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Self-study Textbook for Junior Secondary School

Algebra

First Term (for Year 7)

Published by Chiu Chang Math Books & Puzzles Co.
Provided by Chiu Chang Mathematics Education Foundation

Content

Explanation of Content

I. This book contains chapters on rational numbers, addition and subtraction of integral expressions, simple equations in one unknown, linear inequalities, simultaneous linear equations in two unknowns, multiplication and division of integral expressions, fractorization and fractions. This book is written for use by Year 7 junior secondary students.

II. The exercises in this book are classified into 3 categories, namely practice, exercise and revision exercise

(1)	**Practice**	For use in class practice.
(2)	**Exercise**	For use in homework.
(3)	**Revsion Exercise**	For use in revision. Some questions marked with "*"are challenge questions for use by able students.

III. This book is written in accordance with the secondary school syllabus set by the Mainland China Education Bureau in 1984.

How to self-study using the book

1. First plough through each Chapter and Section carefully.

2. Digest the examples, then close the book and see if you can work out the solution by yourself.

3. Work out the questions set in the Practice, Exercise and Revision Exercise in your notebook. There is no need to copy the question, but student should mark the page reference and show all working steps and details.

4. Student should form the habit of checking the solution after completing each question. After verifying that the answer is correct, place a " ∨ " mark against the question number for record.

5. When the student encounters a question that he does not understand or cannot work out the answer, he should re-read the explanation in the relevant Chapter and Section until he fully understands it.

6. It is advisable for the students to organize themselves into study group of 4-6 persons. Set target plan to monitor the progress. Meet regularly to discuss and check the progress together.

7. This book will require 3 months time (involving about 200 study hours) to complete the learning of all the topics set in the junior secondary school syllabus. Among the topics, Algebra and Geometry can be studied in parallel.

Note: The units of measurement used in this book include

Units of length:	km (kilometer), m (meter), cm (centimeter), mm (millmeter); 1 km = 1000 m, 1 m = 100 cm = 1000 mm;
Units of weight:	T (Tonne), kg (kilogram), g (gram); 1 T = 1000 kg, 1 kg = 1000 g;
Units of capacity:	kL (kiloliter), L (liter), mL (milliliter); 1 kL = 1000 L, 1 L = 1000 mL.

Chapter 1   Rational Numbers

I.   Definition of Retional Numbers

1.1   Positive numbers and negative numbers

In our daily live, we often count objects individually, by using positive integers 1, 2, 3, … : when conveying a message that there is no object at all, we use the figure 0. But when we measure the length or weight of objects, we often encounter values which are not exact integers, in which case we shall have to express the values in fractions and decimals.

Is it possible to describe every single value using only integer, fraction and decimal? Let us look at the following exampes: On one day, the highest temperature is 5°C above zero, the lowest temperature is 5°C below zero (Diagram 1-1). If we limit our knowledge to what we have learnt in Primary school, we can only describe the two temperatures as 5°C without any differentiation.

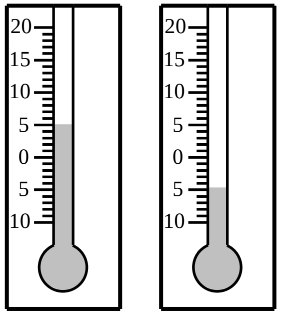


Diagram1-1

Although both values are 5°C, a temperature of 5°C above zero is contrastly different from a temperature of 5°C below zero. Looking at the calibrations on a thermometer, one temperature is above the 0°C marking and the other temperature is below the 0°C marking. To differentiate the two different values, we describe the temperature of 5°C above zero as +5°C (read as positive 5°C); and describe the temperature of 5°C below zero as -5°C (read as negative 5°C). That is to say, we describe a temperature above zero as having positive value, and describe a termperature below zero as having negative value. Using the knowledge we have learnt from Primary school, we describe positive value by prefixing it with a "＋" (read as positive) sign, or leave it without a prefix; we describe negative value by prefixing it with a "－" (read as negative) sign.

There are many examples of objects with contrasting opposite values. For example, location A is 5.2m above sea-leval, and location B is 3.6m below sea-level (Diagram 1-2); Yesterday $8\frac{1}{2}$ T(Tonne) of goods was moved in, today $4\frac{1}{2}$ T of goods is moved out; etc. We can describe the location 5.2 m above see level as +5.2 m, the location 3.6 m below sea level as -3.6 m. For $8\frac{1}{2}$ T of goods moved into a godown, we describe it as $+8\frac{1}{2}$ T, for $4\frac{1}{2}$ T of goods moved out of a godown, we describe it as $-4\frac{1}{2}$ T .

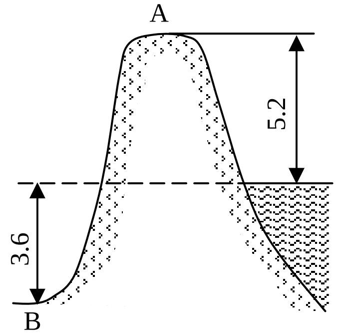


Diagram 1-2

Practice

1. (*Mental*) List some examples with contrasting opposite values.
2. (*Mental*) If we describe moving east 5 km (kilometer) as +5 km, then how to describe moving west 6 km?
3. (*Mental*) If we describe having descended 400 m as -400 m, then how to describe having ascended 800 m?
4. (*Mental*) If we describe an account balance of 10.32 dollars as $+10.32$ dollars, then how shall we describe a loss of 4.15 dollars?

Numbers like $+5$, $+8\frac{1}{2}$, $+5.2$ which carry a positive sign are called **positive numbers** (the positive sign can be abbreviated). Numbers like -5, $-4\frac{1}{2}$, -3.6 which carry negative sign are called **negative number**. 0 is neither a positive number, nor a negative number, it can be described as a neutral number.

Practice

(*Mental*) Read the following number, and say whether it is a positive number or negative number.

$$+6,\ -8,\ 75,\ -0.4,\ 0,\ \frac{3}{7},\ 9.15,\ -\frac{2}{3},\ +1\frac{4}{5}.$$

【**Example**】 All numbers with positive values fall into the set of positive numbers, all numbers with negative values fall into the set of negative numbers. Classify the following number as positive number and negative number and fill it into the appropriate circle one for the set of positive numbers and the other for the set of negative numbers:

$$-11,\ 4.8,\ +73,\ -2.7,\ \frac{1}{6},\ +\frac{7}{12},\ -8.12,\ -\frac{3}{4}.$$

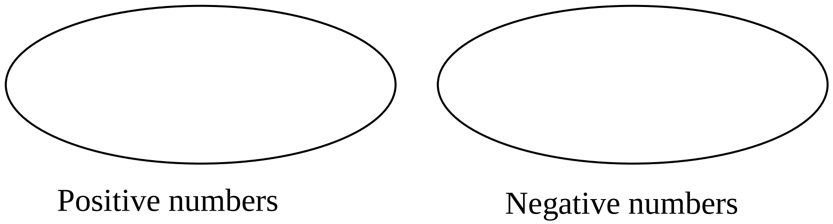


Diagram 1-3

Solution

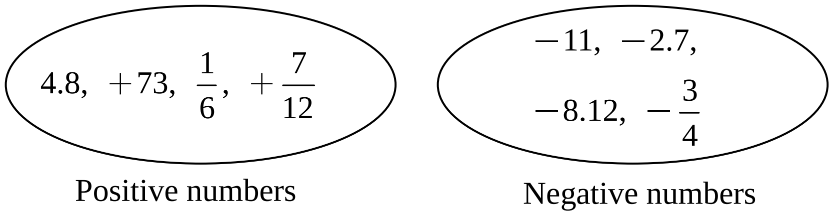


Diagram 1-4

Up till now, the types of numbers we have learnt are:

Positive integers: for example $+1,\ +2,\ +3,\ \ldots$;

Zero: 0;

Negative integers: for example $-1,\ -2,\ -3,\ \ldots$;

Positive fractions: for example $+8\frac{1}{2},\ +5.2$ (that is $+5\frac{1}{5}$), $\frac{2}{3},\ \ldots$;

Negative fractions: for example $-4\frac{1}{2},\ -3.6$ (that is $-3\frac{3}{5}$), $-\frac{6}{37},\ \ldots$;

Positive integers, zero, negative integers are collectively called **integers**; positive fractions, negative fractions are collectively called **fractions**. Integers and fractions are collectively called **rational numbers.**

NOTE:
1. An integer can be regarded as a fraction with denominator 1, therefore the set of fractions includes integers. Sometimes we want to study the difference, we would treat integers and fractions as separate sets, in which case, the set of fractions will not include integers.
2. The term natural number in general use in American does not include 0, whereas in general use in Europe it will include 0. To avoid confusion, it is better to use the term positive integer to mean 1, 2, 3, ...; to use the term non negative number to mean 0, 1, 2, 3, ...

Practice

1. (*Mental*) Examine each number below, is it an integer or fraction? Is it a positive number or negative number?

$$-7,\ 10.1,\ -\frac{1}{6},\ 89,\ 0,\ -0.67,\ 1\frac{3}{5}.$$

2. (*Mental*) List out some positive integers, negative integers, positive fractions, negative fractions.

1.2 Number Axis

In our daily lives, we usually make calibrations on a straight line to indicate values of different magnitudes. For example, the calibration on a thermometer indicates the degree of temperature: the first calibration mark above zero represents 1°C; the second calibration below zero represents -2°C; … Further, the calibration on a ruler indicates lengths of different magnitudes, the calibration on a beam balance indicates weights of different magnitudes, … etc.

Similarly, we can mark points on a straight line to reprent positive and negative numbers.

Draw a straight line, choose a point on the line near the middle and mark it as 0, it is called the **origin**. Regard the right side of it as the positive direction, then the opposite side of it is the negative direction. Mark the positive and negative numbers on the line as shown in diagram 1-5.

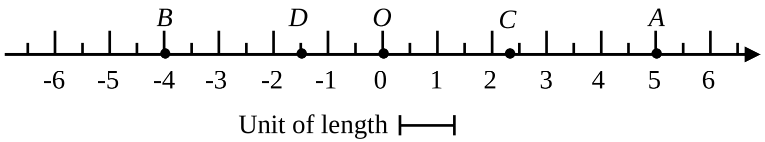


Diagram 1-5

This sort of line with markings of the origin, the positive direction and a unit of length is called a **number axis**.

With this number axis, $+5$ can be marked as point A which is 5 units to the right of the origin, -4 can be marked as point B which is 4 units to the left of the origin, $+2.4$ can be marked as point C which is 2.4 units to the right of the origin, $-1\frac{1}{2}$ can be marked as point D which is $1\frac{1}{2}$ units to the left of the origin, ..., etc. In this manner, all rational numbers can be represented by points on the number axis.

【**Example**】Mark the following number on the number axis:

$$+1,\ -5,\ -2.5,\ +4\frac{1}{2},\ 0.$$

Solution

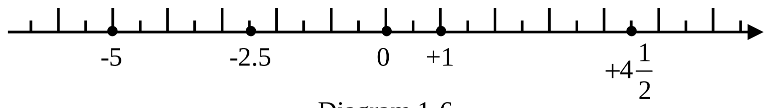


Diagram 1-6

Practice

1. (*Mental*) In the number axis below, what numbers do the points *A*, *B*, *C*, *D*, *E* represent?

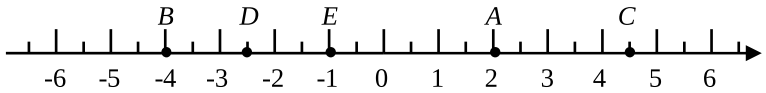


Diagram 1-7

2. Draw a number axis and mark the following number on the number axis:

$$+6,\ 1.5,\ -6,\ 2\frac{1}{2},\ 0,\ 0.5,\ -2\frac{1}{2}.$$

1.3 Opposite numbers

We can see that $+6$ and -6 are two numbers with different signs, one positive and one negative. When marked on the number axis, these two numbrs are represented by two points at equal distance from the origin, one on each side of the origin. The situation for the pair of numbers $2\frac{1}{2}$ and $-2\frac{1}{2}$ is similar.

For two numbers like these which are equal in magnitude but different in signs, we say that one number is the **opposite number** of the other one. +6 is the opposite number of -6, -6 is the opposite number of +6, $+6$ and -6 are mutually opposite numbers. Similarly $2\frac{1}{2}$ and $-2\frac{1}{2}$ are mutual opposite numbers.

The opposite number of 0 is 0.

Practice

1. (*Mental*) What is the opposite number of $+9$? What is the opposite number of -7?
2. (*Mental*) What is the opposite number of -2.4? What is the opposite number of $\frac{3}{5}$?

We know, the values of +2 and 2 are the same, that is to say, $+2 = 2$. Similarly $+(+3)=+3$, $+(-4)=-4$.

-2 is the opposite number of 2, and similarly $-(+3)$ is the opposite number of +3. Therefore $-(+3)=-3$; $-(-4)$ is the opposite number of -4, therefore $-(-4) = 4$.

If we put a "$+$" sign before a number, it does not change the value of the number. But if we put a "$-$" sign before a number, it will turn the number into its opposite number.

$+0 = 0$, $-0 = 0$.

Practice

1. Simplify the sign of the following number:

$$-(+8);\ \ +(-9);\ \ -(-6);\ \ -(+7);\ \ +(+\frac{2}{3}).$$

2. Examine the following pair of numbers, which pair is a pair of equal numbers? Which pair is a pair of opposite numbers?

$+(-8)$ and -8;	$-(-8)$ and -8;
$+(+8)$ and -8;	$-(+8)$ and $+(+8)$;
$-(-8)$ and $+(-8)$;	$-(-8)$ and $+8$;
$-(-8)$ and $+(+8)$;	$+8$ and $+(-8)$.

1.4 Absolute Value

In order to differentiate numbers with contrasting values, we make use of positive number and negative number. For example, one car goes due East for 5 km and a second car goes due West for 4 km. If we regard the eastward direction as positive, we can represent both the direction and distance of the two cars as +5 km and -4 km respectively.

