Chapter 2 Addition and Subtraction of Integral Expressions

I. Integral Expressions

2.1 **Algebraic Expressions**

While in Primary school, we know we can use letters to represent numbers. For example: area of rectangle can be expressed as

$$S = ab,$$

Here letter S represents the area of the rectangle, letters a, b represent respectively the length and width of the rectangle.

We know Laws of Mathematical Operations can be expressed succinctly using letters:

Commutative Law for Addition	$a + b = b + a$;
Associative Law for Addition	$(a + b) + c = a + (b + c)$;
Commutative Law for Multiplication	$ab = ba$;
Associative Law for Multiplication	$(ab)c = a(bc)$;
Distributive Law	$a(b + c) = ab + ac$.

Here, a, b, c represent any rational numbers.

In Algebra, we usually use letters to represent numbers.

For example, a can travels 40 km in one hour, then the distances it travels in 2 hours, 2.5 hours, $1\frac{3}{4}$ hours are respectively:

$$40\times 2 \text{ km},\ 40\times 2.5 \text{ km},\ 40\times 1\frac{3}{4} \text{ km}.$$

If we use letter t to respresent the time in hour traveled by the car, then the distance in km the car has traveled is

$$40t \text{ km}.$$

If we use letter v to represent the distance in km the car travels in one hour, letter t to represent the time in hours the car has traveled, then the distance in km the car has traveled is

$$vt \text{ km}.$$

From the above examples, we can see that using letters to

represent numbers, we can express the numerical values and relationship of numerical values succinctly.

In the above examples, we obtain many expressions involving letters, namely ab, $a+b$, $40r$, vt ⋯ etc. Expressions with numbers and letters linked up by mathematical operations are called **Algebraic Expressions**.

A single number or letter, like -31, 0, x, can also be an algebraic expression.

In Algebra, every letter represents a number. Therefore, some Laws of Mathematical Operations also apply to alegrabic expressions.

Using an algebraic expression to represent a quantity or a quantitative relationship is very important to future learning.

【Example 1】 Write the following in terms of algebraic expression:

(1) Difference between x and 5;

(2) Quotient of b divided by 8;

(3) $\frac{1}{3}$ of x;

(4) 50% of y.

Solution (1) $x-5$; (2) $\frac{b}{8}$; (3) $\frac{1}{3}x$; (4) $\frac{50}{100}y$.

【Example 2】 Find the perimeter l of the rectangle shown in Diagram 2-1.

Solution $l = a\times 2 + b\times 2 = 2a + 2b$.

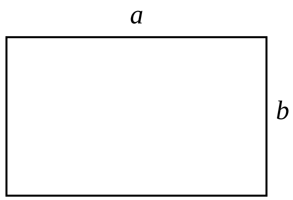


Diagram 2-1

NOTE: When a number is multiplied by a letter, if the mulitiplication sign is to be omitted, then the number should preceed the letter, for example $a\times 2$ is written as $2a$.

Practice

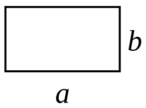
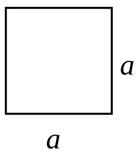
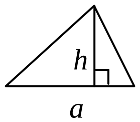
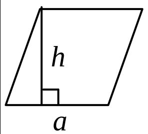
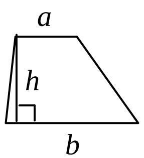
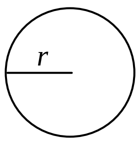
1. Write the following in terms of algebraic expression:

(1) Sum of 15 and S; (2) Difference between x and 3; (3) Product of a times 15;

(4) Quotient of a divided by 15; (5) 70% of y; (6) $\frac{2}{3}$ of a times d;

Practice

2. Complete the table:

Name	Shape	Description of relationship	Meaning of letters	Algebraic Formula
Rectangle		Perimeter = Length × 2 + Width × 2 Area = Length×Width	l : Perimeter s : Area a : Length b : Width	$l = 2a + 2b$ $s = ab$
Square		Perimeter = Length × 4 Area = Length^2	l : Perimeter s : Area a : Length	???
Triangle		Area $= \frac{1}{2} \times$ Base × Height	s : Area a : Base h : Height	???
Parallelogram		Area = Length× Height	s : Area a : Base h : Height	???
Trapezium		Area $= \frac{1}{2} \times$ (Upper Side + Lower Side) × Height	s : Area a : Upper Side b : Lower Side h : Height	???
Circle		Perimeter = 2× π ×Radius Area = π × Radius^2	c : Perimeter s : Area π : Pi r : Radius	???

【Example 3】 Write the following in algebraic expression.

(1) $\frac{2}{5}$ of the sum of x and -1;

(2) A number 4 larger than $\frac{5}{9}$ of a;

(3) A number 5 less than the opposite number of m;

(4) A number 2 greater than the reciprocal of n.

Solution (1) $\frac{2}{5}(x-1)$; (2) $\frac{5}{9}a+4$;

(3) $-m-5$; (4) $\frac{1}{n}+2$.

【Example 4】 Let the number be x. Express the following in terms of x.

(1) Three times of the sum of x and 3;

(2) Subtract -5 from the quotient of 4 divided by x square.

Solution (1) $3(x+3)$; (2) $\frac{4}{x^2}+5$.

【Example 5】 Let the value of A be x and the value of B be y. Write the following in algebraic expression:

(1) Sum of square of A and square of B;

(2) Product of sum of A and B and difference of A and B.

Solution (1) x^2+y^2; (2) $(x+y)(x-y)$.

【Example 6】 Let value of A be x. Express value of B in terms of x.

(1) B is larger than A by 5;

(2) Sum of A and B is 16.

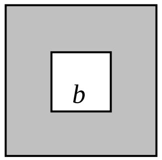
Solution (1) $x+5$;

(2) $16-x$.

Practice

1. Write the following in algebraic expression:
   (1) 2 times of the difference between a and -8;
   (2) Sum of 5% of x and 6% of y;
   (3) a number smaller than the reciprocal of x by 8;
   (4) a number greater than $\frac{3}{4}$ of a by b.
2. Let the value of the number be x. Express the following in terms of x.
   (1) Sum of 8 times the number and 7;
   (2) Difference between cube of the number and -3;
   (3) 2 times of difference between 5 and the number;
   (4) Difference between 7 and quotient of the number dividied by 2.
3. Write the following in algebraic expressions:
   (1) Product of c and the sum of a and b.
   (2) Square of difference between a and b.
   (3) Difference between a square and b square;
   (4) Quotient of product of a and b divided by the difference between a and b.
4. Let the value of A be x and the value of B be y. Express the following in terms of x and y:
   (1) Absolute value of sum of A and B;
   (2) Product of sum of two times A and B, and difference between 2 times A and B;
5. Let the value of A be x. Write the following in algebraic expression.
   (1) B is less than 2 times of A by 6;
   (2) Difference between A and B is 15.

【Example 7】 In Diagram 2-2, the length of the side of the outer square is a cm while the length of the side of the inner square is b cm. Express the area of shaded region in terms of a and b.



a
Diagram 2-2

Solution Area of square with side a cm is a^2 cm^2.
Area of square with side b cm is b^2 cm^2.
So, the area of shaded region is $(a^2 - b^2)$ cm^2.

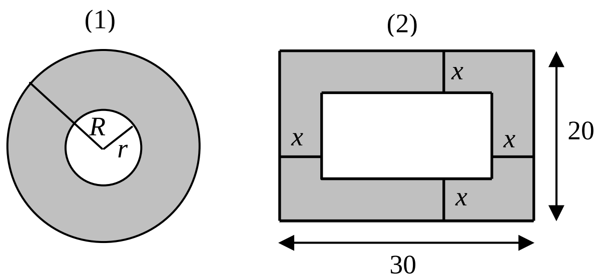
【Example 8】 The distance between Town A and Town B is 245 km. A car travels at a speed of v km per hour from Town A to Town B. Represent the following in terms of algebraic expression:

(1) How long, in hours, does the car take to travel from Town A to Town B?
(2) If the car increases its speed by 3 km per hour, how long, in hours, does it take to travel from Town A to Town B?
(3) After increasing the speed, how much earlier, in hour, does the car arrive?

Solution (1) The car takes $\frac{245}{v}$ hours to travel from Town A to Town B.
(2) After increasing the speed of the car by 3 km per hour, it takes $\frac{245}{v+3}$ hours.
(3) After increasing the speed, the car can arrive $\left(\frac{245}{v} - \frac{245}{v+3}\right)$ hours earlier.

Practice

1. Write down the algebraic expressions of the area of the cross section of the spare part (shaded region).



(No. 1)

Practice

2. A farm has m acres of paddy field and planned to reap S acres each day. Having recruited support labor, it reaps 50 more acres of paddy field per day. Write the following in algebraic expression:
   (1) How many days does it take to complete the reaping originally?
   (2) With increased labor, how many days does it take to complete reaping?
   (3) With increased labor, how much earlier, in days, does it take to complete reaping?
3. There is a class with 50 students. Four-fifths of the students each do a good deeds and the remaining of students each do one more good deeds. Write the total number of good deeds done by the whole class in algebraic expressions.

