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Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

English Version

Elementary Mathematics International Contest

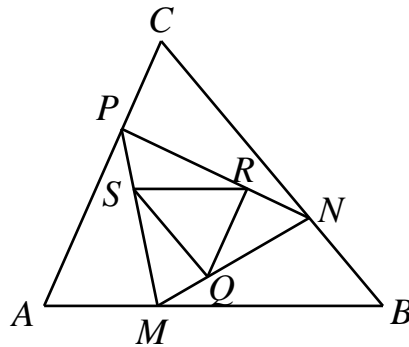
Individual Contest

Time limit: 90 minutes

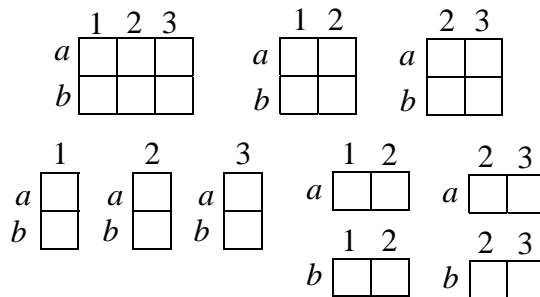
20th July 2011 Bali, Indonesia

1. For any two numbers a and b , $a * b$ means $a + b - \frac{2011}{2}$.
Calculate: $1 * 2 * 3 * \dots * 2010 * 2011$.
2. Suppose 11 coconuts have the same cost as 14 pineapples, 22 mango have the same cost as 21 pineapples, 10 mango have the same cost as 3 bananas, and 5 oranges have the same cost as 2 bananas. How many coconuts have the same cost as 13 oranges?
3. A girl calculates $\frac{1+2}{3} + \frac{4+5}{6} + \dots + \frac{2011+2012}{2013}$ and a boy calculates $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{671}$. What is the sum of their answers?
4. What is the first time between 4:00 and 5:00 that the hour hand and the minute hand are exactly 10° apart?
5. Two squirrels, Tim and Kim, are dividing a pile of hazelnuts. Tim starts by taking 5 hazelnuts. Thereafter, they take alternate turns, each time taking 1 more hazelnut than the other in the preceding turn. If the number of hazelnuts to be taken is larger than what remains in the pile, then all remaining hazelnuts are taken. At the end, Tim has taken 101 hazelnuts. What is the exact number of hazelnuts at the beginning?
6. In how many ways can we pay a bill of \$500 by a combination of \$10, \$20 and \$50 notes?
7. The least common multiple of the numbers 16, 50 and A is 1200. How many positive integers A have this property?

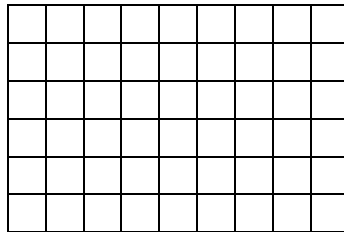
8. In the figure below, $\frac{AM}{MB} = \frac{BN}{NC} = \frac{CP}{PA} = \frac{1}{2}$ and $\frac{MQ}{QN} = \frac{NR}{RP} = \frac{PS}{SM} = \frac{1}{2}$. If the area of $\triangle ABC$ is 360 cm^2 , what is the area of $\triangle QRS$, in cm^2 ?



9. In a 2×3 table, there are 10 rectangles which consist of an even number of unit squares.

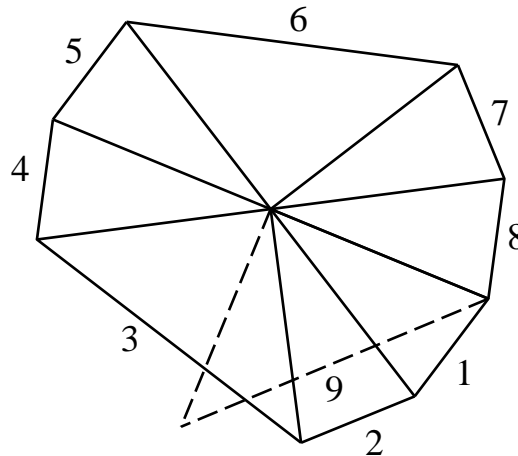


How many rectangles are there in a 6×9 table which consist of an even number of unit squares?

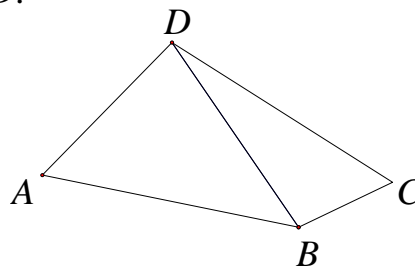


10. Find the smallest positive common multiple of 4 and 6 such that each digit is either 4 or 6, there is at least one 4 and there is at least one 6.

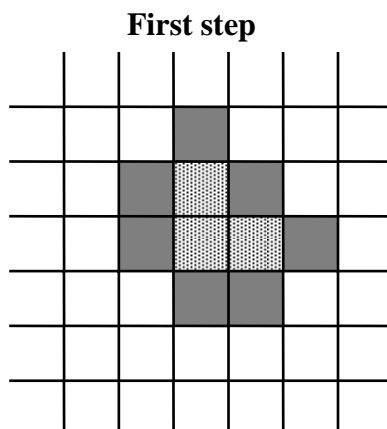
11. We have two kinds of isosceles triangles each with two sides of length 1. The acute triangle has a 30° angle between the two equal sides, and the right triangle has a right angle between the two equal sides. We place a sequence of isosceles triangles around a point according to the following rules. The n -th isosceles triangle is a right isosceles triangle if n is a multiple of 3, and an acute isosceles triangle if it is not. Moreover, the n -th and $(n+1)$ -st isosceles triangles share a common side, as shown in the diagram below. What is the smallest value of $n > 1$ such that the n -th isosceles triangle coincides with the 1-st one?



12. When the digits of a two-digit number are reversed, the new number is at least 3 times as large as the original number. How many such two-digit numbers are there?
13. In the quadrilateral $ABCD$, $AB=CD$, $\angle BCD=57^\circ$, and $\angle ADB + \angle CBD = 180^\circ$. Find the value of $\angle BAD$.



14. Squares on an infinite chessboard are being painted. As shown in the diagram below, three squares (lightly shaded) are initially painted. In the first step, we paint all squares (darkly shaded) which share at least one edge with squares already painted. The same rule applies in all subsequent steps. Find the number of painted squares after one hundred steps.



15. The rows of a 2011×4024 chessboard are numbered from 1 to 2011 from bottom to top, and the columns from 1 to 4024 from left to right. A snail starts crawling from the cell on row 1 and column 1 along row 1. Whenever it is about to crawl off the chessboard or onto a cell which it has already visited, it will make a left turn and then crawl forwards in a straight line. Thus it follows a spiraling path until it has visited every cell. Find the sum of the row number and the column number of the cell where the path ends. (The answer is $3+2=5$ for a 4×5 table.)