Sometimes we are only interested in the distance covered, regardless of the direction, then we can regard the two cars being at 5 km and 4 km respectively. Here the number 5 is the absolute value of +5, and the number 4 is the absolute value of -4.

We can say, **the absolute value of a positive number is the number itself; the absolute value of a negative number is its opposite number; the absolute value of zero is zero.**

For example, 5 is the absolute value of +5, $-(-4) = 4$ is the absolute value of -4. In the same way, the absolute values of both $\frac{1}{3}$ and $-\frac{1}{3}$ are $\frac{1}{3}$.

Refer to the number axis, we notice that the absolute value of a number is just its distance from the origin.

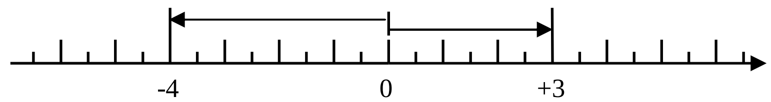


Diagram 1-8

For example, the absolute value of $+3$ is 3 which means that the point representing $+3$ is at a distance of 3 units from the origin; the absolute value of -4 is 4 which means that the point representing -4 is at a distance of 4 units from the origin (Diagram1-8).

To indicate taking the absolute value of a number, the mathematical notation is to put two vertical lines to bracket the number .

For example, the absolute value of +4 is written as $|+4|$, the absolute value of -6 is written as $|-6|$; Use $\left|+\frac{2}{3}\right|$ to represent absolute value of $+\frac{2}{3}$, and use $|-4.5|$ to represent absolute value of -4.5.

【**Example**】 $|+8|=?$ $|-8|=?$ $\left|+\frac{1}{4}\right|=?$ $\left|-\frac{1}{4}\right|=?$

Solution $|+8|=8$; $|-8|=8$; $\left|+\frac{1}{4}\right|=\frac{1}{4}$; $\left|-\frac{1}{4}\right|=\frac{1}{4}$.

Practice

1. (*Mental*) What is the absolute value of the following number?

$$+7,\ -2,\ \frac{3}{4},\ -9.6.$$

2. $|-3|=?$ $\left|+1\frac{1}{2}\right|=?$ $|-1|=?$ $|9|=?$ $|0|=?$ $|-0.4|=?$

1.5 Comparing rational numbers

Comparing $+6$ and $+2$, which number is larger? On the number axis (Diagram 1-9), which of the two numbers $+6$ and $+2$ lies on the right side?

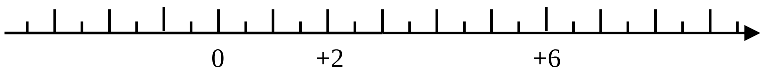


Diagram 1-9

+6 is larger than +2 because on the number axis +6 is on the right side of +2. We can write:

$$+6>+2, \text{ or } +2<+6.$$

Here the "$>$" sign means larger than, and the "$<$" sign means smaller than.

Let us think for a while: Location A is at a height of $+4$ m, and location B is at a height of -10 m (Digram1-10), which location is higher? On the number axis, which of the two numbers $+4$ and -10 is on the right side (Diagram 1-11)?

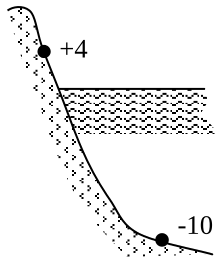


Diagram 1-10

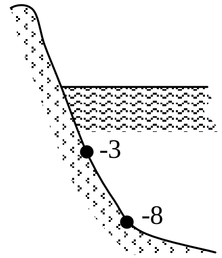


Diagram 1-12

Diagram 1-11

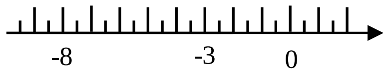


Diagram 1-13

Location A is at a height of -3 m, and location B is at a height of -8 m (Diagram 1-12), which location is higher? On the number axis, which of the two numbers -3 and -8 is on the right side (Diagram 1-13)?

For two rational numbers on the number axis, the number that lies on the right side is larger than the number that lies on the left side.

For example, in Diagrams 1-11 and 1-13, $+4>-10$, $-3>-8$.

Regarding comparison of rational number, we have: **Positive number is larger than zero, negative number is smaller than zero, positive number is larger than negative number; comparing two negative numbers, the one with larger absolute value is smaller.**

Practice

(*Mental*) Compare the magnitude of the following numbers:

10 and 2; -10 and -1; 4 and -12; -3 and 7;
-5 and -20; -18 and 1; 8 and 0; 0 and -100; 0.9 and 1.1;
-0.9 and -1.1; -1.1 and -1.09; $+1.1$ and -1.09.

【Example 1】 Compare the magnitude of $-\frac{2}{3}$ and $-\frac{3}{4}$.

Solution $\because \left|-\frac{2}{3}\right|=\frac{2}{3}=\frac{8}{12}$, $\left|-\frac{3}{4}\right|=\frac{3}{4}=\frac{9}{12}$

Also $\because \frac{8}{12}<\frac{9}{12}$,

$\therefore -\frac{2}{3}>-\frac{3}{4}$.

NOTE: In the above, the symbol "∵" is read as "because"; the symbol "∴" is read as "therefore".

Practice

(*Mental*) Compare the magnitude of each pair of numbers below and explain your reason.

$\frac{7}{10}$ and $\frac{3}{10}$; $-\frac{7}{10}$ and $-\frac{3}{10}$; $\frac{1}{2}$ and $\frac{1}{3}$; $-\frac{1}{2}$ and $-\frac{1}{3}$;
$\frac{1}{5}$ and $\frac{1}{20}$; $-\frac{1}{5}$ and $-\frac{1}{20}$; $\frac{1}{2}$ and $\frac{2}{3}$; $-\frac{1}{2}$ and $-\frac{2}{3}$;
$-\frac{1}{2}$ and $\frac{2}{3}$.

【Example 2】 Use "$>$" to connect the following numbers:

-7, 2, -3.

Solution Arrange the three numbers 2, -3, -7 in descending order. Use "$>$" to connect:

$2>-3>-7$.

Exercise 1

1. Recording the rise of 0.07 m in the water level of a reservoir as $+0.07$ m, how to record the fall of 0.04 m in the water level of reservoir?

2. If we use -50 dollars to mean giving out 50 dollars, what is the meaning of $+200$ dollars?
3. If we regard going north is a positive direction, then what is meant by going -70 m? If we regard going south is a positive direction, then what is meant by going -70 m?
4. Use positive or negative numbers to represent the following contrasting values:
   (1) M Mount Everest stands at 844.43 m above sea level (according to the measurement taken in 2005);
   (2) The deepest location in Pacific Ocean is at 11022 m below sea level.
5. On a certain day, the temperature recorded at the peak weather station at 4 different times of the day are respectively:
   2.2°C below zero, 5.7°C below zero,
   0.4°C below zero, 4.9°C below zero.
   Represent the temperatures using positive and negative numbers.
6. The inflow and outflow record of a food stockhouse are (positive represents inflow):

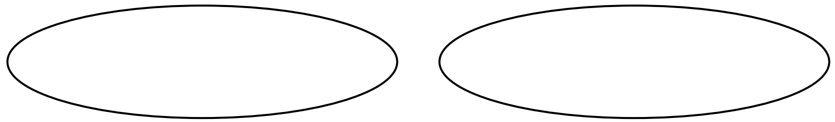
September

Date	14	15	16	17	18	19	20
Inflow (T)	$+82$	-17	-30	$+68$	-25	$+40$	-56

Explain the rational of the daily records.

7. (1) Write down a 3-digit positive number at random;
   (2) Write down a 3-digit negative number at random.
8. The left circle is a set of positive numbers, the right circle is a set of negative numbers, write each number below into the proper circle:

-16, 0.004, $+\frac{1}{8}$, $-\frac{1}{2}$, 9651, 25.8, -3.6, -4, $\frac{3}{5}$.



Positive numbers　　　　Negative numbers

(No. 8)

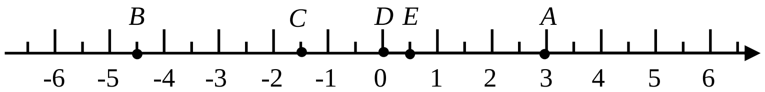
9. Write the number below inside the suitable bracket:

$$1,\ -\frac{4}{5},\ 8.9,\ -7,\ \frac{5}{6},$$
$$-3.2,\ +1008,\ -0.05,\ 28,\ -9.$$

Set of positive integers: {1, }
Set of negative integers: { }
Set of positive fractions: { }
Set of negative fractions: { }

10. Is there a rational number which is not a positive number nor a negative number? If there exists such number(s), how many are there? What are the numbers?

11. In the number axis below, what numbers do the points *A*, *B*, *C*, *D*, *E* represent?



(No. 11)

12. Mark the following numbers on the number axis: $+5.5$, -6, 4, -3.5, 0, 1.5.

13. What is the opposite number of -5? What is the opposite number of $+1$? What is the opposite number of -3? What is the opposite number of 0?

14. What is the number opposite to -1.6? What is the number opposite to -0.2? What number is mutually opposite to $\frac{1}{4}$? Are $\frac{1}{2}$ and -0.5 mutually opposite to each other?

15. Mark on the number axis the points corresponding to the numbers 2, -4.5, 0 and their opposite numbers.

16. Simplify the sign of the following number:
(1) $-(-16)$; (2) $-(+20)$;
(3) $+(+50)$; (4) $-(-3\frac{1}{2})$;
(5) $+(-8.07)$; (6) $-(+\frac{1}{5})$;

17. $|+1|=?$ $|-9|=?$ $\left|-\frac{1}{2}\right|=?$ $|10.5|=?$

18. What is the absolute value of $+3$? What is the absolute value of -3? How many numbers have their absolute value equal to 3? What numbers have their absolute value equal to 4? What numbers have their absolute value equal to 0?

19. $|-5|=-5$ true or false? $|-5|=\frac{1}{2}$ true or false?

20. Compute:
(1) $|-28|+|-17|$; (2) $\left|-5\frac{3}{8}\right|-\left|-3\frac{5}{6}\right|$;
(3) $|-16|\times|-5|$; (4) $|-0.15|\div|-6|$.

21. -5 is larger than -4, true or false? $-\frac{1}{5}$ is larger than $-\frac{1}{4}$, true or false?

22. Compare the magnitude of the following pair of numbers:
(1) -9 and -7; (2) -100 and $+0.01$;
(3) $-\frac{5}{8}$ and $-\frac{3}{8}$; (4) $\frac{4}{5}$ and $\frac{3}{4}$;
(5) -1.9 and -2.1; (6) -0.75 and -0.748;
(7) 0.85 and $-\frac{7}{8}$; (8) $-\frac{3}{11}$ and -0.273.

23. Arrange the three numbers below in ascending order, connect them using the "$<$" symbol:
(1) 3, -5, $-4|$; (2) -9, 16, -11.

24. Compare the magnitude of the following pair of numbers:

(1) $+(-4.8)$ and $-(+4\frac{3}{4})$;

(2) $-(-\frac{3}{4})$ and $-(-\frac{3}{5})$;

(3) $|-4|$ and -4;

(4) $-|-2|$ and $-(-2)$;

(5) $-(-1\frac{1}{3})$ and $\left|+1\frac{2}{3}\right|$;

(6) $-(+3.25)$ and $-|-3.245|$.

25. The following table shows the average January temperature of a certain year of a number of cities in the world, arrange them in descending order according to their temperatures.

Beijing	New York	Taipei	Moscow	Seoul
−4.6°C	3.1°C	15.1°C	−19.4°C	2.4°C

26. The altitude of 4 locations *A*, *B*, *C*, *D* in a coal mine are:

$A(-97.4\text{ m})$, $B(-159.8\text{ m})$,
$C(-136.5\text{ m})$, $D(-71.3\text{ m})$.

Which location is at the highest altitude? which location is at the lowest altitude?

27. The production rate of 4 types of rice, namely type *A*, type *B*, type *C*, type *D* are compared to the production rate of Brand *A* rice (Regard the production rate of Brand *A* rice as positive):

Type A: $+12.4\%$; Type B: -9.8%;
Type C: -6.4%; Type D: $+8.6\%$.

Among the 4 types of rice, which type has the highest production rate? which one has the lowest production rate?

II. Addition and subtraction of Rational Numbers

1.6 Laws for Addition of Rational numbers

Starting from one point and making two moves, what is the result? (assume East is the positive direction) Look at the following examples:

(1) Go east for 5m and then east again for 3m. The result is 8m due east. This is a simple addition.

This is calculating the sum of two east side movements. Using what we have learnt in Primary school, we can answer the question using addition rule:

$$(+5)+(+3)=+8.$$

(2) Go west for 5m and then west again for 3m. The result is 8m due west.

$$(-5)+(-3)=-8.$$

Let us look at the equations (1) and (2) of the two examples, what is the meaning of the + sign in different locations in the equation? What is the relatioship between the sign of the sum and the sign of each number? What is the relationship between the absolute value of the sum and the absolute value of each number?

(3) Go east for 5 m and then west for 3 m (Diagram 1-14).

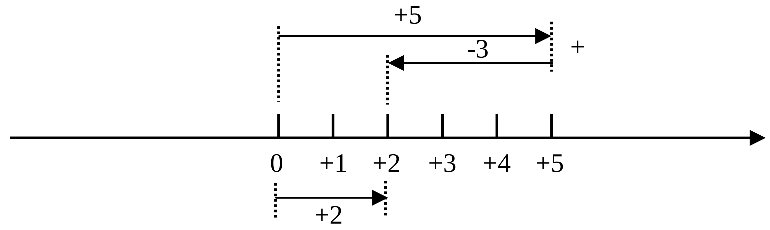


Diagram 1-14

The result is 2 m due east. The reason is that going west for 3 m can be regarded as going east -3 m, therefore once we have learnt rational numbers, we can answer this question using the Law for Addition:

$$(+5)+(-3)=+2.$$

(4) Go east for 3 m and then west for 5 m (Diagram 1-15).

