Chapter 3 Linear Equations in One Unknown

3.1 Equations

In Primary school, we have an early idea of equation. This chapter reviews the Primary school learning, and develops knowledge and method to solve equations.

Let us look at the following relationships:

$$1 + 2 = 3,\ a + b = b + a,\ S = ab,\ 4 + x = 7.$$

Relationship of this type is called an **equality**. In an equality, the two sides of the equal sign are called the left-hand side and the right-hand side.

Equalities have the following characteristics: if both sides of the equality are added, subtracted, multiplied, or divided (divisor cannot be zero) by the same number, the equality is preserved.

Let us examine the last equality of the previous examples.

$$4 + x = 7,$$

Here, the letter x is an unknown. Let us evaluate what value x should take in order for the equality to be true (Diagram 3-1).

$$\boxed{x = ?}$$

$$\downarrow$$

$$4 + x = 7$$

Duagram 3-1

An equality involving unknown is called an **equation**.

$4 + x = 7$ is an equation. In the following equalities x and y are unknowns. Therefore all of them are equations.

$$5-2x=1,\quad y^2+2=4y-1,\quad x-2y=6$$

【Example 1】 List the equation which will satisfy the following condition:

(1) 2 times a number minus 3 equals 5;

(2) Adding 2 to a number and multiplying it by 3 equals 12.

Solution (1) Let the number be x, then, "2 times x minus 3 equals 5" can be expressed by the equation

$$2x-3=5;$$

(2) Let the number be x, then "Adding 2 to a number and multiplying it by 3 equals 12" can be expressed as

$$(x+2)\times 3=12,$$

that is $3(x+2)=12$.

Practice

List the equation satisfying the following specification:

1. A number minus 5 equals 4;
2. 3 times a number plus 2 equals 8.

A value of the unknown that makes the left-hand side ("LHS") and the right-hand side ("RHS") of an equation equal is called a **solution** to the equation. For example, in equation $4 + x = 7$, when x takes the value of 3, the value on the left-hand side equals the value on the right-hand side. In other words, when $x = 3$, both sides of the equation are equal. Therefore $x = 3$ is a solution of the equation $4 + x = 7$. The solution of an equation in one unknown is also called a root of the equation. For example, $x = 3$ is a root of equation $4 + x = 7$.

The procedure to find the solution to an equation is called **solving the equation**.

【Example 2】 Verify whether the following is a solution to the equation $2x-3=5$.

(1) $x=6$; (2) $x=4$.

Solution (1) Substitute $x=6$ into the equation,

$\text{LHS}=2\times 6-3=9$, $\text{RHS}=5$.

$\because$ $\text{LHS}\neq\text{RHS}$,

$\therefore$ $x=6$ is not a solution to equation $2x-3=5$.

(2) Substitute $x=4$ into the equation,

$\text{LHS}=2\times 4-3=5$, $\text{RHS}=5$.

$\because$ $\text{LHS}=\text{RHS}$,

$\therefore$ $x=4$ is a solution to the equation $2x-3=5$.

Legend: "$\neq$" is a symbol for being not equal and is read as "not equal to".

Practice

1. Verify whether the following is a solution to the equation $6(x+3)=30$:

   (1) $x=5$; (2) $x=2$.

2. Verify whether the following is a solution to the equation $3x-1=2x+1$:

   (1) $x=4$; (2) $x=2$.

3.2 Equivalent Equations

Examine the following two equations:

$$x+1=4, \tag{1}$$

$$x+2=5. \tag{2}$$

We know the solution to equation (1) is x = 3 and solution to equation (2) is x = 3. That is, equation (1) and equation (2) have a common solution.

If two equations have a common solution, we say that the two equations are **equivalent equations**. From the above, we know that equation (1) and equation (2) are equivalent equations.

Equation (2) can be written as

$$(x+1)+1=4+1,$$

That is to say, if we add 1 to both sides of equation (1) ,we obtain equation (2) which is an equivalent equation with equation (1).

In general, we have

Equivalent Equation Axiom 1: When both sides of an equation are added (or subtracted) by an equal number or an equal integral expression, the resultant equation is an equivalent equation to the original equation.

Let us examine the following two equations:

$$x+1=3, \tag{3}$$

$$2x+2=6. \tag{4}$$

Here we know, solution to equation (3) is x = 2, solution to equation (4) is x = 2.That is to say, equations (3) and (4) have equal solutions. Therefore they are equivalent equations.

