shown in the figure. The area of the shaded region is … cm^2.



4. The least positive integer a so that $490 \times a$ is a perfect cube number is … .

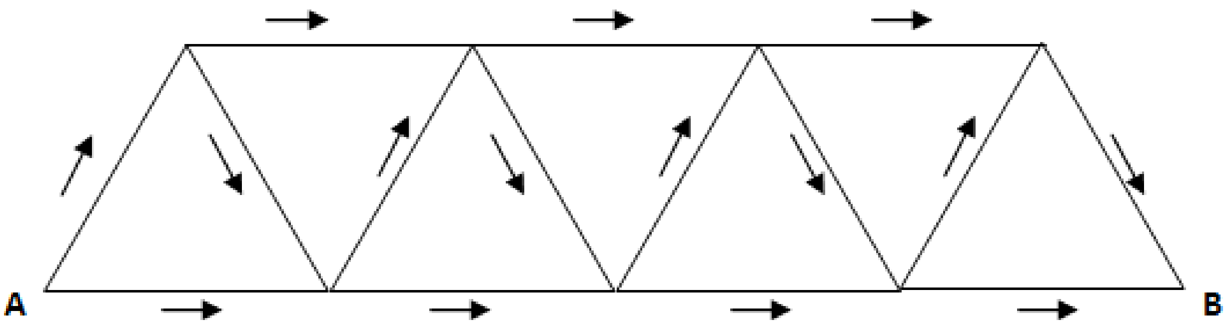
5. A number of cubes are stacked as shown seen in the figure below. The highest level consists of one cube, the second highest level consists of 3×3 cubes, the third highest level consists of 5×5 cubes, and so on.



If there are 2010 cubes to be stacked in this way, such that all levels are complete, some cubes may not be used. There can be as many as … levels.

6. Draw a square inside a circle of radius 1 so that all four vertices lie on the circle. The ratio of the area of the square to the area of the circle is ... : .... [Use $\pi = \frac{22}{7}$.]

7. If we move from point A to B along the directed lines shown in the figure, then the number of different routes is ....



8. The areas of three faces of a rectangular box are $35, 55$ and 77 cm^2. If the length, width and height of the box are integers, then the volume of the box is ... cm^3.

9. In a certain year, the month of January has exactly 4 Mondays and 4 Thursdays. The day on January 1st of that year is ....

10. In an isosceles triangle, the measure of one of its angles is four times the other angle. The measure of the largest possible angle of the triangle, in degree, is ....

11. The average of 2010 consecutive integers is 1123.5. The smallest of the 2010 integers is ....

12. If the length of each side of a cube is decreased by 10%, then the volume of the cube is decreased by ...%.

13. The operations $\circ$ and ⧫ are defined by the following tables.

$\circ$	1	2	3
1	2	1	3
2	2	3	2
3	1	1	3

⧫	1	2	3
1	2	3	2
2	1	2	3
3	3	1	2

For example, $2 \circ 3 = 2$ and 2⧫3 $= 3$. The value of (1⧫2) $\circ\, 3$ is ....

14. The number of zeros in the **end** digits in the product of $1 \times 5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50 \times 55 \times 60 \times 65 \times 70 \times 75 \times 80 \times 85 \times 90 \times 95$ is ....

15. In the addition sentence below

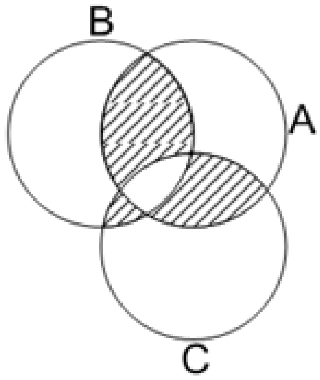
$$ONE + ONE = TWO$$

each letter represents a different digit. The number of all possible digits represented by 'E' is ....

16. A piece of rod is 16 meters long. There is a device that can divide any piece of rod into two equal pieces. We can use this device 8 times. In the end, we will have 9 pieces of rod. The maximum possible difference between the length of the longest piece and the length of shortest piece is ... meters.

17. Rearrange the twelve numbers of the clock $1, 2, 3, \ldots, 12$ around its face so that any two adjacent numbers add up to a triangle number. The triangle numbers are 1, 3, 6, 10, 15, 21 and so on. If 12 is placed in its original position, then the number that should be placed in the opposite position of 12 must be ....

18. In the figure, A, B and C are circles of area of 60 cm^2. One-half the area of A is shaded, $\frac{1}{3}$ area of B is shaded, and $\frac{1}{4}$ area of C is shaded. The total area of the shaded regions is ... cm^2.



19. For any positive integer n, let $d(n)$ be the sum of digits in n.
For example, $d(123) = 1 + 2 + 3 = 6$ and $d(7879) = 7 + 8 + 7 + 9 = 31$.
The value of

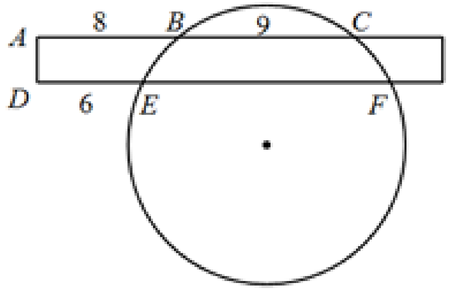
$$d\,(d\,(999\ 888\ 777\ 666\ 555\ 444\ 333\ 222\ 111))$$

is ....

20. The numbers 1447, 1005 and 1231 have something in common. Each is a four-digit number beginning with 1 that has exactly two identical digits. There are ... such numbers.

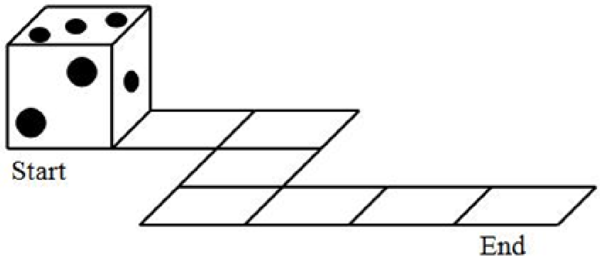
21. Let n be a positive integer greater than 1. By the length of n, we mean the number of factors in the representation of n as a product of prime numbers. For example, the 'length' of the number 90 is 4, since $90 = 2 \times 3 \times 3 \times 5$. The number of odd numbers between 2 and 100 having 'length' 3 is ....

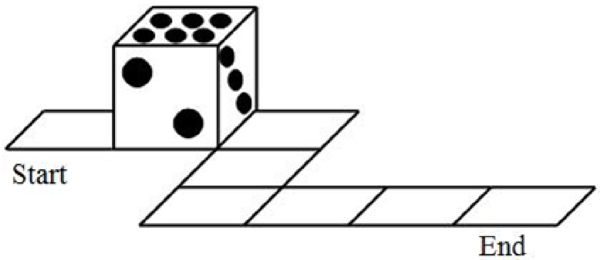
22. A rectangle intersects a circle as shown: $AB = 8\text{cm}, BC = 9\text{cm}$ and $DE = 6\text{cm}$. The length of EF is ...cm.



23. The numbers on each pair of opposite faces on a die add up to 7. A die is rolled without slipping around the circuit shown. At the start the top face is 3.

At the end point, the number displayed on the top face is ....





24. Let A, B and C be three distinct prime numbers. If $A \times B \times C$ is even and $A \times B \times C > 100$, then the smallest possible value of $A + B + C$ is ....

25. The product of $a \times b \times c \times d = 2010$, where a, b, c and d are positive integers and $a < b < c < d$. There are ... different solutions for a, b, c and d.