【Example 9】 Write following in algebraic expression:

(1) 18% saline solution weighs a kg, what is the amount, in kg, of pure salt in the saline solution?

(2) 75% alcoholic solution weighs x g, what is the amount, in g, of pure alcohol?

Remark: 18% saline solution means 1 kg saline solution contains $\frac{18}{100}$ kg of pure salt.;

75% alcohol means 1 g of alcoholic solution contains $\frac{75}{100}$ g of pure alcohol.

Solution (1) For a kg of that saline solution, it contains $\frac{18}{100}a$ kg of pure salt;

(2) For x g of that alcoholic solution, it contains $\frac{75}{100}x$ g of pure alcohol and $\left(x-\frac{75}{100}x\right)=\frac{25}{100}x$ g of water.

Practice

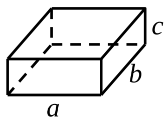
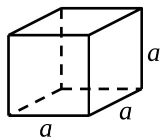
1. Write the following in algebraic expression:
   (1) 25% saline solution weighs m kg, what is the amount, in kg, of pure salt in the saline solution? What is the amount, in kg, of water in the saline solution?
   (2) 72% of sugar solution weighs m kg, what is the amount, in kg, of pure sugar in the sugar solution? What is the amount, in kg, of water in the sugar solution?

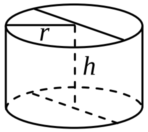
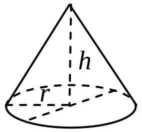
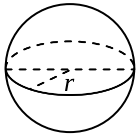
2. A farm originally had 260 acres of paddy field. Its size is increased by 75%. What is the size (in acres) of the farm after the increase?

3. When rough rice is hulled to husked rice, its weight drops to 72%. Now there is rough rice of (G + 10) kg. What is the weight in kg of husked rice produced?

Exercise 7

1. Complete the table

Name	Shape	Literal Formula	Symbol's Meaning	Symbolic Formula
Cuboid		Volume = Length × Width × Height	V : Volume a : Length b : Width c : Height	
Cube		Volume = Side^3	V : Volume a : Side	

Name	Shape	Literal Formula	Symbol's Meaning	Symbolic Formula
Cylinder		Volume = Base Area × Height	V : Volume r : Radiu h : Height	
Cone		Volume $= \frac{1}{3} \times$ Base Area ×Height	V : Volume r : Radius h : Height	
Sphere		Volume $= \frac{4}{3}\pi \times$ Radius^3	V : Volume r : Radius	

2. Each exercise books cost 9 dollars and each pencil costs 6 dollars.
   (1) How much does it cost for 5 exercise books and 4 pencils?
   (2) How much does it cost for 2 exercise books and y pencils?
   (3) How much does it cost for x exercise books and 3 pencils?
   (4) How much does it cost for x exercise books and y pencils?

3. Write the following in algebraic expression:
   (1) Product of 6 times the sum of a and b;
   (2) 2 times the product of x and y;
   (3) Sum of 1 and the product of a and b;
   (4) Quotient of difference between a and 5 divided by b.

4. Write the following in expression:
   (1) Sum of $1\frac{1}{2}$ times of x and 7;
   (2) $\frac{2}{3}$ of b times of y;
   (3) Difference between opposite number of x and -2;
   (4) Quotient of sum of a and b divided by c;

   (5) A number 13 larger than the product of x and y;
   (6) A number 108 less than 160% of a;
   (7) Difference between the quotient of a divided by b and the reciprocal of c;
   (8) Sum of three times x square and 25% of y;
   (9) Difference between m cube and n cube;
   (10) The cube of difference between m and n.

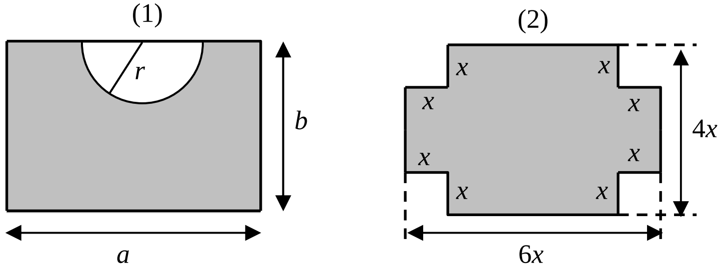
5. Write the following in algebraic expression:
   (1) A number c larger than two times of sum of a and b;
   (2) Square of sum of a and b and c;
   (3) A number c smaller than three times of the difference between a cube and b cube;
   (4) Product of the difference between the sum of a cube, b cube, and c cube and 3 times the product of a, b, and c.

6. Let the number be x. Express the following in terms of x:
   (1) Sum of 13 and two times x square;
   (2) Three times of the difference between the absolute value of -3 and x;
   (3) Difference between x and opposite number of x;
   (4) Quotient of the difference between x cube and 3 divided by x.

7. Let the value of A be x and the value of B be y. Express the following in terms of x and y:
   (1) Three times the product of A and B;
   (2) Product of the square of sum of A and B multiplied by the square of the difference between A and B;
   (3) Difference between two times of A and the quotient of B divided by 3;
   (4) Sum of A square and B square and the product of A and B.

8. Let the value of A be x. Express the value of B in terms of x:
   (1) A is less than B by 10;
   (2) Sum of A and B is 15;
   (3) Three times of A is larger than B by 6;
   (4) Two times of A is less than B by 9.

9. Express the area of shaded regions in terms of a, b, r and x:



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10. A farm owns m acres of vegetable field and n acres of paddy field. Each vegetable field requires a kg of fertilizer per acre and each paddy field needs b kg of fertilizer per acre. Express the total amount, in kg, of fertilizer in terms of a, b, m and n.

11. In a farm, there are m acres of paddy fields which yield a kg of rice per acre and n units of paddy fields which yield b kg of rice.per acre. Express the average yield of each paddy field in terms of m, n, a and b.

12. An automobile factory produces s cars in August. The number of cars produced in September is 5 less than twice number of cars produced in August. Express the number of cars produced in September in terms of s.

13. A cultivator is used to plough 120 acres of fields with a plan to plough x acres of fields each day. How many days does it take to complete ploughing? If the cultivator ploughs 5 more acres of fields each day, how many days does it take to complete ploughing? How many days has the ploughing been shortened after the increase?

14. A factory has to produce a products with a production plan to manufacture b products per day. How many days does it take to complete the manufacturing process? If the production plan is increased by manufacturing d more products each day, how many days will the manufacturing process be shortened?

15. A factory originally has a workers and recruits some more workers this year. The number of new workers is 6% of the original number of workers. How many workers does the factory have now?

16. Mr. Lee's monthly salary is m dollars and 45% of it is spent to pay rent. Write the dollar amount of Mr. Lee's monthly savings in algebraic expression.

17. 20% saline solution weighs n kg. What is the amount, in kg, of pure salt in the saline solution? What is the amount, in kg, of water in the saline solution?

18. In an automobile factory, the number of cars manufactured in 2013 is 5 times of that in 2012. It is known that the number of cars manufactured in 2012 is q. Write the number of cars manufactured in 2013 in algebraic expression.

19. Positive integers which are successively larger than the previous one by 1, such as 14, 15, 16, are called positive consecutive integers. Given three positive consecutive integers, (1) the middle one is m, express the remaining two integers in terms of m; (2) the largest one is n, express the remaining two integers in terms of n.

20. A slow train leaves Station A for Station B, traveling at a speed of 56 km per hour; simultaneously, an express train leaves Sttaion B for Station A, traveling at a speed of 72 km per hour. Two trains meet after t hours. Express the distance between Station A and Station B in terms of t.

21. It is required to pump water out of a pool. Pump A working alone takes a hours to finish pumping, while Pump B working alone takes b hours. Express the following in terms of a and b:
    (1) By using Pump A alone, what fraction of the pool will be pumped in one hour?
    (2) By using Pump B alone, what fraction of the pool will be pumbed in one hour?
    (3) By using both Pump A and Pump B together, what fraction of the pool will be pumped in one hour?

22. It takes a days for Team A to excavate a conduit. Team A excavates for three days and the remaining excavation work will be finished by another team. Express the amount of the remaining excavation work in terms of a.

2.2 **Value of Algebraic Expression**

Given that the base and the height of a triangle is a cm and h cm respectively, we know that the area of the triangle can be expressed as

$$\frac{1}{2}ah \ (\text{cm}^2)$$

Now, using the algebraic expression, we can calculate the area of the triangles shown below (Diagram 2-3).