Equation (4) can be obtained by multiplying both sides of equation (3) by 2.

In general, we also have

Equivalent Equation Axiom 2: When both sides of an equation are multiplied (or divided) by the same non-zero number, the resultant equation is an equivalent equation to the original equation.

Practice

(*Mental*) In accordance to Equal Equation Axioms 1 and 2, explain whether the following equations are equal equations.

(1) $6x-1=1$, $6x=2$; (2) $3x+2=-4$, $3x=-6$;

(3) $15x=25$, $3x=5$; (4) $\frac{2}{3}x=7$, $2x=21$.

3.3 Method to solve Linear Equations in One Unknown

Examine the following equations:

$$4+x=7,\quad 3x+5=7-2x,\quad \frac{y-2}{6}=\frac{y}{3}+1.$$

Each of these equations involves only one unknown, and the degree of the unknown is one. This type of equation is called **a Linear Equation in one unknown**.

Now, we apply Equivalent Equation Axioms to solve linear equations in one unknown.

【**Example 1**】Solve the equation $x-7=5$.

Solution According to Equivalent Equation Axiom 1, by adding 7 to both sides of the equation, we get

$$x-7+7=5+7;$$

Collecting like terms, we get

$$x=12.$$

To verify whether the solution is correct, we can substitute the value of the unknown we have found into the original equation, to check whether the values of both sides of the equation are equal.

Substitute $x=12$ into the original equation,

LHS $=12-7=5$, RHS $=5$, LHS = RHS,

Therefore $x=12$ is a solution to the original equation.

【**Example 2**】Solve the equation $7x=6x-4$.

Solution In accordance with Equivalent Equation Axiom 1, by subtracting both sides of the equation by $6x$, we get

$$7x-6x=6x-4-6x;$$

Collecting like terms , we get $x=-4$.

Checking: Substitute $x=-4$ into the original equation,

LHS $=7\times(-4)=-28$,

RHS $=6\times(-4)-4=-28$,

LHS = RHS,

Therefore $x=-4$ is a solution to the original equation.

In Examples 1 and 2, we have applied Equivalent Equation Axiom 1, to transform the equations as follows (Diagram3-2):

Example 1

$$x\ \boxed{-7} = 5$$
$$x = 5\ \boxed{+7}$$

Example 2

$$7x = \boxed{6x}\ -4$$
$$7x\ \boxed{-6x} = -4$$

Diagram 3-2

Reviewing the changes in two equations before and after transformation, we note that the transformation process is equivalent to changing the sign of a term in one side of the equation and moving it to the other side of the equation. We describe this transformation as **transposing a term**. We must bear in mind that we have to change the sign of the term in transposing it.

When solving equation, we often transpose terms around. We transpose all terms involving the unknown to one side of the equation, transpose all terms not involving the unknown to the other side of the equation. With regard to Examples 1 and 2, we can explain the solution process in the following manner:

(1) $x-7=5$

Transposing the term -7 from left side of the equation to the right side, we get

$$x=5+7.$$

Collecting like terms, we get $x=12$.

(2) $7x=6x-4$

Transposing $6x$ from the right side of the equation to the left side, we get

$$7x-6x=-4$$

Collecting like terms, we get $x=-4$.

Practice

1. Solve the following equation using Equivalent Equation Axiom 1, and write down the checking:

   (1) $x+6=7$; (2) $4x=3x-2$.

2. Check if the following transposing of term is correct or not? If not, which part is incorrect? How should it be corrected?

   (1) From $7+x=13$, we obtain $x=13+7$;

   (2) From $5x=4x+8$, we obtain $5x-4x=8$.

3. Solve the following equation by transposing term, and write down the checking:

   (1) $x+12=34$; (2) $x-15=74$;

   (3) $3x=2x+5$; (4) $7x-3=6x$.

【**Example 3**】Solve the following equation:

(1) $-5x = 70$; (2) $\frac{3}{5}x - 8 = 1$.

Solution (1) $-5x = 70$.

In accordance with Equivalent Equation Axiom 2, dividing both sides of the equation by -5, we get

$$\frac{-5x}{-5} = \frac{70}{-5},$$

That is

$$x = -14.$$

Checking: $\text{LHS} = -5 \times (-14) = 70$, $\text{RHS} = 70$,
$\text{LHS} = \text{RHS}$,

Therefore $x = -14$ is a solution to the equation.

(2) $\frac{3}{5}x - 8 = 1$.