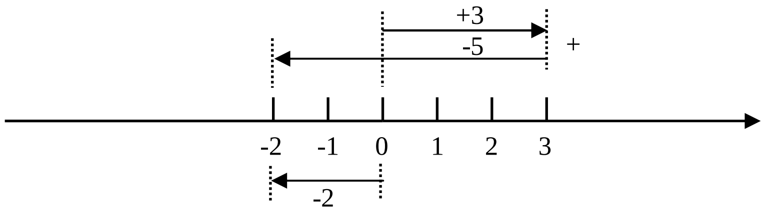


Diagram 1-15

The result is 2m due west. $(+3)+(-5) = -2$.

Let us look at equations (3) and (4) of the two examples, what is the meaning of the plus sign in different locations in the equations? What is the relatioship between the sign of the sum and the plus sign of each number? What is the relationship between the absolute value of the sum and the absolute value of each number?

(5) Go east for 5 m and then west for 5 m. The result is 0 m.

$$(+5)+(-5) = 0.$$

Looking at the above equation, what is the relationship of the two positive numbers?

(6) Go west for 5 m and then east for 0 m. The result is 5 m due west.

$$(-5)+0 = -5.$$

Summarizing the above cases, we have the following Law for Addition of rational numbers:

1. When adding two numbers of the same sign, keep the sign unchanged and add the absolute values of the two numbers together.
2. When adding two numbers with different signs, take the sign as the sign of number with the larger absolute value and subtract the smaller absolute value from the larger absolute value. When adding two opposite numbers, the result is zero.
3. When adding zero to a number, the result is the original number.

Practice

1. (*Mental*) Rise 8 cm, rise further 6 cm, what is the result?
$$(+8)+(+6) = ?$$
2. (*Mental*) Fall 8 cm, fall further 6 cm, what is the result?
$$(-8)+(-6) = ?$$
3. (*Mental*) Rise 8 cm, fall 6 cm, what is the result?
$$(+8)+(-6) = ?$$
4. (*Mental*) Rise 6 cm, fall 8 cm, what is the result?
$$(+6)+(-8) = ?$$

【Example】 Compute: (1) $(-3)+(-9)$;

(2) $(-\frac{1}{2})+(+\frac{1}{3})$.

Solution (1) $(-3)+(-9) = -12$;

(2) $(-\frac{1}{2})+(+\frac{1}{3}) = -\frac{1}{6}$.

Practice

1. (*Mental*) $(+4)+(+7)$; $(-4)+(-7)$; $(+4)+(-7)$; $(+7)+(-4)$; $(+4)+(-4)$; $(+9)+(-2)$ $(-9)+(+2)$; $(-9)+0$; $0+(+2)$; $0+0$.
2. Compute:
$(+12)+(-18)$; $(-42)+(+8)$; $(+84)+(+36)$;
$(-35)+(-25)$; $(-0.9)+(+1.5)$; $(+2.7)+(-3)$;
$(-1.1)+(-2.9)$; $(+2.8)+(+3.7)$; $(+\frac{1}{2})+(+\frac{1}{4})$;
$(-\frac{1}{3})+(+\frac{1}{2})$; $(+\frac{1}{2})+(-\frac{2}{3})$; $(-\frac{1}{4})+(-\frac{1}{3})$.

1.7 Operation Laws for Addition

Compute: $(+30)+(-20)$; $(-20)+(+30)$.
Do you arrive at the same result for the two computations?

Try with two other numbers.

Regarding the addition of rational numbers, there is the following Commutative Law for Addition:

When adding two numbers together, the sum is the same regardless of the order of addition. That is

Commutative Law for Addition: $a + b = b + a$.

Here a, b are any two rational numbers.

Compute: $[(+8)+(-5)]+(-4)$; $(+8)+[(-5)+(-4)]$.

Do you arrive the same result for these two computation?

Try with a different sets of three numbers.

Regarding the addition of rational numbers, there is the following Associative Law for Addition:

When adding three numbers together, the sum is the same if we add the first two numbers first, or add the last two numbers first.

Associative Law for Addition: $(a + b) + c = a + (b + c)$.

Here a, b, c are any three rational numbers.

From the Commutative law of Addition and Associative law for Addition, we can derive the following rule: when adding three of or more rational numbers together, we can add the numbers in any order and under any grouping.

【**Example 1**】Compute $(+16)+(-25)+(+24)+(-32)$.

Solution

$$\begin{aligned}&(+16)+(-25)+(+24)+(-32)\\ &= [(+16)+(+24)]+[(-25)+(-32)]\\ &= (+40)+(-57)\\ &= -17\end{aligned}$$

NOTE: In the above example, we group and add numbers with positive sign and negative sign separately, before adding the subtotals together. This facilitates computation.

【**Example 2**】There are 10 packets of rice, each is marked with reference to a standad weight of 180 kg. If the weight is over 180 kg, the excess will be recorded as a positive number. If the weight is less than 180 kg, the underweight will be recorded as a negative number, the weighting record is as in Diagram1-16. How many kg are the 10 packets of rice being in total over-weight or under-weight? What is the total weight of the 10 packets of rice?

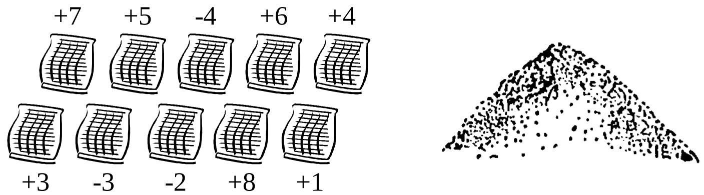


Diagram 1-16

Solution

$$\begin{aligned}&(+7)+(+5)+(-4)+(+6)+(+4)\\ &\quad+(+3)+(-3)+(-2)+(+8)+(+1)\\ &= [(+4)+(-4)]+[(+5)+(-3)+(-2)]\\ &\quad+[(+7)+(+6)+(+3)+(+8)+(+1)]\\ &= 0+0+(+25)\\ &= +25\end{aligned}$$

$$180 \times 10 + 25 = 1825.$$

Answer: They are in total over-weight by 25 kg; The total weight is 1825 kg.

NOTE: In this example, we group the numbers which will add up to zero together, computation is thereby facilitated.

Exercise 2

1. Compute:

(1) $(+3)+(-8)$; (2) $(-3)+(-8)$;

(3) $(+3)+(+8)$; (4) $(-3)+(+8)$;

(5) $(-10)+(+6)$; (6) $(+12)+(-14)$;

(7) $(-5)+(-7)$; (8) $(+6)+(+9)$;

(9) $(-12)+(+18)$; (10) $(-16)+(-9)$.

2. Compute:

(1) $\begin{array}{r} +7 \\ +)\quad -5 \\ \hline \end{array}$ (2) $\begin{array}{r} -6 \\ +)\quad -6 \\ \hline \end{array}$ (3) $\begin{array}{r} -9 \\ +)\quad +8 \\ \hline \end{array}$ (4) $\begin{array}{r} +5 \\ +)\quad -5 \\ \hline \end{array}$

(5) $\begin{array}{r} +\ 4 \\ +)\quad -12 \\ \hline \end{array}$ (6) $\begin{array}{r} +20 \\ +)\quad -\ 8 \\ \hline \end{array}$ (7) $\begin{array}{r} +17 \\ +)\quad +\ 9 \\ \hline \end{array}$ (8) $\begin{array}{r} -10 \\ +)\quad 0 \\ \hline \end{array}$

(9) $\begin{array}{r} 0 \\ +)\quad -7 \\ \hline \end{array}$ (10) $\begin{array}{r} 0 \\ +)\quad 0 \\ \hline \end{array}$

3. In the diagram, add (-3) to each figure in the input box, write the answer in the corresponding output box.

Input				Output
$+5$	$\Rightarrow$	$+(-3)$	$\Rightarrow$	
$+3$	$\Rightarrow$	$+(-3)$	$\Rightarrow$	
$+1$	$\Rightarrow$	$+(-3)$	$\Rightarrow$	
0	$\Rightarrow$	$+(-3)$	$\Rightarrow$	
-1	$\Rightarrow$	$+(-3)$	$\Rightarrow$	
-3	$\Rightarrow$	$+(-3)$	$\Rightarrow$	

(No. 3)

4. For each cell in the table, add the number in the left-most column of the same row and the number in the top-most row of the same column and write the sum in the cell.

$+$	-4	-2	0	$+2$	$+4$
-4		-6			
-2	-6				
0					
$+2$					
$+4$					

(No. 4)

5. Compute:

(1) $(+67)+(-73)$; (2) $(-84)+(-59)$;

(3) $(+33)+(+48)$; (4) $(-56)+(+37)$;

(5) $(+105)+(-76)$; (6) $(-91)+(-24)$.

6. Compute:

(1) $(-0.9)+(-2.7)$; (2) $(+3.8)+(-8.4)$;

(3) $(-0.5)+(+3)$; (4) $(+3.92)+(+1.78)$;

(5) $(+7)+(-3.04)$; (6) $(-2.9)+(-0.31)$.

7. Compute:

(1) $(+\frac{2}{5})+(-\frac{3}{5})$; (2) $(-\frac{1}{3})+(-\frac{2}{3})$;

(3) $(+\frac{1}{2})+(-2\frac{2}{3})$; (4) $(-\frac{1}{2})+(-1\frac{1}{3})$;

(5) $(-\frac{1}{3})+(+\frac{2}{5})$; (6) $(-\frac{5}{6})+(-\frac{1}{8})$.

8. In accordance with the following specification, use the Addition Rule for Rational Numbers to calculate the total number of bags of rice moved in or moved out of the warehouse:

(1) The first day moved in 250 bags, the second day moved out 150 bags;

(2) The first day moved out 100 bags, the second day moved out 120 bags;

(3) The first day moved out 180 bags, the second day moved in 140 bags;

(4) The first day moved in 175 bags, the second day moved in 175 bags.

9. Express the Commutative Law for Addition and Associative Law for Addition using letters, and list one example each using Rational Numbers.

10. Compute:

(1) $(-8)+(+10)+(+2)+(-1)$;

(2) $(+5)+(-6)+(+3)+(+9)+(-4)+(-7)$;

(3) $(-0.8)+(+1.2)+(-0.7)+(-2.1)+(+0.8)+(+3.5)$;

(4) $(+\frac{1}{2})+(-\frac{2}{3})+(+\frac{4}{5})+(-\frac{1}{2})+(-\frac{1}{3})$.

11. First compute the answer to the following two expressions, then compare their magnitudes:

(1) $|(+4)+(+5)|$ and $|+4|+|+5|$;

(2) $|(-4)+(-5)|$ and $|-4|+|-5|$;

(3) $|(+4)+(-5)|$ and $|+4|+|-5|$;

(4) $|(-4)+(+5)|$ and $|-4|+|+5|$.

12. Compute:

(1) $(-17)+(+59)+(-37)$;

(2) $(-18.65)+(-6.15)+(+18.15)+(+6.15)$;

(3) $(-4\frac{2}{3})+(-3\frac{1}{3})+(+6\frac{1}{2})+(-2\frac{1}{4})$;

(4) $(-0.5)+(+3\frac{1}{4})+(+2.75)+(-5\frac{1}{2})$.

13. A person has the following income and outgo on each day in a week (Income is positive):

$+41.28$ dollars, -27.64 dollars, -5 dollars, $+84$ dollars,
-16.8 dollars, -31.09 dollars, $+25.7$ dollars.

After off-setting the income and outgo, what is the total income or outgo in dollars?

14. There are eight baskets of vegetables, with a standard weight of 50 kg per basket, For any overweight basket, the excess weight in kg is recorded with a positive value, for any underweight basket, the deficit weight in kg is recorded with a negative value, The record of the 8 baskets are as follows:

$$+3,\ -6,\ +4,\ -1,\ +2,\ -4,\ -4,\ -5.$$

What is the total overweight or underweight in kg of the 8 baskets of vegetables? What is the total weight in kg of the 8 baskets of vegetables?

1.8 Laws for Subtraction of Rational Numbers

The meaning of substraction is the same as what we have learnt in primary school. Subtraction of rational numbers is the reverse of addition of rational numbers. Subtraction of rational numbers is the process of finding the other addend when you know the sum of two rational numbers and the value of one addend.

Let us look at the following example:

(1) $(+3)-(+5)$.

This computation is to find out what add to $+5$ and the sum is $+3$.

$\because$ $(+5)+(-2)=+3$,

$\therefore$ $(+3)-(+5)=-2$.

From the addition of rational numbers, we know that

$(+3)+(-5)=-2$.

$\therefore$ $(+3)-(+5)=(+3)+(-5)$.

(2) $(+3)-(-5)$.

This computation is to find out what add to -5 gives a sum of $+3$.

$\because$ $(-5)+(+8)=+3$,

$\therefore$ $(+3)-(-5)=+8$.

From the addition of rational numbers, we know that

$(+3)+(+5)=+8$.

$\therefore$ $(+3)-(-5)=(+3)+(+5)$.

Summarizing the above cases, we have the following subtraction rules of rational numbers:

Subtracting a rational number is equivalent to adding its opposite number.

Thus, when computing subtraction of rational numbers, we can change minus sign to add sign, and change the subtrahend to its opposite number, then compute according to the Law for Addition of rational numbers.

【**Example 1**】Compute: (1) $(-3)-(-5)$;

(2) $0-(-7)$.

Solution (1) $(-3)-(-5)=(-3)+(+5)=2$;

(2) $0-(-7)=0+(+7)=7$.

【**Example 2**】How many degrees higher is 7 degrees above zero compared with 3 degrees above zero? How many degrees higher is 7 degrees above zero compared with 3 degrees below zero?

Solution $(+7)-(+3)=(+7)+(-3)=4$;

$(+7)-(-3)=(+7)+(+3)=10$.

Answer: 7 degrees above zero is higher than 3 degrees above zero by 4 degrees；7 degrees above zero is higher than 3 degrees below zero by 10 degrees.