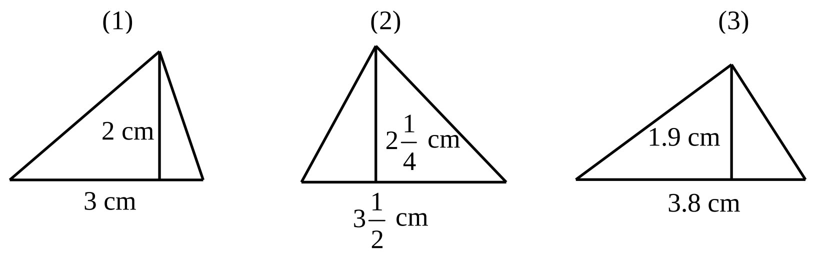


Diagram 2-3

In (1), $a = 3$, $h = 2$,

$$\frac{1}{2}ah = \frac{1}{2}\times 3\times 2 = 3;$$

In (2), $a = 3\frac{1}{2}$, $h = 2\frac{1}{4}$,

$$\frac{1}{2}ah = \frac{1}{2}\times 3\frac{1}{2}\times 2\frac{1}{4} = \frac{1}{2}\times\frac{7}{2}\times\frac{9}{4} = \frac{63}{16} = 3\frac{15}{16};$$

In (3), $a = 3.8$, $h = 1.9$,

$$\frac{1}{2}ah = \frac{1}{2}\times 3.8\times 1.9 = 3.61.$$

Therefore the areas of the three triangles are 3 cm^2, $3\frac{15}{16}$ cm^2, and 3.61 cm^2 respectively.

Substituting values into letters, we can calculate the value of an algebraic expression, the result of the calculation is called **the value of the Algebraic expression**.

It can be seen from the above example, when the values substituted into the letters are changed, the value of the algebraic expression also changes. Therefore, the value of an algebraic expression depends on the values substituted into the letters.

Although we can substitute various values into the letters, we should not substitute unreasonable values into the letters, otherwise the value of the algebraic expression would not have practical meaning. Take the above algebraic expression $\frac{1}{2}ah$ as an example. Since it represents the area of a triangle, so the values of a and h cannot be zero or negative. Consider the algebraic expression $\frac{12}{x}$ as another example, the divisor x cannot take the value zero, otherwise the division is invalid.

【Example 1】 Find the value of $-2x+5$ according to the following value of x:

(1) $x = 4$; (2) $x = 0$; (3) $x = -5$.

Solution (1) When $x = 4$, $-2x+5 = -2\times 4+5 = -3$;

(2) When $x = 0$, $-2x+5 = -2\times 0+5 = 5$;

(3) When $x = -5$, $-2x+5 = -2\times(-5)+5 = 15$.

Observed from Example 1, when a value is assigned to x, it fixes the value for the algebraic expression $-2x+5$ (Diagram 2-4).

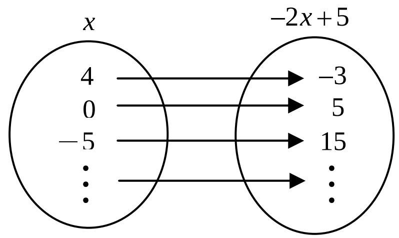


Diagram 2-4

Practice

Complete the table:

x	-2	-1	0	1	2
$4x-5$	-13				
x^2+2			2		

【**Example 2**】 When $a=-2$, find the value of $2a^3-\frac{1}{2}a^2+3$.

Solution When $a=-2$,

$$2a^3-\frac{1}{2}a^2+3=2\times(-2)^3-\frac{1}{2}\times(-2)^2+3$$
$$=-16-2+3$$
$$=-15$$

【**Example 3**】When $x=\frac{1}{2}$, $y=-2$, find the value of the following algebraic expression:

(1) $2x^2-y+3$; (2) $\frac{4x-2y}{xy}$.

Solution (1) When $x=\frac{1}{2}$, $y=-2$,

$$2x^2-y+3=2\times\left(\frac{1}{2}\right)^2-(-2)+3$$
$$=5\frac{1}{2}$$

(2) When $x=\frac{1}{2}$, $y=-2$,

$$\frac{4x-2y}{xy}=\frac{4\times\frac{1}{2}-2\times(-2)}{\frac{1}{2}\times(-2)}=\frac{2+4}{-1}=-6.$$

【**Example 4**】When (1) $x=-5$, $y=3$; (2) $x=5$, $y=-3$, find the value of

$$|x|+|y|-2|x|\bullet|y|$$

Solution (1) When $x=-5$, $y=3$,

$$|x|+|y|-2|x|\bullet|y|=|-5|+|3|-2|-5|\bullet|3|$$
$$=5+3-2\times5\times3$$
$$=-22$$

(2) When $x=5$, $y=-3$,

$$|x|+|y|-2|x|\bullet|y|=|5|+|-3|-2|5|\bullet|-3|$$
$$=5+3-2\times5\times3$$
$$=-22$$

Practice

1. When $x=-2$, compute the value of x^3-3x^2+2x+7.
2. When $x=-3$, $y=4$, compute the value of $x^2+3xy-y^2-5$.
3. When $a=2$, $b=-1$, $c=-1\frac{1}{2}$, compute the value of the following algebraic expression:

   (1) a^2-b^2+2bc; (2) $\frac{2c}{a+b}$.
4. When $x=-3$, $y=4$, compute the value of the following algebraic expression:

   (1) $|x|+3|y|$; (2) $|x+3y|$.

【**Example 5**】Diagram 2-5 shows that the cross-section of a channel is a trapezium. Express its area in terms of *a*, *b* and *h* and calculate the area when *a* = 2.8, *b* = 0.8, *h* = 1 (unit: m).

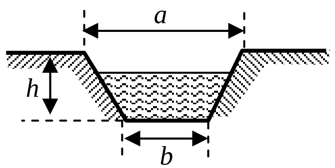


Diagram 2-5

Solution Since the cross-section of the channel is a trapezium, and the upper base, lower base and height are *a*, *b* and *h* respectively, so the area of the cross-section is

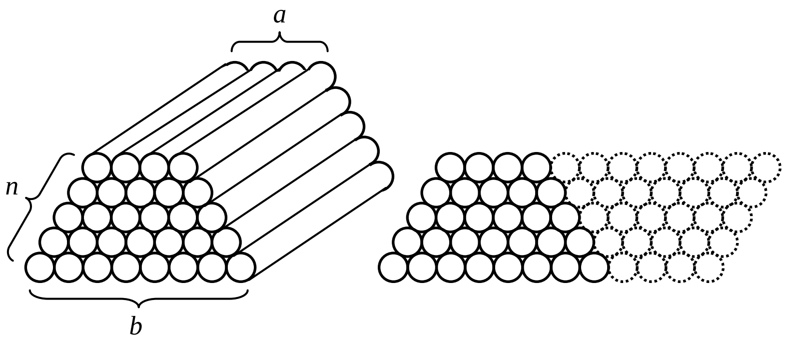
$$\frac{1}{2}(a+b)h.$$

When $a = 2.8$, $b = 0.8$, $h = 1$,

$$\begin{aligned}\frac{1}{2}(a+b)h &= \frac{1}{2}(2.8+0.8)\times 1\\ &= \frac{1}{2}\times 3.6\times 1\\ &= 1.8\end{aligned}$$

Answer: Area of cross-section is 1.8 m^2.

【**Example 6**】Workers often place the cylindrical steel tubes as shown in Diagram 2-6(1). The lower layer has one more steel tube than the upper one. By counting the number of steel tubes at the top *a*, the number of tubes at the bottom *b* and the number of layer *n*, then the formula $\frac{n(a+b)}{2}$ can be used to find out the total number of steel tubes. When $n = 6$, $a = 4$, $b = 8$, find the total number of steel tubes.



(1) (2)

Diagram 2-6

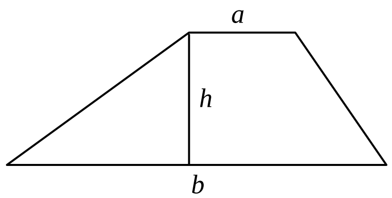
Solution When $n = 6$, $a = 4$, $b = 8$,

$$\frac{n(a+b)}{2} = \frac{6\times(4+8)}{2} = 36.$$

Answer: When $n = 6$, $a = 4$, $b = 8$, there are 36 steel tubes.

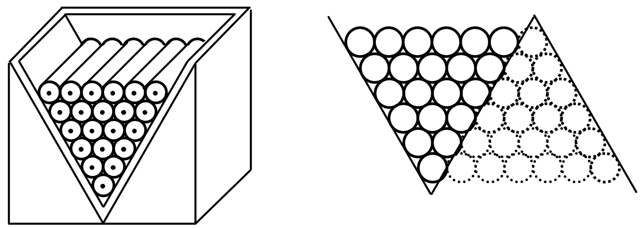
Practice

1. The cross-section of a dam is a trapezium. Express its area in terms of a, b and h. When $a = 2$, $b = 13$, $h = 3$, calculate its area.