Transposing term, we get

$$\frac{3}{5}x = 1 + 8.$$

Collecting like terms, we get

$$\frac{3}{5}x = 9.$$

Dividing both sides of the equation by $\frac{3}{5}$ (or multiply by $\frac{5}{3}$), we get

$$\frac{3}{5}x \bullet \frac{5}{3} = 9 \bullet \frac{5}{3}.$$

That is

$$x = 15.$$

Checking: $\text{LHS} = \frac{3}{5} \times 15 - 8 = 1$, $\text{RHS} = 1$, $\text{LHS} = \text{RHS}$,

Therefore $x = 15$ is a solution to the equation.

In Example 3, we observe that the equation $-5x = 70$ and equation $\frac{3}{5}x = 9$, are both equations of the form $ax = b$ (here a and b are known literals, and $a \neq 0$), this type of equation is called the simplest equation. Applying Equivalent Equation Axiom 2, dividing both sides of the equation by the coefficient of the unknown, we can obtain a solution to the equation

$$x = \frac{b}{a}.$$

Practice

Solve the following equation , and write down the checking:

(1) $15x = 45$;	(2) $-2x = 30$;	(3) $-18x = -3$;
(4) $3.5x = 7$;	(5) $9x = 0$;	(6) $32 = 8x$;
(7) $\frac{x}{5} = 3$;	(8) $6x = 16 - 2x$;	(9) $\frac{2}{5}x - 4 = 12$;
(10) $4 - \frac{3}{7}y = 13$;	(11) $13 = \frac{t}{2} + 3$;	(12) $\frac{1}{2} = \frac{1}{3} + 2x$.

【**Example 4**】Solve the equation $5x + 2 = 7x - 8$.

Solution Transposing term, we get

$$2 + 8 = 7x - 5x.$$

Collecting like terms, we get

$$10 = 2x.$$

That is

$$2x = 10.$$

Divide both sides of the equation by 2

$$x = 5.$$

(Mentally check the solution by yourself.)

【**Example 5**】Solve the equation $2(x-2) - 3(4x-1) = 9(1-x)$.

Solution Removing the bracket, we get

$$2x - 4 - 12x + 3 = 9 - 9x.$$

Transposing term, we get

$$2x - 12x + 9x = 9 + 4 - 3.$$

Collecting like terms, we get

$$-x = 10.$$

Multiply both sides of equation by -1, we get

$$x = -10.$$

(Mentally check the solution by yourself.)

Practice

Solve the following equation, and mentally check the result:

(1) $2x + 5 = 25 - 8x$. (2) $\frac{x}{2} - 7 = 5 + x$.

(3) $5(x + 2) = 2(2x + 7)$. (4) $3(2y + 1) = 2(1 + y) + 3(y + 3)$.

【**Example 6**】Solve the equation $\frac{5y - 1}{6} = \frac{7}{3}$.

To solve the equation, we can apply Equivalent Equation Axiom 2, multiplying both sides of the equation by the LCM of the denominators (LCM of 6 and 3 in this example is 6), we can remove both denominators in the equation.

Solution Removing the denominators, we get

$$\frac{5y - 1}{6} \times 6 = \frac{7}{3} \times 6,$$

That is

$$5y - 1 = 14.$$

Transposing term, we get

$$5y = 14 + 1.$$

Collecting like terms, we get

$$5y = 15.$$

Dividing both sides of equation by 5, we get

$$y = 3.$$

【**Example 7**】Solve the equation $\frac{2x - 1}{3} - \frac{10x + 1}{6} = \frac{2x + 1}{4} - 1$.

In this example, the LCM of the denominators 3, 6, 4 is 12.

Solution Removing denominators, we get

$$4(2x - 1) - 2(10x + 1) = 3(2x + 1) - 12.$$

Removing brackets, we get

$$8x - 4 - 20x - 2 = 6x + 3 - 12.$$

Transposing terms, we get

$$8x - 20x - 6x = 3 - 12 + 4 + 2.$$

Collecting like terms, we get

$$-18x = -3.$$

Dividing both sides of equation by -18, we get

$$x = \frac{1}{6}.$$

Practice

Solve the following equation:

(1) $\frac{7x - 5}{4} = \frac{3}{8}$. (2) $\frac{3 - x}{2} = \frac{x - 4}{3}$. (3) $\frac{2x - 1}{6} - \frac{5x + 1}{8} = 1$.