Practice

1. (*Mental*) $(+8)-(+5)$; $(+8)-(-5)$; $(+6)-(+9)$; $(+6)-(-9)$; $(-6)-(+4)$; $(-6)-(-4)$; $(-7)-(+8)$; $(-7)-(-8)$.
2. (*Mental*) $(+9)-(+4)$; $(+9)-(-4)$; $(+4)-(+9)$; $(+4)-(-9)$; $(-9)-(+4)$; $(-9)-(-4)$; $(-4)-(+9)$; $(-4)-(-9)$.
3. (*Mental*) $(+5)-(-5)$; $(-5)-(-5)$; $(+5)-(+5)$; $(-5)-(+5)$; $0-(-5)$; $(-5)-0$; $0-(+9)$; $(+5)-0$.

1.9 Converting a mixed operation of Additions and Subtractions to a pure Addition computation

In this expression $(-20)-(+5)+(+3)-(-7)$，there is a mixture of additions and subtractions。According to the rule for subtraction, it can be re-written as:

$$(-20)+(-5)+(+3)+(+7)\text{[1]}.$$

This changes the whole expression from a mixed operation of additions and subtractions to a pure addition computation.

Hence the mixed operation of additions and subtractions can be re-written uniformly as additions.

We can simplify an expression by abbreviating the addition sign。For example

$$(-20)+(-5)+(+3)+(+7)$$

can be re-written with addition sign abbreviated as follows:

$$-20-5+3+7.$$

It is read as “the sum of negative 20, negative 5, positive 3 and positive 7”. In fact, the expression can be treated as

$$(-20)-(+5)+(+3)+(+7).$$

Therefore the expression can be and read as “negative 20, minus 5, plus 3, plus 7”.

Practice

Change the expression, $(-8)-(+4)+(-6)-(-1)$, from a mixed operation of additions and subtractions to a pure addition computation, and then abbreviate the addition sign.

【**Example**】 Compute:

(1) $12+7-5-30+2$;

(2) $(+\frac{1}{3})-(+\frac{1}{2})+(-\frac{3}{4})-(-\frac{2}{3})$.

Solution (1)
$$\begin{aligned}12+7-5-30+2 &= 12+7+2-5-30\\ &= 21-35\\ &=-14;\end{aligned}$$

(2) $(+\frac{1}{3})-(+\frac{1}{2})+(-\frac{3}{4})-(-\frac{2}{3})$

[1] This way of converting a mixed operation of additions and subtractions into a pure addition computation is sometimes called an **algebraic sum**.

$$=\frac{1}{3}-\frac{1}{2}-\frac{3}{4}+\frac{2}{3}$$
$$=\frac{1}{3}+\frac{2}{3}-\frac{1}{2}-\frac{3}{4}$$
$$=1-1\frac{1}{4}$$
$$=-\frac{1}{4}.$$

NOTE: Here, we applied the Commutative Law for Addition, We know this is possible because in algebra, a mixture of additions and subtractions can be converted to a pure addition.

Practice

Compute: (1) $5-8$; (2) $-4+7-6$; (3) $6+9-15+3$.

Exercise 3

1. Compute: $(+2)-(+4)$; $(+2)-(-4)$; $(-4)-(+2)$; $(-4)-(-2)$; $(+15)-(+7)$; $(+15)-(-7)$; $(-15)-(+17)$; $(-15)-(-17)$.

2. Compute: $(-8)-(+8)$; $(-8)-(-8)$; $(+8)-(-8)$; $(+8)-(+8)$; $0-(+6)$; $(+6)-0$; $0-(-6)$; $(-6)-0$.

3. Compute:

$$\begin{array}{r} +5 \\ -)\ -3 \\ \hline \end{array} \quad \begin{array}{r} -5 \\ -)\ +3 \\ \hline \end{array} \quad \begin{array}{r} +5 \\ -)\ -5 \\ \hline \end{array} \quad \begin{array}{r} -5 \\ -)\ -5 \\ \hline \end{array} \quad \begin{array}{r} -5 \\ -)\ 0 \\ \hline \end{array}$$

$$\begin{array}{r} -5 \\ -)\ +5 \\ \hline \end{array} \quad \begin{array}{r} -5 \\ -)\ -3 \\ \hline \end{array} \quad \begin{array}{r} +5 \\ -)\ 0 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ -)\ -3 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ -)\ +3 \\ \hline \end{array}$$

4. In this diagram, subtract $(+9)$ from each number in the input box, write the answer in the output box.

Input				Output
$+10$	$\Rightarrow$	$-(+9)$	$\Rightarrow$	
$+8$	$\Rightarrow$	$-(+9)$	$\Rightarrow$	
$+5$	$\Rightarrow$	$-(+9)$	$\Rightarrow$	
0	$\Rightarrow$	$-(+9)$	$\Rightarrow$	
-4	$\Rightarrow$	$-(+9)$	$\Rightarrow$	
-9	$\Rightarrow$	$-(+9)$	$\Rightarrow$	

(No. 4)

5. In the diagram, subtract (-9) from each number in the input box, write the answer in the output box.

Input				Output
$+12$	$\Rightarrow$	$-(-9)$	$\Rightarrow$	
$+9$	$\Rightarrow$	$-(-9)$	$\Rightarrow$	
$+6$	$\Rightarrow$	$-(-9)$	$\Rightarrow$	
0	$\Rightarrow$	$-(-9)$	$\Rightarrow$	
-4	$\Rightarrow$	$-(-9)$	$\Rightarrow$	
-9	$\Rightarrow$	$-(-9)$	$\Rightarrow$	

(No. 5)

6. Compute:
   (1) $(+16)-(+47)$; (2) $(+28)-(-74)$;
   (3) $(-37)-(-85)$; (4) $(-112)-(+98)$;
   (5) $(-131)-(-129)$; (6) $(+341)-(+249)$.

7. Compute:

(1) $(+1.6)-(-2.5)$; (2) $(+0.4)-(+1)$;
(3) $(-3.8)-(+7)$; (4) $(-5.9)-(-6.1)$;
(5) $(-2.3)-(+3.6)$; (6) $(+4.2)-(+5.7)$.

8. Compute:

(1) $(+\frac{2}{5})-(-\frac{3}{5})$; (2) $(-\frac{2}{5})-(-\frac{3}{5})$;
(3) $(+\frac{1}{2})-(+\frac{1}{3})$; (4) $(-\frac{1}{2})-(+\frac{1}{3})$;
(5) $(-1)-(-\frac{1}{2})$; (6) $(-1)-(+1\frac{1}{2})$.

9. Compute:

(1) $(+8)+(+6)$; (2) $(+8)-(+6)$;
(3) $(-8)-(-6)$; (4) $(-8)+(-6)$;
(5) $(+8)+(-6)$; (6) $(-8)+(+6)$;
(7) $(-8)-(+6)$; (8) $(+8)-(-6)$;
(9) $(-6)+(+8)$; (10) $0-(-8)$;
(11) $0+(-6)$; (12) $0-(+8)$.

10. Compute:

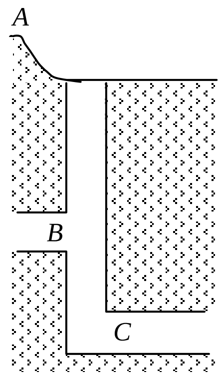
(1) $(+2.9)+(-1.7)$; (2) $(-3.1)-(+7)$;
(3) $(-4.5)-(-2.6)$; (4) $(-0.06)+(-0.47)$;
(5) $(+\frac{1}{2})-(-\frac{1}{3})$; (6) $(-1\frac{1}{5})+(+\frac{3}{5})$;
(7) $(+2\frac{2}{3})+(+1\frac{1}{2})$; (8) $(+\frac{3}{4})-(+\frac{5}{6})$;
(9) $0-(+\frac{2}{3})$; (10) $0-(-\frac{4}{7})$.

11. Within a day, the difference between the highest temperature and the lowest temperature is called "diurnal difference". Compute the diurnal difference in the following table.

Unit of temperature: °C

month/date	12/1	12/2	12/3	12/4	12/5	12/6	12/7	12/8	12/9	12/10
Highest temperature	10	12	11	9	7	5	6	8	7	7
Lowest temperature	2	1	0	-1	-4	-5	-5	-3	-4	-6
Diurnal difference										

12. Refer the diagram, taking the ground level as the reference point, the altitude of point A is $+2.5$ m, the altitude of point B is -17.8 m, the altitude of point C is -32.4 m. How much is point A higher than point B? How much is point B relative to point C? How much higher is point A relative to point C?



(No. 12)

13. Re-write the following expression by abbreviating the addition sign:

(1) $(+10)+(-8)$;
(2) $(-3)-(-7)+(-6)$;
(3) $(+15)+(-30)-(+14)-(-25)$.

14. Compute:

(1) $3-8$; (2) $-4+7$; (3) $-6-9$;
(4) $8-12$; (5) $-15+7$; (6) $0-2$;
(7) $-5-9+3$; (8) $10-17+8$;
(9) $-3-4+19-11$; (10) $-8+12-16-23$.

15. Compute:

(1) $-4.2+5.7-8.4+10$; (2) $6.1-3.7-4.9+1.8$;
(3) $\frac{1}{3}-\frac{2}{3}+1$; (4) $-\frac{1}{4}+\frac{5}{6}+\frac{2}{3}-\frac{1}{2}$.

16. Re-write the following expression abbreviating the addition sign, and compute its value:

(1) $(+12)-(-18)+(-7)-(+15)$;

(2) $(-40)-(+28)-(-19)+(-24)-(-32)$;

(3) $(+4.7)-(-8.9)-(+7.5)+(-6)$;

(4) $(-\frac{2}{3})+(-\frac{1}{6})-(-\frac{1}{4})-(+\frac{1}{2})$.

17. Compute:

(1) $|(+3)-(+4)|$, $|+3|-|+4|$;

(2) $|(-3)-(-4)|$, $|-3|-|-4|$;

(3) $|(-3)-(+4)|$, $|-3|-|+4|$;

(4) $|(+3)-(-4)|$, $|+3|-|-4|$.

18. Compute:

(1) $-216-157+348+512-678$;

(2) $81.26-293.8+8.74+111$;

(3) $-4\frac{2}{3}+1\frac{11}{12}-17\frac{1}{4}-2\frac{17}{18}$;

(4) $2.25+3\frac{3}{4}-12\frac{5}{12}-8\frac{3}{8}$.

III. Multiplication and Division of Rational Numbers

1.10 Laws for Multiplication

Let's read the following problems:

Probem 1: The water level in a pool is rising at a speed of 3 cm per hour, how many cm will it rise in 2 hours?

The problem can be solved by multiplication:

$$3 \times 2 = 6 \text{ (cm)} \quad (1)$$

Water level will rise 6 cm (Diagram1-18).

Problme 2: The water level in a pool is falling at a speed of 3 cm per hour, how many cm will it fall in 2 hours?

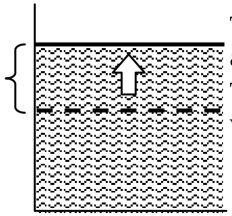


Diagram 1-18

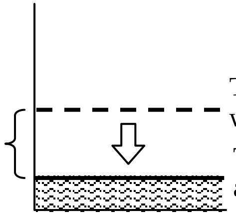


Diagram 1-19

Obviously water level will fall by 6 cm (Diagram1-19).

As mentioned earlier, we use positive number to represent raised level, and negative number to represent fallen level, using multiplication to solve this problem, then the formula should be:

$$(-3) \times 2 = -6 \text{ (cm)} \quad (2)$$

Compare formula (2) with formula (1), we can see that when the factor "3" is replaced by its opposite number "-3", the product "6" is also replaced by its opposite number "-6":

Replace a factor by its opposite number, the product is also replaced by its opposite number.

Following this rule, we compute:

$$3 \times (-2) = ?$$

Compare this with formula (1), when the factor "2" is replaced by its opposite number "-2", applying the above rule, the product "6" is also replaced by its opposite number "-6", i.e.

$$3 \times (-2) = -6 \quad (3)$$

Finally, let us compute:

$$(-3) \times (-2) = ?$$

Compare this with formula (3), here we replace a factor "3" by its opposite number "-3", therefore the product "-6" is also replaced by its opposite number "6", hence

$$(-3) \times (-2) = 6 \quad (4)$$

From Formulas (1)~(4) above, what is the relationship between the sign of the product and the sign of its factors？What is the relationship between the absolute value of the product and the absolute value of the factors?

Furthermore，if one of the factors is replaced by 0, then the product is also 0. For example, $(-3) \times 0 = 0$.

From the above, we can conclude the Law for Multiplication of rational numbers:

When two numbers are multiplied together, the sign of the product is positive if the numbers are of the same sign, the sign of the product is negative if the two numbers are of different signs, the value of the product is equal to the product of the absolute values of the two numbers.
When any number is multiplied by zero, the product is always zero.

Practice

1. Compute:
   $(+8) \times (-5)$; $(-37) \times (-3)$; $(-25) \times (+4.8)$;
   $(-1) \times (-1.5)$; $(+\frac{4}{7}) \times (-\frac{1}{2})$; $(-\frac{5}{12}) \times (-\frac{8}{15})$;
   $(+1\frac{1}{2}) \times (-\frac{2}{3})$; $(-8) \times 0$.
2. When the temperature increases by 1°C, the mercury level in a thermometer rises by 2 mm. What is the rise in mercury level in mm when the temperature rises by 12°C? What is the change in mercury level when the temperature increases by −15°C?

Evaluate whether the product of the following computation is positive or negative?

$$(-2) \times (+3) \times (+4) \times (+5);$$
$$(-2) \times (-3) \times (+4) \times (+5);$$
$$(-2) \times (-3) \times (-4) \times (+5);$$
$$(-2) \times (-3) \times (-4) \times (-5);$$

Can you find out the governing rule?

When several non-zero rational numbers are multiplied together, the sign of the product is dependent on the number of negative factors in the product. When there are in total an odd number of negative factors in the product, the sign of the product is negative; if not, the sign is positive.