(No. 1)

2. The Diagram below shows a V-shaped pencil case. The first layer can hold 1 pencil and the second layer can hold 2 pencils. Accordingly, each layer can hold one more pencil than the lower one. By counting the number of pencils placed at the top layer *n*, the formula $\frac{n(n+1)}{2}$ can be used to find out the total number of pencils in the pencil case. When n=6, n=11, calculate the total number of pencils in the pencil case.



(1) (2)

(No. 2)

Exercise 8

1. Given the following value of *x*, compute the value of $\frac{3}{5}x-3$:

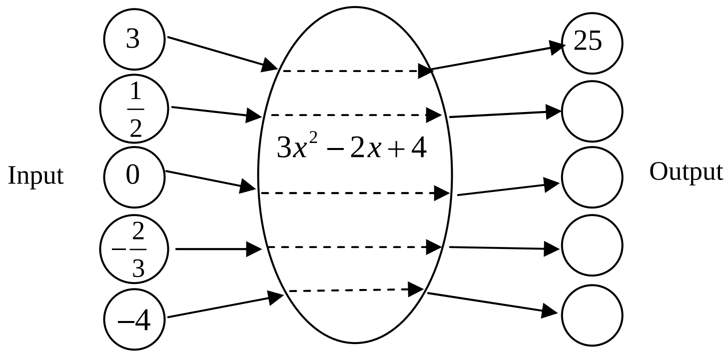
   (1) $x=5$; (2) $x=1\frac{1}{5}$; (3) $x=0$;

   (4) $x=-1$; (5) $x=-2\frac{1}{2}$.

2. For each value of *a* given below, find the value of $3a^3-2a^2+a+5$:

   (1) $a=2$; (2) $a=0$; (3) $a=-1$; (4) $a=\frac{1}{3}$.

3. Use *x* as an input value, find the output value below:



(No. 3)

4. When $x=-1$, $y=6$, compute the value of the algebraic expression below.

   (1) $3x+2y$; (2) xy^2; (3) $(xy)^2$;

   (4) $x+y^2$; (5) $(x+y)^2$.

5. When $x=-1$, $y=6$, compute the value of the algebraic expression below:

   (1) $\frac{x+y}{x-y}$; (2) $\frac{x^2-y^2}{xy-y}$; (3) $\frac{x^2+y^2}{x+y}$; (4) $\frac{x^3-y^3}{x-y}$.

6. When $a=2$, $b=-3$, $c=-1$, compute the value of the algebraic expression below:

   (1) $3a-4b+c$; (2) b^2-4ac;

   (3) $a^2-b^2+2bc-c^2$; (4) $\frac{c}{a+b}$;

   (5) $a^3+b^3+c^3-3abc$; (6) $(a-b)(b-c)(c-a)$.

7. Fill in the following table:

x	-4	$-3\frac{1}{2}$	-2	$-1\frac{2}{3}$	0	$1\frac{2}{3}$	2	$3\frac{1}{2}$	4
$\lvert x\rvert$									
$\lvert x-1\rvert-3$									
x^2+1									
x^2-x+1									

8. When $m=25.87$, $n=19.04$, calculate the algebraic value below:

   (1) m^2-n^2; (2) m^3+n^3.

9. Take $\pi=3.14$, calculate the volume of a sphere with diameter $D=69.5$ cm. (Give the answer correct to 2 significant figures. Volume of a sphere is $V=\frac{1}{6}\pi D^3$).

10. (1) Every even number can be expressed in terms of $2n$ (*n* is an integer). When *n* = 0, 1, 2, 3, find the even numbers accordingly.

    (2) Every odd number can be expressed in terms of $2n+1$(*n* is an integer). When *n* = 0, 1, 2, 3, find the odd numbers accordingly.

11. If the pit distance of mango trees is *a* m, separation distance is *b* m, then the number of mango tree pits $\frac{600000}{ab}$ per acre. When *a* = 4, *b* = 5, compute the number of mango tree pits.

12. There are m teams in a round robin tournament (in which each team meets all other teams once in turn). The formula $\frac{m(m-1)}{2}$ can be used to find out the number of games required. Now there are 4 teams playing in the tournament, what is the number of games required? What if there are 8 teams? Or 10 teams?

13. A light bulb of a watt will consume $\frac{at}{1000}$ kilowatt of electricity if switched on for t hours. If on average the light bulb is switched on for 3 hours each day, what is the amount of electricity, in kilowatt, that can be saved by using a 25 watt bulb instead of using a 40 watt butt?

14. There is a cylindrical barn with radius 2.5 m and height 3.5 m. Using the the formula $V=\pi r^2 h$, calculate the weight of grain (in kg) that can be stored in the barn (1 m^3 of grain weighs about 1150 kg).

2.3 **Integral Expressions**

Look at the algebraic expressions below:

$$2x,\ -\frac{3}{4}a^2,\ \frac{4x^2y^3}{7},\ ab^3,\ -x^2yz.$$

These are algebraic expressions, formed by the product of numbers and letters. These algebraic expressions are called **monomials**.

Single number or letter, for example, -5, x, is also a monomial.

The numerical factor of the monomial $2x$ is 2. The numerical factor of the monomial $-\frac{3}{4}a^2$ is $-\frac{3}{4}$. The numerical factor of the monomial $\frac{4x^2y^3}{7}$ (can be regarded as $\frac{4}{7}x^2y^3$) is $\frac{4}{7}$. The numerical factor (or literal factor) of a monomial is called **the coefficient of the monomial**, abbreviated as **coefficient**. The coefficients of $2x$, $-\frac{3}{4}a^2$, $\frac{4x^2y^3}{7}$ are respectively 2, $-\frac{3}{4}$, $\frac{4}{7}$.

When the coefficient of a monomial is 1 or -1, '1' is usually omitted. For example, $1ab^3$ is written as ab^3 and $-1x^2yz$ is written as $-x^2yz$.

For monomial $2x$, the index of x is 1; for $-\frac{3}{4}a^2$, the index of a is 2; for $\frac{4x^2y^3}{7}$, the sum of the indicies of x and y is 2 + 3 = 5; for $-x^2yz$, the sum of the indicies of x, y and z is 2 + 1 + 1 = 4. In a monomial, the sum of indicies of all letters is called **the degree of the monomial**. For example, $2x$ is a degree one monomial, $-\frac{3}{4}a^2$ is a degree two monomial, $\frac{4x^2y^3}{7}$ is a degree five monomial and $-x^2yz$ is a degree four monomial.

Practice

1. (*Mental*) Consider the following algebraic expression, which one is a monomial and which one is not?

$$-2x^3,\ ab,\ 1+x,\ \frac{4ab^2}{5},\ -y,\ 6x^2-\frac{1}{2}x+7.$$

2. What is the degree of the following monomials? What are their coefficients?

$$8x,\ -2a^2bc,\ xy^2,\ -t^2,\ \frac{3x^2y}{10},\ \frac{5}{7}vt,\ -10xyz.$$

Look at the algebraic expressions below:

$$4x-5,\ 6x^2-\frac{1}{2}x+7,\ a^2+ab+b^2.$$

For these algebraic expressions, $4x-5$ is the sum of monomials $4x$ and -5; $6x^2-\frac{1}{2}x+7$ is the sum of monomials $6x^2$, $-\frac{1}{2}x$ and $+7$; and a^2+ab+b^2 is the sum of monomials a^2, $+ab$ and $+b^2$.

The sum of monomials is called a **polynomial**. In a polynomial, each monomial is called a **term**. For example, in the polynomial $6x^2-\frac{1}{2}x+7$, $6x^2$, $-\frac{1}{2}x$ and $+7$ are terms. Take note to pay particular attention to include the sign of the term. For example, the second term of the polynomial $6x^2-\frac{1}{2}x+7$ is $-\frac{1}{2}x$, not $\frac{1}{2}x$.

A polynomial is also described by the number of terms it contains. For example, $4x-5$ is a binomial, $6x^2-\frac{1}{2}x+7$, a^2+ab+b^2 are trinomials.

In a polynomial, **the degree** of a polynomial refers to the term with the highest degree. For example, the degree of $4x-5$ is 2 and the degree of both $6x^2-\frac{1}{2}x+7$ and a^2+ab+b^2 is 3.

In a polynomial, the term which does not contain any letter is called the **constant term**. For example, in polynomial $4x-5$, -5 is the constant term. In polynomial $6x^2-\frac{1}{2}x+7$, $+7$ is the constant term.