【Example 1】 Compute: $(-3) \times (+\frac{5}{6}) \times (-1\frac{4}{5}) \times (-\frac{1}{4})$.

Solution

$$(-3) \times (+\frac{5}{6}) \times (-1\frac{4}{5}) \times (-\frac{1}{4})$$
$$= -3 \times \frac{5}{6} \times \frac{9}{5} \times \frac{1}{4} = -1\frac{1}{8}.$$

NOTE: When several non-zero rational numbers are multiplied together, we first decide on the sign of the product, then multiply the absolute values of the rational numbers together.

Think for a while, how to compute $(+7.8) \times (-8.1) \times 0 \times (-19.6)$?

When several rational numbers are multiplied together, if one of the rational numbers is zero, then the product is zero.

【Example 2】 Compute: (1) $8+5 \times (-4)$;
(2) $(-3) \times (-7)-9 \times (-6)$.

Solution (1) $8+5 \times (-4) = 8+(-20) = -12$;
(2) $(-3) \times (-7)-9 \times (-6) = 21-(-54) = 75$.

NOTE: In a mixed operation with addition, subtraction, multiplication and division, if there is no bracket specifying the order of computation, we compute multiplications and divisions first, before additions and subtractions.

Practice

Compute:

(1) $(-5)\times(+8)\times(-7)\times(-0.25)$;

(2) $(-6)-(-3)\times\frac{1}{3}$;

(3) $(-1)\times(+8)+3\times(-2)$;

(4) $1+0\times(-1)-(-1)\times(-1)-(-1)\times 0\times(-1)$;

(5) $3\times 5\times 7-(-3)\times(-5)\times(-7)-(-3)\times(-5)\times 7+3\times(-5)\times 7$.

1.11 Operation Laws for Multiplication

What are the laws of multiplication we have learnt in primary school?

While the laws of addition that we have learnt in primary school are applicable to rational numbers, would the laws of multiplication be applicable to rational numbers? Let's read the following examples

$$(+5)\times(-6)= -30;\ (-6)\times(+5)= -30.$$

therefore $(+5)\times(-6)=(-6)\times(+5)$.

$$[(+3)\times(-4)]\times(-5)=(-12)\times(-5)=60;$$
$$(+3)\times[(-4)\times(-5)]=(+3)\times(+20)=60.$$

therefore $[(+3)\times(-4)]\times(-5)=(+3)\times[(-4)\times(-5)]$.

Try with a different set of numbers。In general, we have:

In multiplication, even we exchange the positions of the two numbers, the product is still the same.

Commutative Law for Multiplication: $ab = ba$.

In multiplying 3 numbers together, it does not matter whether we multiply the first 2 numbers first or multiply the last 2 numbers first, the product is the same.

The Associative Law for Multiplication: $(ab)c$ =

In this above we write $a\times b$ as ab. In situation where it does not cause any confusion, we usually simplify the expression by replacing the "×" sign by " • ", or even omitting it.

Read another example:

$$5\cdot[(+3)+(-7)]=5\cdot(-4)= -20,$$
$$5\cdot(+3)+5\cdot(-7)=15+(-35)= -20,$$

therefore $5\times[(+3)+(-7)]=5\times(+3)+5\times(-7)$.

Trying with a different set of numbers, you will arrive at the same conclusion. In general, we have:

Multiply a sum of 2 numbers by a third number, is the same as multiplying each of the two numbers by the third number, and then adding up the two products.

Distributive Law for Multiplication: $a(b+c) = ab+ac$.

Practice

Write down the law of operation of the following equation? How can the law be expressed using letters?

1. (*Mental*) $(-4)\cdot 8=8\cdot(-4)$.
2. (*Mental*) $(3+9)+(-5)=3+[9+(-5)]$.
3. (*Mental*) $(-6)\cdot[7+2]=(-6)\cdot 7+(-6)\cdot 2$.
4. (*Mental*) $(5\times 4)\times 6=5\times(4\times 6)$.
5. (*Mental*) $(-8)+(-9)=(-9)+(-8)$.

【Example 1】 Compute: $(\frac{1}{4}+\frac{1}{6}-\frac{1}{2})\times 12$.

Solution

$$(\frac{1}{4}+\frac{1}{6}-\frac{1}{2})\times 12=\frac{1}{4}\times 12+\frac{1}{6}\times 12-\frac{1}{2}\times 12$$
$$=3+2-6$$
$$=-1.$$

【**Example 2**】Compute: $9\frac{18}{19} \times 15$.

Solution $9\frac{18}{19} \times 15 = (10 - \frac{1}{19}) \times 15 = 150 - \frac{15}{19} = 149\frac{4}{19}$.

NOTE: Sometimes we can apply the law of operation to simplify or facilitate the computation.

Practice

Compute: (1) $(-85)(-25)(-4)$; (2) $(-\frac{7}{8}) \times 15 \times (-1\frac{1}{7})$;

(3) $(\frac{9}{10} - \frac{1}{15}) \times 30$; (4) $\frac{24}{25} \times 7$.

1.12 Laws for Division

Similar to what we have learnt about division in Primary school, division of rational numbers is the inverse process of multiplication, division of rational numbers is the process of finding a factor when the product is known and the other factor is known.

Let's see the results of the following different expressions.

From $(+3) \times (+2) = +6$, we get $(+6) \div (+2) = +3$.

From $(+3) \times (-2) = -6$, we get $(-6) \div (-2) = +3$.

From $(-3) \times (+2) = -6$, we get $(-6) \div (+2) = -3$.

From $(-3) \times (-2) = +6$, we get $(+6) \div (-2) = -3$.

Furthurmore, from $0 \times (-2) = 0$, we get $0 \div (-2) = 0$.

Summing up the above, we have the following Law for Division of rational numbers:

In division, the sign of the quotient is positive if the dividend and the divisor both have the same sign, the sign of the quotient is negative if they have different signs, the value of the quotient is obtained from the division of the absolute values of the two numbers.

If the dividend is zero, the quotient is always zero, regardless of the value of the divisor.

NOTE: The divisor cannot be zero.

Practice

1. (*Mental*) $(-18) \div (+6)$; $(-63) \div (-7)$; $(+36) \div (-3)$; $(+32) \div (-8)$; $(-54) \div (-9)$; $0 \div (-8)$.
2. Compute: $(+84) \div (-7)$; $(-96) \div (-16)$; $(-6.5) \div (+0.13)$; $(+8) \div (-0.02)$; $(-\frac{3}{5}) \div (-\frac{2}{5})$; $(-\frac{7}{8}) \div (+\frac{3}{4})$.

In a fraction $\frac{3}{4}$, if we toggle the numerator and denominator, we get an inverse number $\frac{4}{3}$. This is same the quotient of 1 divided by $\frac{3}{4}$. In general, the quotient of 1 divided by a number, is called **the reciprocal of the number**.

For example, the reciprocal number of $\frac{3}{4}$ is $\frac{4}{3}$, the reciprocal number of $\frac{4}{3}$ is $\frac{3}{4}$. Further, the reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$, the reciprocal of 2 is $\frac{1}{2}$, the reciprocal of -2 is $-\frac{1}{2}$.

NOTE: Zero does not have reciprocal. (Why?)

Practice

(*Mental*) Write the reciprocal of the following number:

$\frac{5}{6}$, $-\frac{4}{7}$, 0.2, $\frac{1}{3}$, -5, $1\frac{1}{3}$, -1

By converting the divisor into its reciprocal, we can change a division computation into a multiplication computation.

【**Example 1**】Compute: $-3.5 \div \frac{7}{8} \times (-\frac{3}{4})$.

Solution $-3.5 \div \frac{7}{8} \times (-\frac{3}{4}) = \frac{7}{2} \times \frac{8}{7} \times \frac{3}{4} = 3$.

【**Example 2**】Compute: (1) $8+32\div(-4)$;

(2) $-9 \cdot (-2)-15\div(-3)$.

Solution (1) $8+32\div(-4) = 8+(-8) = 0$;

(2) $-9 \cdot (-2)-15\div(-3) = 18-(-5) = 23$.

Practice

Compute: (1) $-0.25 \div (-\frac{2}{3}) \times (-1\frac{3}{5})$;

(2) $14+56 \div (-7)$;

(3) $-81 \div 3+27 \div (-9)$.

Exercise 4

1. Compute: $(-8) \times (-7)$, $(+12) \times (-5)$,
$(-36) \times (-1)$, $(-25) \times (+16)$.

2. In the diagram，multiply the number in the input (left) box by (-3)，write the answer in the output box (right).

Input		Output
8	⇒ $\times (-3)$ ⇒	
6	⇒ $\times (-3)$ ⇒	
3	⇒ $\times (-3)$ ⇒	
0	⇒ $\times (-3)$ ⇒	
-1	⇒ $\times (-3)$ ⇒	
-5	⇒ $\times (-3)$ ⇒	

(No. 2)

3. For each cell in the table, fill in the product of the left-most column of its row multipled by the number in the top-most row of its column.

×	-3	-2	-1	0	1	2	3
3							9
2						4	
1					1		
0							
-1							
-2							
-3							

(No. 3)

4. Compute:
(1) $2.9 \times (-0.4)$;
(2) $(-30.5) \times 0.2$;
(3) $(+100) \times (-0.001)$;
(4) $(-4.8) \times (-1.25)$;
(5) $(-7.6) \times 0.03$;
(6) $(-4.5) \times (-0.32)$.

5. Compute:
(1) $\frac{1}{4} \times (-\frac{8}{9})$; (2) $(-\frac{5}{6}) \times (-\frac{3}{10})$;
(3) $(-2\frac{4}{15}) \times 25$; (4) $(-0.3) \times (-1\frac{3}{7})$.

6. $(-1) \times (-5)$ =? $-(-5)$ =?
Are $(-1) \times (-5)$ and $-(-5)$ equal?

7. Compute:
(1) $(-2)(+3)(-4)$; (2) $(-6)(-5)(-7)$;
(3) $0.1 \times (-0.001) \times (-1)$;
(4) $(-100) \times (-1) \times (-6) \times (-0.5)$;
(5) $(-17) \times (-49) \times 0 \times (-8) \times (+37)$.

8. Compute:
(1) $-9 \times (-6)-18$; (2) $5+23 \times (-2)$;
(3) $-12 \times 4-(-8) \times 6$; (4) $8 \cdot (-9)-7 \cdot (-15)$;
(5) $(-\frac{2}{3}) \times \frac{1}{2} + \frac{1}{3} \times (-4)$.

9. For each 100 m rise in altitude, the temperature is lowered by 0.6°C. Now the ground temperature is 19°C, what will be the temperature in °C at the altitude of 4000 m?

10. Express the Commutative Law for Addition, Associative Law for Addition, Commutative Law for Multiplication, and Associative Law for Multipliation using letters.

11. Compute:

(1) $(-4\frac{1}{20})(+1.25)(-8)$; (2) $(-10)(-8.24)(-0.1)$;

(3) $(-\frac{5}{6})(+2.4)\ (+\frac{3}{5})$;

(4) $(\frac{7}{9}-\frac{5}{6}+\frac{3}{4}-\frac{7}{18})\times 36$;

(5) $-\frac{3}{4}\times(8-1\frac{1}{3}-0.04)$; (6) $71\frac{15}{16}\times(-8)$.

12. Compute:

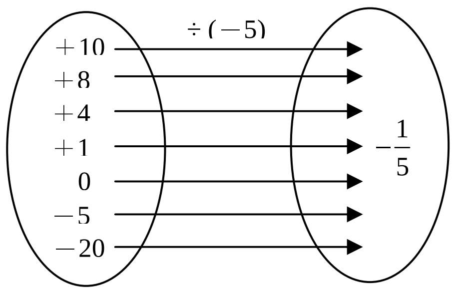
(1) $-91\div 13$; (2) $-56\div(-14)$; (3) $(-42)\div 0.6$;

(4) $-25.6\div(-0.064)$; (5) $16\div(-3)$;

(6) $1\div(-\frac{2}{3})$; (7) $\frac{4}{5}\div(-1)$; (8) $-3\frac{1}{7}\div\frac{11}{12}$;

(9) $-0.25\div\frac{3}{8}$; (10) $-\frac{1}{4}\ \div(-1.5)$.

13. For each number in the left circle, divide it by (-5) and write the result in the corresponding position in second circle..



(No. 13)

14. Fill in the table:

Original number	$\frac{7}{8}$	$-\frac{4}{5}$	-2.5	$\frac{1}{6}$	-2	$-3\frac{1}{3}$	1
Its reciprocal							

15. Compute:

(1) $(-\frac{3}{4})\times(-1\frac{1}{2})\div(-2\frac{1}{4})$;

(2) $-6\div(-0.25)\times\frac{11}{14}$.

16. Compute:

(1) $-8+4\div(-2)$;

(2) $6-(-12)\div(-3)$;

(3) $3\cdot(-4)+(-28)\div 7$;

(4) $(-7)(-5)-90\div(-15)$;

(5) $(-48)\div 8-(-25)(-6)$;

(6) $42\times(-\frac{2}{3})+(-\frac{3}{4})\div(-0.25)$.

17. Somebody measures the outdoor temperature at 8:00 am every day, and records as follows:

Temperate unit: °C November 2012

Date	1	2	3	4	5	6	7	8	9	10	First 10 days'a average
Temperature	6	6.5	7	4	2.5	3	1	1.5	−2	−3	
Date	11	12	13	14	15	16	17	18	19	20	Middle 10 days' average
Temperature	−1	0	1.5	0.5	−2	−3.5	−4	−1	2	1	

Date	21	22	23	24	25	26	27	28	29	30	Last 10 days' average
Temperature	1.5	0.5	0	−3	−5	−2	−1	−4	−5.5	−7.5	

Average for the month:___________

Compute the average temperatue at 8:00 am for the first 10 days, middle 10 days and last 10 days of November.