For convenience of computing, we can apply the Commutative Law for Additions to re-arrange the terms in a polynomial in descending order (or in ascending order) according to the index of its term with respect to a letter. For example, the polynomial

$$x^3+5x-6-4x^2$$

can be arranged starting from the term with the largest index of x to the amallest

$$x^3-4x^2+5x-6,$$

or starting from the term with the smallest index of x to the largest

$$-6+5x-4x^2+x^3.$$

Arranging the polynomial by starting with the term with the largest index of a letter is called **arranging in descending order**. Arranging the polynomial by starting with the term with the smallest index of a letter is called **arranging in ascending order**. For example, if the polynomial

$$3x^2y+4xy^2-x^3-5y^3$$

is arranged in ascending orders of y, it becomes

$$-x^3+3x^2y+4xy^2-5y^3,$$

if it is arranged in descending orders of y, then it becomes

$$-5y^3+4xy^2+3x^2y-x^3.$$

Monomials and polynomials are categorically called **integer expressions**.

Practice

1. (*Mental*) For the following algebraic expression, which one is a polynomial? Which one is not? Why?

$$\frac{3}{5}x-x^2+1,\ 3ab+\frac{a}{b},\ \frac{a+c}{b},\ a^2+2ab+b^2,$$

$$x^2-y^2,\ 3x-\frac{1}{3},\ 8x+y.$$

2. (*Mental*) How many terms does the following polynomial contain? What is the degree of the polynomial?

$$2x-8,\ a+b-c,\ -x^2-\frac{3}{5}x+\frac{3}{4},\ x^2-2xy+y^2,$$

$$m^3-1,\ a^3+ab+b.$$

3. (1) Arrange the polynomial $a^4-7a+6+3a^5-4a^3$ in descending order of a;
   (2) Arrange the polynomial $3x^3y-y^4+5xy^3-x^4$ in ascending order of x.

Practice

4. Arrange the following polynomial in descending order of *x*, then re-arrange in ascending order of *x*:

   (1) $12x-10x^2+8$; (2) x^2+y^2+2xy;

   (3) $3x^2y-5xy^2+y^3-2x^3$; (4) $6+2x^2+5x-9x^5+7x^3$.

Exercise 9

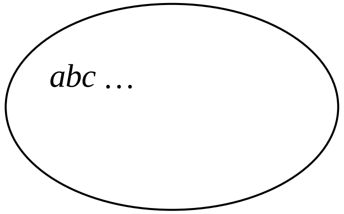
1. For the following polynomial, which one is a monomial? Write it inside the set of monomials in the Diagram.

   abc, $-2x^3$, $x+y$,

   -15, $-m$, $3x^2+4x-2$,

   $xy-\frac{1}{2}a$, $x^4+x^2y^2+y^4$,

   a^2-ab+b^2, $3ab^2$.



The set of monomials

(No. 1)

2. What is the coefficient of a monomial? What is the coefficient of each of the monomials below?

$$15a^2,\ xy,\ \frac{2}{3}a^2b^3,\ 0.11m^3,\ -a^2bc,\ \frac{3x^2y}{5}.$$

3. How to calculate the degree of a monomial? What is the degree of each of the respective monomials shown in Question 2?

4. In the following polynomial, write down each of the terms separately:

   (1) $4x^2-\frac{1}{2}$; (2) $a^3+ab^3+b^4$;

   (3) $a^4+b^4-a^2b^2$; (4) $-x^3-\frac{3x^2y}{4}+2x^2y^2-y^5$.

5. How to calculate the degree of a polynomial? What is the degree of the each of the polynomial shown in Question 4?

6. For each of the following polynomial, what is the degree and what is the number of terms?

   (1) $3x^3-\frac{1}{4}$; (2) $3a-2b$;

   (3) $3x^2-2x+1$; (4) a^6+b^6.

7. Arrange the following polynomial in descending order of *x*, and then re-arrange it in ascending order of *x*:

   (1) $13x-3x^2-2x^3-6$; (2) x^2+y^2-2xy;

   (3) $3x^2y-3xy^2+y^3-x^3$; (4) $\frac{2}{3}ax^4+\frac{1}{2}bx-\frac{1}{6}cx^3+\frac{5}{12}d$.

II Addition and Subtraction of Polynomials

2.4 **Like Terms**

Look at the polynomials below:

$$\underline{4xy^2}\ \underline{\underline{+3x^3}}-6x^3y\ \underline{-5xy^2}\ \underline{\underline{\underline{+7}}}\ \underline{\underline{+4x^3}}\ \underline{\underline{\underline{-10}}}\ \underline{\underline{-x^3}}$$

The first term $4xy^2$ and the fourth term $-5xy^2$ contains the same letters *x* and *y*, and the degree of each letter is the same in both terms. Likewise, the terms containing the same letters and the same powers of each letter are called **like terms**. All constant terms are like terms. Considering the above polynomial, $4xy^2$ and $-5xy^2$ are like terms, $+3x^3$, $+4x^3$ and $-x^3$ are like terms, $+7$ and -10 are like terms, but $-6x^3y$ has no like terms in this case.

Practice

1. (*Mental*) Are the following pairs of terms like terms? Why?

   (1) $2x^2y$ and $5x^2y$; (2) $\frac{1}{3}ab^3$ and $-\frac{4}{3}ab^3$;

   (3) $4abc$ and $4ab$; (4) $0.2x^2y$ and $0.2xy^2$;

   (5) mn and $-mn$; (6) $\frac{1}{4}st$ and $5ts$;

   (7) $12x^2y^2$ and $-12x^2y^3$; (8) $2x^2$ and $2x^3$;

   (9) a^3 and 5^3; (10) -125 and 12.

2. Find the like terms in the polynomial below:

   (1) $5x^2y-3y^3-x-4+x^2y+2x-9$;

   (2) $4ab-7a^2b^2-8ab^2+5a^2b^2-9ab+a^2b^2$.

In polynomials, the like terms can be combined. For example, in the polynomial

$$3x-4y+y+2x$$

$3x$ and $2x$ are like terms and $-4y$ and y are like terms. According to the distributive law,

$$3x+2x=(3+2)x=5x,$$
$$-4y+y=(-4+1)y=-3y.$$

Combining like terms into one term is called **grouping like terms**. In grouping like terms, **add the coefficients of the like terms to form the coefficient of the grouped term while keeping the letters and the indicies of the letters unchanged.**

【Example 1】 Group the like terms:

(1) $3x^3-x^3$;

(2) xy^2-5xy^2.

Solution (1) $3x^3-x^3=(3-1)x^3=2x^3$;

(2) $xy^2-5xy^2=(1-5)xy^2=-4xy^2$.

【Example 2】 Group the like terms in the polynomial

$$4x^2-8x+5-3x^2+6x-2.$$

Solution $\underline{4x^2}\ \underline{\underline{-8x}}+5\underline{-3x^2}\ \underline{\underline{+6x}}-2 = (4-3)x^2+(-8+6)x+(5-2)$

$= x^2-2x+3$.

【Example 3】 Group the like terms in he polynomial

$$4a^2+3b^2+2ab-4a^2-2b^2-b^2.$$

Solution $\underline{4a^2}\ \underline{\underline{+3b^2}}+2ab\underline{-4a^2}\ \underline{\underline{-2b^2}}\ \underline{\underline{-b^2}}$

$= (4-4)a^2+(3-2-1)b^2+2ab$

$= 2ab$.

Practice

1. Group the like terms:

   (1) $5x+4x$; (2) $-7ab+6ab$; (3) $-5x^2+\frac{1}{5}x^2$;

   (4) $-12vt+12vt$; (5) $mn+mn$; (6) $-ab+3ab$.

2. Is the following result of grouping like terms correct?

   (1) $3a+2b=5ab$; (2) $5y^2-2y^2=3$;

   (3) $4x^2y-5xy^2=-x^2y$; (4) $a+a=2a$;

   (5) $7ab-7ba=0$; (6) $3x^2+2x^3=5x^5$.

3. Group like terms:

   (1) $6x-10x^2+12x^2-5x$;

   (2) $7a^2-2ab+2a^2+b^2+3ab-2b^2$;

   (3) $x^5-x^3+4x^2-2x^5-2x^2+3x^4$;

   (4) $5xy-3x^2+y^2-4xy+4x^2-2y^2$.

【Example 4】 Compute the value of $2x^2-5x+x^2+4x-3x^2-2$ if $x=\frac{1}{2}$.

Solution $2x^2-5x+x^2+4x-3x^2-2 = (2+1-3)x^2+(-5+4)x-2$

$= -x-2$

When $x=\frac{1}{2}$, the expression $= -\frac{1}{2}-2 = -2\frac{1}{2}$.

When dealing with polynomial, if there are like terms, it is easier to group the like terms first before substituting the values to computie the value of the polynomial.

【**Example 5**】Find the value of $3a+abc-\frac{1}{3}c^2-3a+\frac{1}{3}c^2$ if $a=-\frac{1}{6}$, $b=2$ and $c=-3$.

Solution $3a+abc-\frac{1}{3}c^2-3a+\frac{1}{3}c^2 = (3-3)a+abc+\left(-\frac{1}{3}+\frac{1}{3}\right)c^2$

$$= abc$$

When $a=-\frac{1}{6}$, $b=2$ and $c=-3$,

the expression = $\left(-\frac{1}{6}\right)\times 2\times(-3) = 1$.

If the coefficients of two like terms are opposite numbers, then these two terms will cancel out after the grouping of like terms.

Practice

Find the value of the following polynomial:

1. $3a+2b-5a-b$, here $a=-2$, $b=1$;
2. $5x^2+4-3x^2-5x-5+6x$, here $x=-3$;
3. $3ab-5ab^3+0.5a^3b-3ab^2+5ab^3-4.5a^3b$, here $a=1$, $b=1\frac{1}{2}$;
4. $4xy-3x^2-xy+y^2+x^2-3xy-2y+2x^2$, here $x=1\frac{13}{15}$, $y=-1$;
5. $\frac{1}{2}x^2-\frac{1}{4}x+0.2x^3+0.25x-0.5x^2-\frac{1}{5}x^3$, here $x=\frac{12}{13}$.

2.5 **Removing Brackets**

Look at the computation below:

$$13\underline{+(7-5)}=13+2=\underline{\underline{15}};$$

$$\underline{13+7}-5=20-5=\underline{\underline{15}};$$

$$8a\underline{+(5a-a)}=8a+4a=\underline{\underline{12a}};$$

$$\underline{8a+5a}-a=13a-a=\underline{\underline{12a}}.$$

Observe that:

$$13+(7-5)=13+7-5\ ; \qquad (1)$$

$$8a+(5a-a)=8a+5a-a\ 。 \qquad (2)$$

Look at the computation below:

$$13\underline{-(7-5)}=13-2=\underline{\underline{11}};$$

$$\underline{13-7}+5=6+5=\underline{\underline{11}};$$

$$8a\underline{-(5a-a)}=8a-4a=\underline{\underline{4a}};$$

$$\underline{8a-5a}+a=3a+a=\underline{\underline{4a}}.$$

Observe that:

$$13-(7-5)=13-7+5\ ; \qquad (3)$$

$$8a-(5a-a)=8a-5a+a\ 。 \qquad (4)$$

From (1), (2), (3) and (4), we can deduce the rule of removing brackets:

if the bracket is prefixed by a '＋' sign, then remove the bracket without changing the sign of any term inside the bracket;

if the bracket is prefixed by a '－' sign, then remove the brackets and reverse the sign of all terms inside the bracket.

【**Example 1**】Remove the bracket:

(1) $a+(-b+c-d)$; (2) $a-(-b+c-d)$.

Solution (1) $a+(-b+c-d)=a-b+c-d$;

(2) $a-(-b+c-d)=a+b-c+d$.

【**Example 2**】Remove the brackets first, then group like terms:

(1) $2y+(-2y+5)-(-3y+2)$;

(2) $4a+2(a-c)$.

Solution (1) $2y+(-2y+5)-(-3y+2) = 2y-2y+5+3y-2$;

$= 3y+3$

(2) $4a+2(a-c)=4a+(2a-2c)=4a+2a-2c=6a-2c$.

【**Example 3**】Simplify $(5a-3b)-3(a^2-2b)$.

Solution $(5a-3b)-3(a^2-2b) = 5a-3b-3a^2+6b$;

$= 5a-3a^2+3b$.

Practice

1. Remove the brackets:
   (1) $a+(b+c)$; (2) $a-(b+c)$; (3) $a+(-b-c)$;
   (4) $a-(-b-c)$; (5) $(a+b)+(c+d)$; (6) $-(a+b)-(c+d)$;
   (7) $(a-b)-(-c+d)$; (8) $-(a-b)+(-c-d)$.
2. In the following equation, is the removal of brackets correct? Correct the mistake if there is any.
   (1) $a^2-(2a-b+c)=a^2-2a-b+c$;
   (2) $a-(-b+c-d)=a+b+c-d$;
   (3) $-(x-y)+(xy-1)=-x-y+xy-1$.
3. Simplify:
   (1) $5a+(3x-3y-4a)$; (2) $3x-(4y-2x+1)$;
   (3) $7a+3(a+3b)$; (4) $(x^2-y^2)+4(2x^2-3y)$;
   (5) $(a^2-2ab+b^2)-(a^2-b^2)$; (6) $5(x-2)-3(2x-1)$.

2.6 **Adding Brackets**

Observe from removing the brackets:

$$a+(b-c)=a+b-c$$

$$a-(b-c)=a-b+c$$

On the contrary, the above equations can be written as:

$$a+b-c=a+(b-c)$$

$$a-b+c=a-(b-c)$$

So we can see that adding brackets will have to obey the following rule:

if the bracket is prefixed by a '+' sign, then every term inside the bracket remains unchanged; if the bracket is prefixed by a '−' sign, then every term inside the bracket will have its sign reversed.

With the rule of adding brackets, we can add a bracket to a polynomial or part of the polynomial without changing its value.

【**Example 1**】Wihtout changing the value of $3a-2b+c$, add the brackets according to the following instructions:

(1) Add a pair of brackets prefixed with a '+' sign to the polynomial;

(2) Add a pair of brackets prefixed with a '−' sign to the polynomial.

Solution (1) $3a-2b+c=+(3a-2b+c)$;

(2) $3a-2b+c=-(-3a+2b-c)$.

【**Example 2**】Wihtout changing the value of x^3-5x^2-4x+9, add the brackets according to the following instructions:

(1) Add a pair of brackets prefixed with a '+' sign to group the last two terms;

(2) Add a pair of brackets prefixed with a '−' sign to group the last two terms.

Solution (1) $x^3-5x^2-4x+9=x^3-5x^2+(-4x+9)$;

(2) $x^3-5x^2-4x+9=x^3-5x^2-(4x-9)$.

Practice

1. Fill in the bracket with appropriate terms:
   (1) $a+b+c-d=a+(\qquad\qquad)$;
   (2) $a-b+c-d=a-(\qquad\qquad)$;
   (3) $a-b-c-d=a-b+(\qquad\qquad)$;
   (4) $a+b-c+d=a+b-(\qquad\qquad)$.

Practice

2. (1) Without changing the value of $m^2+mn-5m-5n$, add a pair of brackets prefixed with a '+' sign to group the first two terms and a pair of brackets prefixed with a '−' sign to group the last two terms.

(2) Without changing the value of $m^2+mn-5m-5n$, add a pair of brackets prefixed with a '−' sign to group the first two terms and a pair of brackets prefixed with a '+' sign to group the last two terms.

Exercise 10

1. Are the following pairs of terms like terms?

(1) $\frac{1}{3}x^2y$ and $-3x^2y$; (2) $0.2a^2b$ and $0.2ab^2$;

(3) $11abc$ and $9ab$; (4) $3m^2n^3$ and $-n^3m^2$;

(5) $5xy$ and $25yx$; (6) $4xy^2z$ and $4x^2yz$;

(7) 125 and $-4\frac{1}{8}$; (8) 6^2 and x^2.

2. Group like terms:

(1) $15x+4x-10$; (2) $-6ab-ab+8ab$;

(3) $-p^2-p^2-p^2$; (4) $1\frac{2}{3}+\frac{5}{6}-\frac{1}{2}$;

(5) $\frac{1}{3}x^3-\frac{5}{6}x^3+\frac{1}{2}x^3$;

(6) $-4a^2b+5a+5a^2b+2a-3$;

(7) $\frac{1}{4}x-0.3y-\frac{1}{2}x+0.3y$; (8) $m+m+m-n^2-n^2$;

(9) $11x^2+4x+1-x^2-4x-5$;

(10) $5x^2+4-3x^2-5x+6x^3+3x$.

3. Treat $(a+b)$ or $(x-y)$ as one term, group the like terms in the following polynomial:

(1) $4(a+b)+2(a+b)-(a+b)$;

(2) $3(x-y)^2-7(x-y)+8(x-y)^2+6(x-y)$.

4. For the following polynomial, first group the like terms and then find its value:

(1) $3c^2-8c+2c^3-13c^2-2c-2c^3+3$, here $c = 4$;

(2) $3y^4-4x^2y-4y^4+2x^3y$, here $x=-2$, $y=\frac{2}{3}$.

5. In the following equation, are the removing of brackets correct? Correct the mistake if there is any.

(1) $a^2-(2a+b+c)=a^2-2a+b+c$;

(2) $a+(-3x+2y-1)=a-3x+2y-1$;

(3) $(a+1)-(-b+c)=a+1-b-c$;

(4) $-(2x-y)+(z-1)=-2x-y+z-1$.