IV. The Power of Rational Number

1.13 The Power of Rational Number

Let us compute: 1. The area of a square with side 7 cm long; 2. The volume of a cube with side 5 cm long (Diagram 1-20).

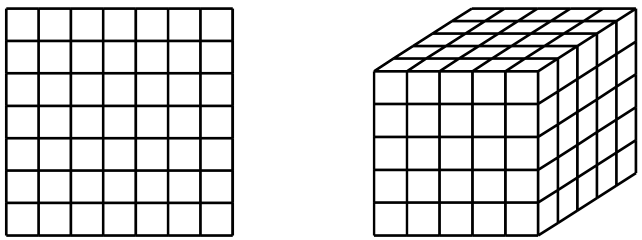


Diagram 1-20

$7 \times 7 = 49\ (\text{cm}^2)$; $5 \times 5 \times 5 = 125\ (\text{cm}^3)$.

Both are multiplication of a number by itself.

To simplify the notation for a factor multiplying by itself several times, we just write the factor once, and at the right upper corner of the factor we write the number of times it is multiplied by itself. For example,

$$7 \times 7 \text{ written as } 7^2,\ 5 \times 5 \times 5 \text{ written as } 5^3.$$

Similarly, $(-2)(-2)(-2)(-2)$ written as $(-2)^4$,

$$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \text{ written as } \left(\frac{3}{4}\right)^5.$$

Practice

1. (*Mental*) How to represent $8 \times 8 \times 8$?
2. (*Mental*) How to represent $(-6)(-6)(-6)(-6)$?
3. (*Mental*) What is the meaning of 0.1^2?
4. (*Mental*) What is the meaning of $\left(-\frac{2}{5}\right)^3$?

If factor a is mutilplied by itself n times, that is, $a \bullet a \bullet \cdots \bullet a$, then it can be written as a^n.

The multiplication of a factor by itself n times is called **power**. The result is called **exponentiation**. In a^n, a is called the **base** and n is called the **index**, and a^n is read as a raised to the power of n or a raised to the n^{th} power.

For example, in 9^4, 9 is the base, 4 is the index, and 9^4 is read as 9 to the power of 4.

Practice

1. (*Mental*) How to read the number 10^2? What is the base in the number 10^2? What is the index in the number 10^2?
2. (*Mental*) How to read the number 7^3? What is 3 called in the number 7^3? What is 7 called in the number 7^3?

Power of 2 is called **squared**, and power of 3 is called **cubed**. 10^2 is read as 10 squared and 7^3 is read as 7 cubed.

A number can be treated as a number to the power of 1. Example, 5^1, but we usually omit writing the index 1.

Practice

1. Compute: 2^3; 3^2; 0.1^3; 5^4; $\left(\frac{2}{3}\right)^2$; 1.2^3; $\left(1\frac{1}{2}\right)^3$; 9^1.
2. Compute: $(+2)^1$; $(+2)^2$; $(+2)^3$; $(+2)^4$; $(+2)^5$;
   $(-2)^1$; $(-2)^2$; $(-2)^3$; $(-2)^4$; $(-2)^5$.

Think for a while: A positive number raised to the power of 2, to the power of 3, … is the result positive or negative? A negative number raised to the power of 2, to the power of 3, … is the result positive or negative? what is the rule?

A positive number raised to any power is still positive; A negative number will become positive if raised to a power of even index, but will remain negative if raised to a power of odd index.

【**Example**】 Compute: (1) $(-3)^4$; (2) -3^4;
(3) 3×2^3; (4) $(3\times2)^3$;
(5) -2×3^4; (6) $(-2\times3)^4$;
(7) $8\div2^2$; (8) $(8\div2)^2$.

Solution (1) $(-3)^4 = 81$;

(2) $-3^4 = -81$;

(3) $3\times2^3 = 3\times8 = 24$;

(4) $(3\times2)^3 = 6^3 = 216$;

(5) $-2\times3^4 = -2\times81 = -162$;

(6) $(-2\times3)^4 = (-6)^4 = 1296$;

(7) $8\div2^2 = 8\div4 = 2$;

(8) $(8\div2)^2 = 4^2 = 16$.

NOTE: In a combination of power, multiplication and division, you should compute the power first before multiplication and division. If there is a pair of brackets, perform the computation inside the bracket first.

Practice

Compute:

1. -8^2;
2. $(-8)^2$;
3. 4×2^2;
4. $(4\times2)^2$;
5. -3×2^3;
6. $(-3\times2)^3$;
7. $(6\div3)^2$;
8. $6\div3^2$.

1.14 Mixed Operation of Rational Numbers

In an expression which composes a mixed operation of additions, subtractions, multiplications, divisions, and taking powers, then the computation must follow the following sequence:

The order of compution is: First compute the power, then multiplication and division, and lastly addition and subtraction. If there are brackets, perform the computation inside the brackets first.

【**Example 1**】Compute: $-1\frac{1}{2}+\frac{1}{3}+\frac{5}{6}-1\frac{1}{4}$.

Solution

$$\begin{aligned}-1\frac{1}{2}+\frac{1}{3}+\frac{5}{6}-1\frac{1}{4} &= -1-\frac{1}{2}+\frac{1}{3}+\frac{5}{6}-1-\frac{1}{4}\\ &= -1-1+\frac{-6+4+10-3}{12}\\ &= -2+\frac{5}{12}\\ &= -1\frac{7}{12}\end{aligned}$$

Practice

Compute:

1. $1.6+5.9-25.8+12.8-7.4$.
2. $-5\frac{1}{2}+8\frac{2}{3}-12\frac{5}{6}$.

【**Example 2**】Compute: $2\frac{1}{5}\times\left(\frac{1}{3}-\frac{1}{2}\right)\times\frac{3}{11}\div1\frac{1}{4}$.

Solution

$$\begin{aligned}2\frac{1}{5}\times\left(\frac{1}{3}-\frac{1}{2}\right)\times\frac{3}{11}\div1\frac{1}{4}&=2\frac{1}{5}\times\left(-\frac{1}{6}\right)\times\frac{3}{11}\div1\frac{1}{4}\\&=\frac{11}{5}\times(\frac{-1}{6})\times\frac{3}{11}\times\frac{4}{5}\\&=-\frac{2}{25}.\end{aligned}$$

Practice

Compute:

1. $-2.5\times(-4.8)\times0.09\div(-0.07)$.
2. $2\frac{1}{4}\times\left(-\frac{6}{7}\right)\div\left(\frac{1}{2}-2\right)$.

【**Example 3**】Compute: $-10+8\div(-2)^2-(-4)\times(-3)$.

Solution

$$\begin{aligned}&-10+8\div(-2)^2-(-4)\times(-3)\\=&-10+8\div4-12\\=&-10+2-12\\=&-20.\end{aligned}$$

Practice

Compute:

1. $-9+5\times(-6)-(-4)^2\div(-8)$.
2. $2\times(-3)^3-4\times(-3)+15$.

Exercise 5

1. Write the following in power form:

$$6\times6\times6\times6,\ (-3)(-3)(-3)(-3)(-3),$$
$$1.1\times1.1,\ \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}.$$

2. Write the following in a multiplicative form:

$$3^4,\ 4^3,\ (-7)^2,\ 6.74^4,\ \left(-\frac{1}{3}\right)^5.$$

3. In 4^2, what is the index? In $(-1)^2$, what is the index? In 7.5^1, what is the index? In a^n, what is the index?

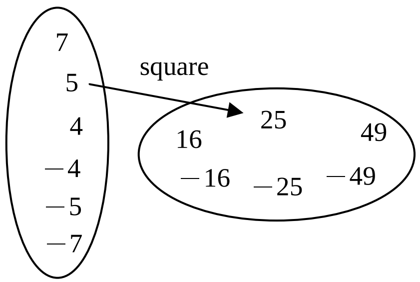
4. In 2^7, what is the base number? How about in $\left(-\frac{1}{2}\right)^5$? How about in 1^3? How about in a^n?

5. Compute: 2^5; 5^2; $(-1)^7$; 1.1^3; 0^5; $\left(\frac{1}{2}\right)^4$; $\left(-1\frac{5}{6}\right)^2$.

6. (1) Ae 3^2 and 3×2 equal? Why?
   (2) Are 3^2 and 2^3 not equal? Why?

7. What is the square of 3? What is the square of -3? How many number when squared equals 9? Is there any rational number when squared equals -9?

8. Draw a line to link each number in the first circle to its squared number in the second circle.



(No. 8)

9. $1.2^2 = ?$ $\quad \begin{cases} 12^2 = ? & 120^2 = ? \\ 0.12^2 = ? & 0.012^2 = ? \end{cases}$

If the decimal point of the base number is shifted 1 position to the left or to the right, how would the decimal point of the squared value be changed? What if the decimal point of the base number is shifted 2 positions to the left or to the right?

10. $1.2^3 = ?$ $\quad \begin{cases} 12^3 = ? & 120^3 = ? \\ 0.12^3 = ? & 0.012^3 = ? \end{cases}$

If the decimal point of the base number is shifted 1 position to the left or to the right, how would the decimal point of the cubed value be changed? What if the decimal point of the base number is shifted 2 positions to the left or to the right?

11. Compute:

(1) $(-2)^3$; (2) $-(-2)^3$; (3) $4 \bullet (-2)^2$;

(4) $(-3)^4(-3)^4$; (5) -2×3^3 ; (6) $(-2\times 3)^3$;

(7) $(6\div 3)^3$; (8) $6\div 3^3$ 。

12. Compute:

(1) $(-2)^4$; (2) $-(-2)^4$;

(3) $4 \bullet (-2)^3$; (4) $(-2)^2(-3)^2$;

(5) $-9\div(-3)^2$; (6) $(-9\div 3)^3$;

(7) $\left(-1\frac{1}{15}\right)^2$; (8) $(-1.7)^2$; (9) $-(-0.8)^3$;

(10) $-\left(-\frac{1}{2}\right)^4$; (11) -5.25^2; (12) $(-5)^3 \bullet \left(-\frac{3}{5}\right)$.

13. Compute:

(1) $-2^2-(-3)^2$; (2) $4-5\bullet\left(-\frac{1}{2}\right)^3$;

(3) $-2^3-3\bullet(-1)^3-(-1)^4$;

(4) $-2^4+(3-7)^2-2\bullet(-1)^2$;

(5) $-2\bullet(0.1)^3(-0.2)^2+(-0.8)$;

(6) $1\frac{1}{2}\times\left[3\bullet\left(-\frac{2}{3}\right)^2-1\right]-\frac{1}{3}\times(-2)^3$;

(7) $-2^3\div\frac{9}{4}\times\left(-\frac{2}{3}\right)^2$;

(8) $-1^4-(1-0.5)\times\frac{1}{3}\times\left[2-(-3)^2\right]$.

14. Compute:

(1) $-\frac{1}{2}+1\frac{1}{5}-2\frac{7}{10}$;

(2) $2.28-3.76+1\frac{1}{2}-\frac{3}{4}$;

(3) $-1\frac{2}{3}\times\left(0.5-\frac{2}{3}\right)\div 1\frac{1}{9}$;

(4) $17-8\div(-2)+4\bullet(-5)$;

(5) $-2\frac{1}{2}+5\frac{3}{5}(-2)\times\left(-\frac{5}{14}\right)$;

(6) $4\bullet(-3)^2\,4-5\times(-3)+6$;

(7) $(-56)\div(-12+8)+(-2)\times 5$;

(8) $-3-\left[-5+\left(1-0.2\times\frac{3}{5}\right)\div(-2)\right]$;

(9) $1\div(-1)+0\div 4-(-4)(-1)$;

(10) $18+32\div(-2)^3-(-4)^2\times 5$;

(11) $(-5)(+8)-(-2)^2(-6)+(-3)^4\div(-27)$;

(12) $(0.01-0.03)^3-(2\times 0.04^2-0.0015)$.

1.15 Approxiate value and significant figures

Read the followings:

(1) There are 48 students in Class 2 of Year 7;

(2) There are 126 sets of machine in the factory;

In the above 48, 126 are exact and accurate measurements;

(3) The distance of the moon from the earth is about 380 thousand km;

(4) David Lee is about 1.57 m tall;

In the above statements, 380 thousand, 1.57 are not exact values. They are approximate values close to the actual values.

The 380 thousand km is a figure obtained from rounding, the actual distance between the moon and the earth is between 375 thousand and 385 thousand km (Diagram 1-21).

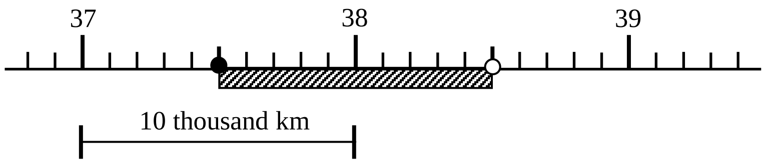


Diagram 1-21

Similarly David Lee's height is about 1.57 m, which means he is taller than 1.565 m and shorter than 1.575 m (Diagram 1-22).

The 380 thousand km, is accurate up to the ten thousand unit; where 1.57 is accurate up to the hundredth unit (or 0.01). In general, the approximate number can be rounded at any position, and it is only accurate to the position it is rounded.

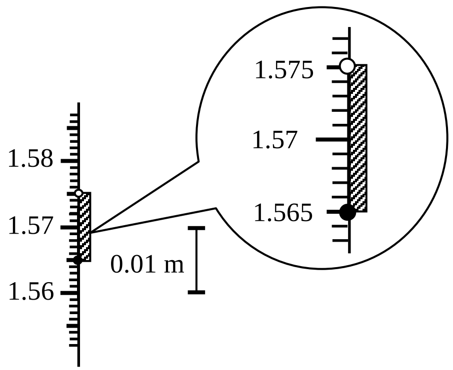


Diagram 1-22

Here, counting from the leftmost non-zero digit, to the position it is rounded, all the digits are called **significant**

digits. Take the 380 thousand km as example, the leftmost non-zero digit is 3, the position it is rounded is 8, the total count is 2, therefore the number 380 thousand, has two significant digits 3 and 8. Look at the 1.57 m example, the leftmost non-zero digit is 1, the position it is rounded is 7, total significant figures are 1, 5, 7.

【Example 1】 The following approximate value is a rounded number, what is the value of its rounding position? How many significant figures has it got?

(1) 1 billion;

(2) 5.07 million;

(3) 43.8;

(4) 0.002;

(5) 0.03086;

(6) 24 thousand.

Solution (1) 1 billion, accurate to the 100 million, there are 2 significant digits 1, 0;

(2) 507 thousand, accurate to the thousand, there are 3 significant digits 5, 0, 7;

(3) 43.8, accurate to the tenth (i.e. 0.1), there are 3 significant digits 4, 3, 8;

(4) 0.002, accurate to the thousandth (i.e. 0.001), there is 1 significant digit 2;

(5) 0.03086, accurate to the hundred thousandth (i.e. 0.00001), there are 4 significant digits 3, 0, 8, 6;

(6) 24 thousand, accurate to the thousand, there are 2 significant digits 2, 4.

Practice

(*Mental*) The ratio a circle's circumference to its diameter, $\pi = 3.14159265\cdots$. Rounding to approximate value of 3.14, what is the rounding accuracy? How many significant digits are there? How about rounding to 3.142？How about rounding to 3.1416?

【**Example 2**】Round the following numbers according to accuracy required:

(1) 0.85149 (accurate to the thousandth);
(2) 47.6 (accurate to the unit digit);
(3) 0.02076 (round to 3 significant digits);
(4) 1.5972 (round to the hundredth).

Solution (1) $0.85149 \approx 0.851$;
(2) $47.6 \approx 48$;
(3) $0.02076 \approx 0.0208$;
(4) $1.5972 \approx 1.60$.

NOTE: For (4), the rounding to 1.60, is different from 1.6, the 0 must be kept to indicate it is a rounded number and accurate to the hundredth. For example, Mr. Wong is about 1.60 m tall, that means he is between 1.595 m and 1.605 m, accurate to the 0.01 m (Diagram 1-23); and Miss Cheung is about 1.6 m, that means she is between 1.55 m to 1.65 m, and accurate to the 0.1 m (Diagram 1-24).

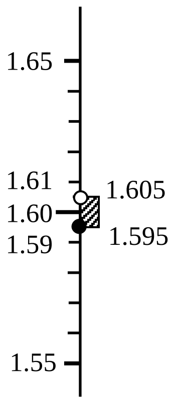


Diagram 1-23

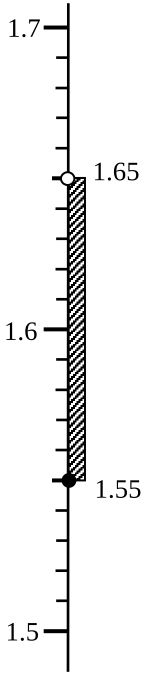


Diagram 1-24

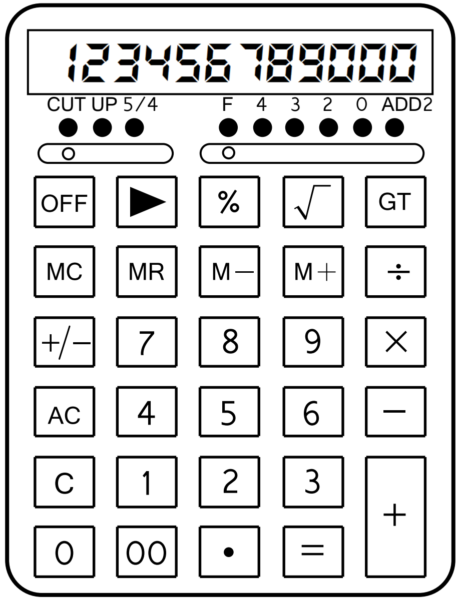
> ***Practice***
>
> Round the following numbers according to accuracy required:
> 1. 56.32 (round to 3 significant digits).
> 2. 0.6648 (round to the hundredth).
> 3. 0.7096 (accurate to the thousandth).

1.16 **Use Calculator to compute square and power**

Nowadays, calculator is a very convenient and practical computing tool. Through continuing improvemets, calculation power is now inbuilt in computers, watches, and mobile phones. Calculation functions have also been extended beyond simple computation of

addition, subtraction, multiplication, division to square, square root, *n*th power, *n*th root and even to advanced mathematics computations.

How to use a simple calculator to compute square and power? There are many different models of calculators, and each may require slightly different operating steps. Herebelow we introduce an example of operation using one of the simplest calculators.



To use a calculator to compute some mixed operations, just follow the order to input the numbers and operation symbols, after completing all the input, press the equal button, and the result of calculaton will be displayed.

【**Example 1**】Use a calculator to compute $5-3$.

Solution Press the buttons in the following sequence $\boxed{5}$, $\boxed{-}$, $\boxed{3}$, $\boxed{=}$, the calculator will display the result 2 on the screen.

【**Example 2**】Use a calculator to compute 3.35^2.

Solution Press the buttons in the following sequence $\boxed{3}$, $\boxed{\bullet}$, $\boxed{3}$, $\boxed{5}$, $\boxed{\times}$, $\boxed{=}$, the calculator will display the answer 11.2225 on the screen.

NOTE: There is a button on the upper part of a calculator, which is used to select the number of decimal places to be displayed. F means full decimal display, CUT means round down, UP means round up, 5/4 means round off. So when F is selected，the display is 11.2225. When CUT is selected, you have to choose the number of decimal places to take，if 3 is selected, 11.222 is displayed, if 2 is selected, then 11.22 is displayed, if 0 is selected then 11 is displayed; when UP is selected, and if number of decimal places 3 is selected, then 11.223 is displayed, if 2 is selected, then 11.23 is displayed, if 0 is selected then 12 is displayed; when 5/4 is selected, and if number of decimal places 3 is selected then 11.223 is displayed, if 2 is selected, then 11.22 is displayed, if 0 is selected then 11 is displayed.

【**Example 3**】Use a calculator to compute 3^2+4^2.

Solution Press the buttons in the following sequence $\boxed{3}$, $\boxed{\times}$, $\boxed{M+}$, $\boxed{4}$, $\boxed{\times}$, $\boxed{M+}$, $\boxed{MR}$, the calculator will display the answer 25 on the screen.

Practice

Use a calculator to compute 2.29^2, 2.15^2, 2.07^2, 2.3^2.

【**Example 4**】Use a calculator to compute 2.468^2 (accurate to 3 decimal places).

Solution Choose 5/4, and select 3 decimal places， then press the following buttons in sequence $\boxed{2}$, $\boxed{\bullet}$, $\boxed{4}$, $\boxed{6}$, $\boxed{8}$, $\boxed{\times}$, $\boxed{=}$, the calculator will display the answer 6.091 on the screen.

Practice

Use a calculator to compute 2.291^2, 2.157^2, 2.073^2, 2.307^2 (accurate to 3 decimal places).

【**Example 5**】Use a calculator to compute 246.8^2 (accurate to the unit digit), compute 0.2468^2 (accurate to 4 decimal places).

Solution First choose 5/4 and 0 decimal place，then press the following buttons in sequence $\boxed{2}$, $\boxed{4}$, $\boxed{6}$, $\boxed{\bullet}$, $\boxed{8}$, $\boxed{\times}$, $\boxed{=}$, 60910 is displayed on screen; Then select 4 decimal places, and press the following buttons in sequence $\boxed{0}$, $\boxed{\bullet}$, $\boxed{2}$, $\boxed{4}$, $\boxed{6}$, $\boxed{8}$, $\boxed{\times}$, $\boxed{=}$, 0.0609 is displayed on the screen.

Practice

Use a calculator to compute 22.91^2, 0.2157^2, 207.3^2, 0.02307^2 (accurate to 3 decimal places).

【**Example 6**】Use a calculator to compute 5.19^3 (accurate to 3 decimal places).

Solution First choose 5/4, and select 3 decimal places， then press the following buttons in sequence $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{=}$, 139.798 is displayed on screen.

NOTE: You cannot press buttons in the following sequence $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{\times}$, $\boxed{=}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{=}$, because the first time you press $\boxed{=}$ the approximate value is used, therefore the final result is not correct.

NOTE: If engineering calculator is used，you can press the following buttons in sequence $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{y^x}$, $\boxed{3}$, $\boxed{=}$.

Practice

Use a calculator to compute 5.37^3, 5.06^3, 5.21^3, 5.4^3 (accurate to 3 decimal places).

【**Example 7**】Use a calculator to compute

(a) 5.263^3; (b) 5.268^3; (c) 5.194^3; (d) 5.198^3.

(accurate up to 1 decimal place).

Solution Calculators do not have an option for accuracy to one decimal place, we have to adjust it manually, to avoid the effect of round off, we choose CUT and 2 decimal places, press the following buttons in sequence:

(a) $\boxed{5}$, $\boxed{\bullet}$, $\boxed{2}$, $\boxed{6}$, $\boxed{3}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{2}$, $\boxed{6}$, $\boxed{3}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{2}$, $\boxed{6}$, $\boxed{3}$, $\boxed{=}$.
Here the calculator displays 145.78 on the screen, so the answer is 145.8 after rounding the result to one decimal place;

(b) $\boxed{5}$, $\boxed{\bullet}$, $\boxed{2}$, $\boxed{6}$, $\boxed{8}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{2}$, $\boxed{6}$, $\boxed{8}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{2}$, $\boxed{6}$, $\boxed{8}$, $\boxed{=}$.
Here the calculator displays 146.19 on the screen, so the answer is 146.2 after rounding the result to one decimal place;

(c) $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{4}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{4}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{4}$, $\boxed{=}$.
Here the calculator displays 140.12 on the screen, so the answer is 140.1 after rounding the result to one decimal place;

(d) $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{8}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{8}$, $\boxed{\times}$, $\boxed{5}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{9}$, $\boxed{8}$, $\boxed{=}$.
Here the calculator displays 140.44 on the screen, so the answer is 140.4 after rounding the result to one decimal place.

NOTE: For (d) if 5/4 and 2 decimal place is selected, the answer displayed on the screen will be 140.45, and rounding off to one decimal place will be 140.5, which is a wrong answer. So we must select CUT.

Practice

Use a calculator to compute

(a) 5.373^3; (b) 5.069^3; (c) 5.215^3; (d) 5.398^3.

(accurate up to 1 decimal place).

【**Example 8**】The volume of a sphere can be calculated by this formula

$$\text{Volume of a sphere} = \frac{4}{3}\times\pi\times(\text{Radius})^3.$$

Use a calculator to compute the volume of a sphere with radius 0.89m (accurate to one decimal place, $\pi = 3.14$).

Solution We have to calculate the value of $\frac{4}{3}\times 3.14\times 0.89^3$.

Select CUT and 2 decimal places，press the following buttons in sequence $\boxed{4}$, $\boxed{\times}$, $\boxed{3}$, $\boxed{\bullet}$, $\boxed{1}$, $\boxed{4}$, $\boxed{\times}$, $\boxed{\bullet}$, $\boxed{8}$, $\boxed{9}$, $\boxed{\times}$, $\boxed{\bullet}$, $\boxed{8}$, $\boxed{9}$, $\boxed{\times}$, $\boxed{\bullet}$, $\boxed{8}$, $\boxed{9}$, $\boxed{\div}$, $\boxed{3}$, $\boxed{=}$, 2.95 is displayed on screen. So the answer is 3.0 after rounding the result to one decimal place.

Answer: The answer displayed on the screen is 2.95, adjusted to 3.0 for rounding to one decimal place.

Practice

1. Use a calculator to compute 3.14×0.95^2 (accurate up to 2 decimal places).
2. Volume of sphere = $\frac{1}{6}\times\pi\times(\text{Diameter})^3$. Use calculator to compute the volume of a sphere with radius 2.35 m (accurate to 1 decimal place, take $\pi = 3.14$).

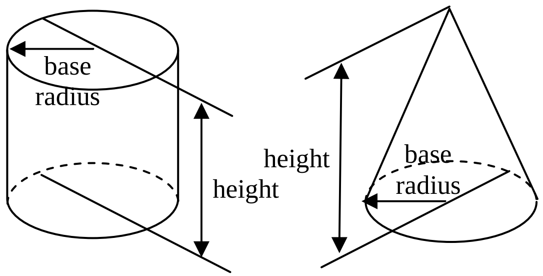
Exercise 6

1. For the approximate value obtained from rounding off, what is the accuracy level? how many significant digits are there?
   (1) 18.32; (2) 35; (3) 0.708; (4) 6.409; (5) 54.80;
   (6) 0.0074; (7) 89.3; (8) 0.0540; (9) 5.02; (10) 2.00.

2. Find the approximate value of the following number by rounding off its value according to the accuracy required in the bracket:
   (1) 12.17, 0.009403, 8.607 (accurate to 3 significant digits);
   (2) 2.768, 3.4017, 92.598 (accurate to the hundredth);
   (3) 19.74, 8.965, 0.409 (accurate to the tenth).
3. Use a calculator to compute the following square:
   (1) 1.98^2; (2) 8.79^2; (3) 4.08^2; (4) 6.3^2;
   (5) 3.168^2; (6) 3.186^2; (7) 5.064^2; (8) 7.707^2;
   (9) 45.6^2; (10) 0.087^2; (11) 604^2; (12) 0.538^2;
   (13) 0.02108^2; (14) 750.6^2; (15) 30.48^2;
   (16) 0.8008^2; (17) $(-2.49)^2$; (18) -56.7^2.
4. Use a calculator to compute the following cube:
   (1) 8.57^3; (2) 1.709^3; (3) 6.43^3; (4) 9.58^3;
   (5) 4.384^3; (6) 2.173^3; (7) 7.058^3; (8) 8.009^3;
   (9) 11.74^3; (10) 0.356^3; (11) $(-0.0489)^3$; (12) -699.8^3;
5. Use a calculator to compute:
   (1) $4.75^2+2.93^2$; (2) $8.27^3-6.42^3$;
   (3) $0.746^2-0.985^2$; (4) $91.08^3+64.37^3$;
   (5) $4\times 3.986^2-10\times 3.986-9$.
6. Use a calculator to compute (accurate to 2 decimal places):
   (1) 3.14×1.77^2; (2) $\frac{1}{3}\times 3.14\times 0.57^2$;
   (3) $\frac{4}{3}\times 3.14\times 1.9^3$.
7. Use a calculator to compute (accurate to 3 decimal places):
   (1) Area of square with side 0.846 m
   (Area of a square $=(\text{side})^2$);
   (2) Volume of cube with side 2.95 m
   (Volumn of a cube $=(\text{side})^3$).