6. Simplify:

(1) $(2x-3y)+(5x-4y)$;

(2) $(8a-7b)-(4a-5b)$;

(3) $a-(2a+b)+2(a-2b)$;

(4) $3(5x-4)-2(3x+5)$;

(5) $(8x-3y)-[4x+(3y-z)]+2z$;

(6) $-4x^2+[5x-8x^2-(-13x^2+4x)+2]-1$;

(7) $2a-\{-3b+[4a-(3a-b)]\}$;

(8) $-[-(-a^2)-b^2]-[+(-b^2)]$.

7. Fill in the bracket with appropriate terms:

(1) $4x^2-3x+6=+(\qquad\qquad)$;

(2) $4x^2-3x+6=-(\qquad\qquad)$;

(3) $a^2-ab-3a+3b=a^2-ab+(\qquad\qquad)$;

(4) $a^2-ab-3a+3b=a^2-ab-(\qquad\qquad)$;

(5) $a^2 - b^2 - (b - a) = a^2 - b^2 + (\qquad\qquad)$;

(6) $a^4 + (-a^2 + 2a - 1) = a^4 - (\qquad\qquad)$.

8. Fill in the bracket with appropriate terms:

(1) $(a+b+c)(a-b-c) = [a + (\qquad)][a - (\qquad)]$;

(2) $(a-b+c)(a+b-c) = [a - (\qquad)][a + (\qquad)]$;

(3) $(-a+b+c)(a+b-c) = [b - (\qquad)][b + (\qquad)]$;

(4) $(a+b-c-d)(a-b+c-d)$

$= [(a-d) + (\qquad)][(a-d) - (\qquad)]$.

9. According the instructions below, add brackets to the polynomial

$$m^4 - 2m^2n^2 - 2m^4 + 2n^2 + n^4$$

Group the terms with degree four in a bracket and prefix it with a '＋' sign. Group the terms with degree two in a bracket and prefix it with a '－' sign.

2.7 Addition and Subtraction of Integral Expressions

The operation of addition and subtraction of integral expressions is indeed grouping of like terms. During the operation, if there are brackets, remove them first by using the rule of removing brackets and then group like terms together.

【Example 1】 Find the sum of monomials $5x^2y$, $-2x^2y$, $2xy^2$ and $-4x^2y$.

Solution $5x^2y + (-2x^2y) + 2xy^2 + (-4x^2y)$

$= 5x^2y - 2x^2y + 2xy^2 - 4x^2y$

$= -x^2y + 2xy^2$

【Example 2】 Find the sum of $3x^2 - 6x + 5$ and $4x^2 + 7x - 6$.

Solution $(3x^2 - 6x + 5) + (4x^2 + 7x - 6)$

$= 3x^2 - 6x + 5 + 4x^2 + 7x - 6$

$= 7x^2 + x - 1$

【Example 3】 Find the difference between $2x^2 + xy + 3y^2$ and $x^2 - xy + 2y^2$.

Solution $(2x^2 + xy + 3y^2) - (x^2 - xy + 2y^2)$

$= 2x^2 + xy + 3y^2 - x^2 + xy - 2y^2$

$= x^2 + 2xy + y^2$

Practice

1. Mentally compute the sum of the monomials:

(1) $-3x$, $-2x$, $-5x^2$, $5x^2$; (2) $-\frac{1}{2}n$, $\frac{3}{5}n^2$, $\frac{4}{5}n$.

2. Mentally compute the difference between the first monomial and the second monomial:

(1) $3ab$, $-2ab$; (2) $-4x^2$, $\frac{1}{2}x$.

3. Compute:

(1) $2xy - 3yx - (-6x^2y^2) - 2y^2x^2$;

(2) $(-3ab) + (-4a^2) + 3a^2 - (-5ab)$.

4. Find the sum of $3a^2 + b^2 - 5ab$ and $4ab - b^2 + 7a^2$.

5. Find the difference between $x^2 - 3xy + 2y^2$ and $3x^2 - 7xy - 3y^2$.

6. Compute:

(1) $(-x + 2x^2 + 5) + (-3 + 4x^2 - 6x)$;

(2) $(3a^2 - ab + 1) - (-4a^2 + 6ab + 7)$.

【Example 4】 Compute $3a - (2a - 4b - 6c) + 3(-2c + 2b)$.

Solution $3a - (2a - 4b - 6c) + 3(-2c + 2b)$

$= 3a - (2a - 4b - 6c) + (-6c + 6b)$

$= 3a - 2a + 4b + 6c - 6c + 6b$

$= a + 10b$

【**Example 5**】Simplify the polynomial, then compute its value.

$$\frac{1}{2}x-\left(2x-\frac{2}{3}y^2\right)+\left(-\frac{3}{2}x+\frac{1}{3}y^2\right),$$

where $x=-2$ and $y=\frac{2}{3}$.

Solution $\frac{1}{2}x-\left(2x-\frac{2}{3}y^2\right)+\left(-\frac{3}{2}x+\frac{1}{3}y^2\right)$

$=\frac{1}{2}x-2x+\frac{2}{3}y^2-\frac{3}{2}x+\frac{1}{3}y^2$

$=-3x+y^2$.

When $x=-2$ and $y=\frac{2}{3}$,

The value of the expression = $-3\times(-2)+\left(\frac{2}{3}\right)^2=6\frac{4}{9}$.

Practice

1. Compute:
   (1) $x-(1-2x+x^2)+(-1+3x-x^2)$;
   (2) $(8xy-3x^2)-5xy-2(3xy-2x^2)$.
2. Simplify the following polynomial, then compute its value:
   (1) $2x-y+(2y^2-y^2)-(x^2+2y^2)$, where $x=1$, $y=-2$;
   (2) $5(3a^2b-ab^2)-(ab^2+3a^2b)$, where $a=\frac{1}{2}$, $b=\frac{1}{3}$.

Exercise 11

1. Compute:
   (1) $4x^3-(-6x^3)+(-9x^3)$;
   (2) $-3x^2y-(-3xy^2)+3x^2y+xy^2$;
   (3) $-3x^2-4xy-6xy-(-y^2)-(+2x^2)-3y^2$;
   (4) $-\frac{2}{3}ab+\frac{3}{4}a^2b-ab+\left(-\frac{3}{4}a^2b\right)-1$.

Compute (Question 2~12) :

2. $(7x^2+3-2x)+(-4-6x-2x^2)$.
3. $(2x^2-3x-1)+(-5+3x-x^2)$.
4. $(5a+4c+7b)+(5c-3b-4a)$.
5. $(8xy-x^2+y^2)-(x^2-y^2+8xy)$.
6. $(3a^2-b^2-2ab)+(b^2-ab-2a^2)$.
7. $(11x^3-2x^2)+2(x^3-x^2)$.
8. $(2x^2-1+3x)-4(x-x^2+1)$.
9. $5(a^2b-3ab^2)-2(2a^2b-7ab^2)$.
10. $-3(a^3b+2b^2)+(3a^3b-13b^2)$.
11. $3x^2-[7x-(4x-3)-2x^2]$.
12. $3x^2y+\left\{xy-\left[3x^2y-\left(4xy^2+\frac{1}{2}xy\right)\right]-4x^2y\right\}$.
13. Simplify the following polynomial, then compute its value:
    (1) $(-x^2+5+4x^3)+(-x^3+5x-4)$, if $x=-2$;
    (2) $(5a^2b+4b^3-2ab^2+3a^3)-(2a^3-5ab^2+3b^3+2a^2b)$, if $a=-2$, $b=3$.
14. (1) Find the polynomial which when added by $3x^2y-3xy^2$ equals x^3-y^3;
    (2) Find the polynomial which when subtracted by a^2+ab equals $-2ab+\frac{1}{4}b^2$.

15. Given that $A = x^3 + x^2 + 2x + 1$ and $B = x + 2x^2$, compute:

(1) $A + B$; (2) $B + A$; (3) $A - B$; (4) $B - A$.

Are the results of (1) and (2) the same? How about the results of (3) and (4)?

Chapter summary

I. This chapter mainly covers the concept of algebraic expressions, integral expressions, monomials (integral expressions with a single term) and polynomials (integral expressions with multiple terms).

II. Algebraic expression builds on the foundation of representing numbers by letters (variables). Since letters are used to represent numbers, so quantity and quantitative relationship can be succinctly presented in terms of letters. This facilitates problem investigation and computation, and brings forth significant advancement in mathematics.

Developing from direct numerical calculation to abstracting quantitative relationships in terms of algebraic expressions, this process expands the power of mathematics from solving particular problem to deriving relationship for solving general situation. Substituting numbers into the relevant letters of general solution of algebraic expression yields the detailed solution of the problem, this process demonstrates the power of mathematics in solving general problem and applying it to obtain answers to specific applications.

III. Integral expressions, in the form of monomials and polynomials are fundament in the study of algebraic expressions. The concept of terms, powers, coefficients, etc will need to be distinguished and clearly understood. Pay attention to determining what constitutes like terms: Firstly, like terms must contain same letters; secondly, like terms must contain same index of each of the same letters. Both conditions must be satisfied in determining like terms. The rule for grouping like terms is: keep the letters (variables) and their indicies unchanged, and attach a coefficient equal to the sum of the coefficients of all the like terms.

IV. Removing and adding brackets are very common operations in algebra, but care must be exercised to ensure that the value of the oringal algebraic expression remains unchanged.

V. Addition and subtraction of integral expressions requires grouping of like terms. Note that during the addition and subtraction operation, brackets, if any, will need to be removed.

The result of adding and subtracting integral expressions is still an integral expression.

Revision Exercise 2

1. Express the following in terms of letters:
   (1) Nature of fractions;
   (2) Law for Multiplication and Division of fractions;

2. Fill in the table below:

a	-7	-4	0	$1\frac{1}{2}$	$2\frac{3}{4}$	$3\frac{2}{3}$
$-a$						
$\lvert -a \rvert$						
$\lvert a-1 \rvert$						
$-\lvert a-1 \rvert$						

3. Fill in the blanks:
   (1) If letter a represents a positive number, then $-a$ represents a ( ) and $|a|$ represents a ( );
   (2) If letter a represents a negative number, then $-a$ represents a ( ) and $|a|$ represents a ( );
   (3) If letter a represents zero, then $-a$ represents a ( ) and $|a|$ represents a ( ).

4. (1) If letter a represents a positive number or zero, then is $|a|$ equal to a? Is $|a|$ equal to $-a$?
   (2) If letter a represents a negative number, then is $|a|$ equal to a? Is $|a|$ equal to $-a$?

5. What is an algebraic expression? What is the value of an algebraic expression? Explain your answer with examples.

6. What is a monomial? What is a polynomial? What is an integral expression? Explain your answer with examples.

7. (1) What is coefficient? What is the degree of a monomial? Explain your answer with examples.
   (2) Write down the coefficients and degrees of the following monomials.

$$45x^2y^2,\ -\frac{1}{2}a^2b,\ \frac{m^2n^2}{3},\ x,\ -x^n.$$

   (3) What is a term in a polynomial? What is the degree of a polynomial? Explain your answer with examples.
   (4) Write down the terms and degrees of the following polynomials:
   x^2+y-1, $3x-4$, $a^2+2ab+b^2$, $x^3+y^3+3x^2y+3xy^2$.

8. Represent the following in terms of algebraic expression:
   (1) A factory manufacture toys at the cost of production of a dollars each. Now the cost of production is reduced by p%. Find the present cost of production;
   (2) A farm has n acres of crop to be harvested. The original plan is to harvest m acres a day. Now the plan is revised to increase the harvest rate by 5 acres a day. Find how many days earlier can the whole farm be harvested;
   (3) Every year, a company pays each of its staff a salary equal to $\frac{1}{5}$ of the manager's salary S dollars minus 200 dollars. find the annual salary of each staff;
   (4) A car manufacturer produced a cars in the first month. It produced x% more cars in the second month compared with the first month. It produced x% more cars in the third month compared with the second month. Find the number of cars produced in the third month.

9. For each value of x given below, compute the value of the algebraic expression $3x^3+2x^2-x+3$:
   (1) $x=-2$; (2) $x=0$; (3) $x=3$; (4) $x=2\frac{1}{2}$.

10. When $x=-\frac{1}{3}$, compute the value of the algebraic expression x^3+1 and the algebraic expression $(x+1)^2$.

11. When $x=2$, $y=-4$, find the value of the following algebraic expression:
   (1) x^2+y^2; (2) $(x-y)^2$;
   (3) $x^2-2xy+y^2$; (4) $\frac{2x+y}{x-y}$.

12. For the values of a, b given below, compute the values of the algebraic expression a^2+b^2 and algebraic expression $(a+b)^2$:
   (1) $a=3$, $b=-2$; (2) $a=-3$, $b=2$;
   (3) $a=0.5$, $b=-0.5$; (4) $a=8$, $b=-7\frac{1}{2}$.

13. A farmer had n cows two years ago. Last year there was growth of 15% over the number of cows two years ago, Use an algebraic expression to represent the number of cows last year. When n = 3640, compute the number of cows the farmer had in the last year.

14. Water flows through the cross section of an irrigation canal which is in the form of a trapezium. The width of water level is a m, the width of the canal base is b m, the depth of water is h m. If the water flow speed is v m/second, when a = 1.2, b = 0.8, h = 0.6, v = 0.4, how much water (in m^3) flow through the cross section in 1 second?

15. What is meant by like terms? How to group like terms together? Give examples to illustrate.

16. Using examples, explain the rules for removing and adding bracket.

17. Group the like terms in the following expression:

(1) x^2y-3x^2y;

(2) $10y^2+y^2$;

(3) $-\frac{1}{2}a^2bc+\frac{1}{2}a^2bc$;

(4) $\frac{1}{4}mn-\frac{1}{3}mn+7$;

(5) $7ab-3a^2b^2+7+8ab^2+3a^2b^2-3-7ab$;

(6) $3x^2-3x^2-y^2+5y+x^2-5y+y^2$.

18. Simplify the following expression:

(1) $(4a^3b-10b^3)+(-3a^2b^2+10b^3)$;

(2) $\left(\frac{1}{2}xy-5xy^2\right)+\left(\frac{1}{3}xy^2-\frac{1}{2}xy\right)$;

(3) $5a^2-[a^2+(5a^2-2a)-2(a^2-3a)]$;

(4) $15+3(1-a)-(1-a+a^2)+(1-a+a^2-a^3)$.

19. In the following equation, fill in appropriate terms inside the bracket:

(1) $2x+x^2-y^2=2x+(\qquad\qquad)$;

(2) $4-x^2+2xy-y^2=4-(\qquad\qquad)$;

(3) $(a-2b+c)(a+2b-c)=[a-(\qquad\quad)][a+(\qquad\quad)]$;

(4) $x^2-x+6=+(\qquad\qquad)=-(\qquad\qquad)$.

20. Given $A=9x^2+8y^3-16xy^2$, $B=3x^3-4y^2+2xy^2$. Compute:

(1) $A+B$;

(2) $A-B$.

21. (1) There is a polynomial which when subtracting $3x^4-x^3+2x-1$ equals $5x^4+3x^2-7x+2$. Find the polynomial;

(2) A polynomial $10a^2-ab$ is added to the polynomial $7a^2+4ab-b^2$. Find the polynomial

22. Compute:

(1) $(4a^2bc-3ab)+(-5a^2bc+2ab^2)$;

(2) $(6m^2-4mn-3n^2)-(2m^2-4mn+n^2)$;

(3) $(2a^3+5a^2+2a-1)-(3-8a+2a^2-6a^3)$;

(4) $3x^2-\left[5x-\left(\frac{1}{2}x-3\right)+2x^2\right]$;

(5) $(5x^3-2x^2+3)-(1-2x+x^2)+3(-1+3x-x^2)$;

(6) $(-x^2+4+3x^4-x^3)+(2x^3-7x^4+6x-5)-(x^2+2x-x^4-5)$.

23. Compute the value of the following expression:

(1) $(3x^2-4)-(2x^2-5x+6)+(x^2-5x)$, where $x=-1\frac{1}{2}$;

(2) $3x^2y-[2x^2y-(2xyz-x^2z)-4x^2z)-xyz$, where $x=-2$, $y=-3$, $z=1$.

24. There are five consecutive integers, the middle one is a, express the remaining numbers in terms of a. Express the sum of the 5 consecutive numbers in terms of a. When $a = 34$, compute the value of the sum?

25. Therefore are 3 consecutive even numbers, the middle one is $2n$, find the algebraic expression for the sum of squares of the three numbers. When $n = 2$, compute the value of the sum of squares of the three numbers.

26. (1) $6\times10^3+9\times10^2+4\times10+8=?$
    (2) Express 35 in the form of $a\times10+b$ (a, b are any integers between 0 and 9).
    (3) Express 712 in the form of $a\times10^2+b\times10+c$ (a, b, c are any integers between 0 and 9).
    (4) A 2-digit number has its tens digit equal a, and its unit digit equals b, write the 2-digit number in terms of a and b.

27. Write down any 5 consecutive positive integers and compute its sum. Can the sum be divisible by 5? Find an algebraic expression to represent the sum of 5 consecutive positive integers. What algebraic expression when multiplied by 5 equals to this sum?

28. Write down any 2-digit number. Obtain a new number by interchanging the tens digit and the unit digit. Can the sum of the new number and the original number be divisible by 11? Use an algebraic expression to represent the 2-digit number. Obtain a new number by interchanging the tens digit and the unit digit. Find an algebraic expression to represent the sum of the new number and the original number. What algebraic expression when multiplied by 11 equals the sum?

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