8. Use a calculator to compute (accurate to 3 decimal places, take $\pi = 3.14$):
   (1) Area of circle with radius 4.8 m
   (Area of a circle $= \pi \times (\text{Radius})^2$);
   (2) Volume of sphere with radius 0.37 m
   (Volumn of a sphere $= \frac{1}{6} \times \pi \times (\text{Diameter})^3$);
   (3) Surface area of sphere with radius 0.96 m
   (Surface area of sphere $= 4 \times \pi \times (\text{Radius})^2$).

9. Use a calculator to compute (accurate to 3 decimal places, take $\pi = 3.14$):
   (1) Volume of circular pillar with height 0.82 m, base radius 0.47 m (Volume of circular pillar = area of base $\times$ height);
   (2) Volume of circular cone with height 7.36 cm and base radius 2.7 cm
   (Volume of circular cone $= \frac{1}{3} \times$ base area $\times$ height).



(No. 9)

Chapter summary

I. This chapter mainly covers the concept and computation of rational numbers.

II. "Mathematics arises from the needs of our daily life". The concept of positive and negative numbers, reflected the large volume of contradicting values we encounter in our daily life.

III. Rational numbers consists of positive integers, zero, negative integers, positive fractions, negative fractions. Rational numbers can be represented by points on the number axis.

IV. Law for Addition: When two rational numbers are added together, if the sign of the numbers are the same, take the sum of the absolute values of the two numbers and leave the sign unchanged; if the signs of the numbers are different, take the difference between the two absolute values, and attach it with the sign of the number with larger absolute value.

Law for Multiplication: When two rational numbers are multiplied together, the sign of the product is positive if the numbers are of same signs, the sign of the product is negative if the numbers are of unequal signs, the value of product is the product of the absolute values of the two numbers.

Subtracting a number is equivalent to adding its opposite number.

Dividing by a number is equivalent to multiplying by its inverse number (or reciprocal).

V. The Computation Laws of rational numbers are:
Commutative law for addition $a + b = b + a$;
Associative law for addition $(a + b) + c = a + (b + c)$;
Commutative law for multiplication $ab = ba$;
Associative law for multiplication $(ab)c = a(bc)$;
Dsitributive law for multiplication $a(b + c) = ab + ac$.

VI. The product of a number multiplied by itself n times is called nth power of the number, i.e. $\underbrace{a \bullet a \bullet \cdots \bullet a}_{n \text{ terms}} = a^n$

Revision Exercise 1

1. Compute:
   (1) $376+489$; (2) $742-145$; (3) 64×28;
   (4) $893 \div 19$; (5) $5487+694$; (6) $4503-784$;
   (7) 325×48; (8) $4623 \div 87$; (9) $9+27 \div 3$;
   (10) $18-10 \div 2$; (11) $9+27 \div 3$; (12) $12-2 \times 5$;
   (13) $36 \times 7-48 \div 2$; (14) $117 \div 13+64 \times 25$.

2. Compute:
   (1) $15.8+2.74$; (2) $4.2-0.39$;
   (3) 3.5×0.68; (4) $12.96 \div 0.072$.

3. Compute:
   (1) $\frac{3}{5}+\frac{1}{5}$; (2) $\frac{5}{6}-\frac{1}{6}$; (3) $\frac{1}{2}+\frac{2}{3}$;
   (4) $\frac{2}{3}-\frac{1}{4}$; (5) $\frac{1}{6}+\frac{3}{10}$; (6) $3\frac{1}{4}-2\frac{5}{6}$.

4. Compute:
   (1) $\frac{3}{5}\times\frac{1}{5}$; (2) $\frac{5}{6}\div\frac{1}{6}$; (3) $\frac{7}{8}\times\frac{4}{5}$;
   (4) $\frac{8}{9}\div\frac{5}{6}$; (5) $1\frac{1}{2}\times\frac{1}{6}$; (6) $\frac{3}{4}\div 2\frac{1}{2}$.

5. Compute:
   (1) $31\%+1.5\%$; (2) $1+0.5\%$; (3) $27\%-12.4\%$;
   (4) $1-35\%$; (5) $32 \times 2.4\%$; (6) $100 \times 0.1\%$;
   (7) $1 \div 25\%$; (8) $0.75 \div 15\%$.

6. Compare the magnitude of the following pair of numbers:
   (1) $\frac{4}{9}$ and $\frac{8}{9}$; (2) $\frac{1}{10}$ and $\frac{1}{100}$; (3) $\frac{7}{11}$ and $\frac{7}{22}$;
   (4) $\frac{5}{7}$ and $\frac{7}{9}$; (5) 0.78 and 0.87; (6) $\frac{3}{4}$ and 0.7.

7. Separate the rational numbers 6.4, -9, $\frac{2}{3}$, $+10$, $-\frac{3}{4}$, -0.02, 1, -1, $7\frac{1}{3}$, -8.5, 25, -100 into 4 sets, according to its value whether it is a positive integer, negative integer, positive fraction, negative fraction.

8. Among the rational numbers -3, $+8$, $-\frac{1}{2}$, $+0.1$, 0, $\frac{1}{3}$, -10.5, -0.4, which of them belong to the set of positive integers? which ones belong to the set of fractions? which of them belong to the set of positive numbers? which of them belong to the set of negative numbers?

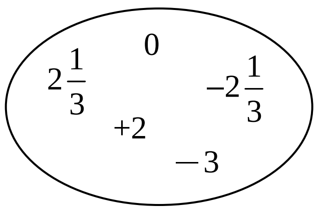
9. Mark the following points on the number axis. Arrange the numbers in descending order and link them using the ">" symbol:
$$+3,\ -5,\ +5\frac{1}{2},\ -2\frac{1}{2},\ -4,\ +4,\ 0.$$

10. Arrange the numbers in ascending order and link them using the "<" symbol:
$$-4\frac{1}{2},\ \frac{2}{3},\ 0.6,\ -0.6,\ -4.2,\ 5.$$

11. Water will freeze into ice at temperature of 0°C, alcohol will free at temperature of -117°C, mercury will freeze at temperature of -39°C. Which one has the highest freezing temperature? Which one has the lowest freezing termperature?

12. Mark on the number axis all integers larger than -5 and smaller than $+5$.

13. In the diagram, the set of rational numbers contains 5 numbers. Find the largest number and the smallest number.



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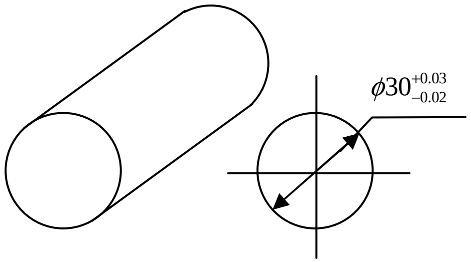
14. Is it true that the absolute value of a number is always positive? Why?

15. Among the rational numbers, is there a number which is the smallest? Is there a number with the smallest absolute value? Is there a number which is the smallest positive integer? Is there a number which is the smallest negative integer? If there is one, what is the number?

16. To produce an axial, the diagram specification of its diameter is $\phi 30^{+0.03}_{-0.02}$, where $\phi 30$ means the diameter standard is 30 mm, the small print of $+0.03$ means that any axial with diameter larger than the standard by 0.03 mm will be disqualified, and the small print of -0.02 means that any axial with diameter less than the standard by 0.02 mm will be disqualfied. What is the maximum diameter of axial that will pass the quality check? What is the smallest diameter of axial that will pass the qualify check?



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17. 30 bags of rice are being weighed, the record of their weights are as follows:

183, 178, 181, 180, 179, 185, 176, 180, 180, 176,
184, 177, 175, 186, 184, 181, 185, 174, 177, 185,
180, 186, 179, 184, 178, 183, 182, 186, 180, 184.

Setting the weight of 80 kg as standard, the excess in kg for overweight bag is marked as positive value, the deficit in kg for underweight bags is marked as negative value. Find the total overweight or underweight of the 30 bags of rice. Find the total weight of the 30 bags of rice.

18. For the 10 team members of a school's volley ball team, their heights are:

1.73 m, 1.74 m, 1.70 m, 1.76 m, 1.80 m,
1.75 m, 1.77 m, 1.79 m, 1.74 m, 1.72 m.

Compute the average height of the team members.

19. What is the sum of integers 9 and -13? What is the absolute value of the sum? What is the sum of their absolute values? Compare the magnitude of the three values.

20. Fill in the table for the opposite number and the reciprocal:

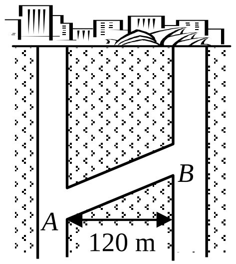
Original number	5	-6	$\frac{2}{3}$	1	-0.5	-1
Opposite number						
Reciprocal						

21. Write any number and its opposite number. Find their sum and their product?

22. (1) Under what circumstances will the product of two rational numbers be a positive number? be a negative number? be zero?
(2) Under what circumstances will the quotient of the division of one rational number by another rational number be 1? be -1? be without meaning?

23. In a cold storage, the temperature is at room temperature of -2°C. Now there is a stock of food requiring to be stored at a temperature of -23°C. If the cold storage can reduce its temperature by 4°C per hour, how many hours will it take for the cold storage to reach the required temperature?

24. In a coal mine, the altitude of a point A inside the well is -174.8 m. Going from A to B, the horizontal distance is 120 m. For every horizontal distance of 10 m, the altitude is raised by 0.4 m. Find the altitude of B.



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25. Compute:

(1) $5 \div 0.1$; (2) $5 \div 0.01$;
(3) $5 \div 0.001$; (4) $5 \div (-0.1)$;
(5) $5 \div (-0.01)$; (6) $5 \div (-0.001)$;
(7) $0.2 \div 0.1$; (8) $0.02 \div 0.01$;
(9) $0.002 \div 0.001$; (10) $(-0.3) \div 0.1$;
(11) $(-0.03) \div 0.01$; (12) $(-0.003) \div 0.001$.

26. For each of the following equations, find an appropriate number to fill in the bracket:

(1) $(+5) + (\quad) = +3$,
$(+5) + (\quad) = -3$,
$(-5) + (\quad) = +3$,
$(-5) + (\quad) = -3$;

(2) $(+3) \times (\quad) = -6$,
$(-3) \times (\quad) = -6$,
$(+6) \times (\quad) = +3$,
$(+6) \times (\quad) = -3$;

(3) $(-3) + (\quad) = 0$,
$(-5) \times (\quad) = 0$.

27. For each of the following equations, is it possible to find an appropriate number to fill in the bracket?

(1) $0 \times (\quad) = 5$;
(2) $0 \times (\quad) = 0$.

28. How many rational numbers when squared equal to 4? Is there a rational number when squared equal to -4? How many rational numbers when cubed equal to 8? Is there a rational number when cubed equal to -8?

29. Compute:

(1) $\left(1\frac{3}{4}-\frac{7}{8}-\frac{7}{12}\right)\times\left(-1\frac{1}{7}\right)$;

(2) $(-81)\div 2\frac{1}{4}+\frac{4}{9}\div(-16)$;

(3) $\frac{2}{5}\div\left(-2\frac{2}{5}\right)-\frac{8}{21}\times\left(1\frac{3}{4}\right)-0.25$;

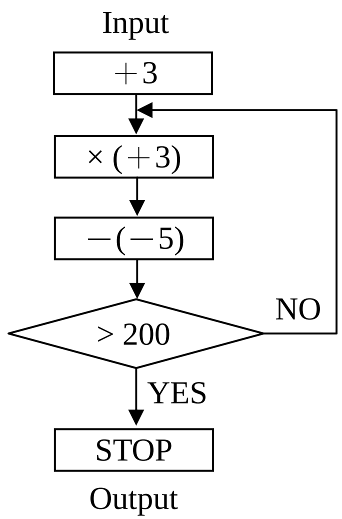
(4) $3(-2.5)(-4)+5(-6)(-3)^2$;

(5) $\{0.85-[12+4\times(3-10)]\}\div 5$;

(6) $2^2+(-2)^3\times 5-(-0.28)\div(-2)^2$;

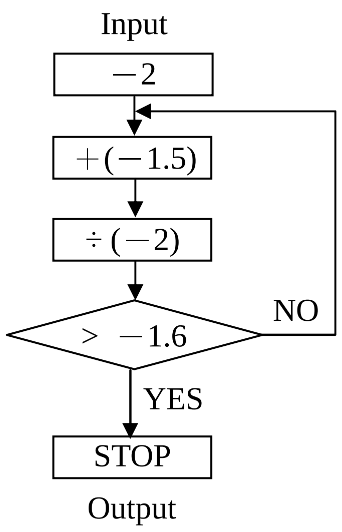
(7) $\left[(-3)^3-(-5)^3\right]\div[(-3)-(-5)]$.

30. Follow the instruction in the flow chart below, perform the computation and write the result in the table (For example in the first cycle, computing $(+3) \times (+3) - (-5)$ we get 14, which is less than 200; in the second cycle, compute $14 \times (+3) - (-5)$, etc.):



No. of Counting	Result
1	14
2	
3	
4	

31. Follow the instruction in the flow chart below，perform the computation and write the result in the table:



No. of Counting	Result
1	
2	
3	

(This chapter is translated to English by courtesy of Mr. Hymen LAM and reviewed by courtesy of Mr. SIN Wing Sang, Edward.)