(4) $\frac{2}{7}(3x + 7) = 2 - 1.5x$. (5) $\frac{30}{100}x + \frac{70}{100}(200 - x) = 200 \times \frac{54}{100}$.

The general procedure to solve a linear equation in one unknown is:

1. **Remove the denominators;**
2. **Remove the brackets;**
3. **Transpose the terms;**
4. **After collecting like terms, the equation is transformed to its simplest form as $ax = b$ ($a \neq 0$);**
5. **Divide both sides of the equation by the coefficient of the unknown, we get the solution to the equation as $x = \frac{b}{a}$.**

To solve the equation, sometimes it may not be possible to follow the above procedure rigidly. We need to adapt the procedure flexibly to suit the problem in finding the solution. Once we are familiar with the steps, we can simplify the procedure in solving linear equations (including verifying the results).

【**Example 8**】Solve the equation $\frac{x}{0.7}-\frac{1.7-2x}{0.3}=1$.

The denominators in this example involve decimal numbers, we can convert the decimal numbers to integers before solving it.

Solution The original equation can be transformed into

$$\frac{10x}{7}-\frac{17-20x}{3}=1.$$

Removing the denominators, we get

$$30x-7(17-20x)=21.$$

Removing the brackets, transposing terms and collecting like terms, we get

$$170x=140.$$

Dividing both sides of the equation by 170, we get

$$x=\frac{14}{17}.$$

【**Example 9**】In the formula for the area of trapezium $S=\frac{1}{2}(a+b)h$, given that S = 120, b = 18 and h = 8, find a.

Solution Substituting S = 120, b = 18, h = 8 into the equation, we get

$$120=\frac{1}{2}\bullet(a+18)\bullet 8.$$

Solving the equation:

$$30=a+18$$

$$a=12$$

Practice

1. Solve the equation:

(1) $2.4-\frac{y-4}{2.5}=\frac{3}{5}y$; (2) $\frac{x-2}{0.2}-\frac{x+1}{0.5}=3$.

2. In formula for the area of trapezium $S=\frac{1}{2}(a+b)h$,

(1) S = 30, a = 6, h = 4, find b;
(2) S = 60, a = 8, b = 12, find h.

Exercise 12

1. Fill in the empty space using the appropriate equal "=" or not equal "$\neq$" signs:

(1) $5+3$____$12-5$; (2) $8+(-4)$____$8-(+4)$;
(3) $1+5\times(-2)$____-12; (4) $2\times(3+4)$____$2\times3+4$.

2. List the equation satisfying the following condition:

(1) Subtract one from a number and multiply by 2. The result is 4;
(2) Multiply a number by 3andtake away 4.The result is 6;
(3) Three times the sum of a number and 6 equals 21;
(4) The sum of $\frac{1}{2}$ a number and $\frac{1}{3}$ of the number equals 5.

3. In the following question, check whether each of the figure in the bracket is a solution of the equation that precedes it:

(1) $3x=x+3$, ($x=2$, $x=\frac{3}{2}$);
(2) $y=10-4y$, ($y=1$, $y=2$, $y=3$);
(3) $(x-2)(x-3)=0$, ($x=0$, $x=2$, $x=3$);
(4) $x(x+1)=12$, ($x=3$, $x=4$, $x=-4$).

4. In accordance with the Equivalent Equation Axiom 1, explain whether the following equations are equivalent equations:
   (1) $2x-1=3$, $2x=4$; (2) $4x=1+x$, $3x=1$.

5. In accordance with the Equivalent Equation Axiom 2, explain whether the following equations are equal equations:
   (1) $\frac{x+1}{3}=4$, $x+1=12$; (2) $\frac{3}{4}(x-4)=3x$, $x-4=4x$.

6. Apply Equivalent Equation Axiom 1 to solve the following equation, and write down the checking:
   (1) $x+15=24$; (2) $3x=4+2x$;
   (3) $2x-7=x$; (4) $5y+8=4y$;
   (5) $1.8x=0.8x-1.2$; (6) $\frac{7}{4}z-\frac{1}{2}=\frac{3}{4}z$.

7. Solve the following equation by transposing terms, and write down the checking:
   (1) $2x+3=x-1$; (2) $8x-2=7x-2$;
   (3) $3x-4+2x=4x-3$; (4) $10y+7=12y-5-3y$;
   (5) $2.4x-9.8=1.4x-9$; (6) $\frac{11}{9}z+\frac{2}{7}=\frac{2}{9}z-\frac{5}{7}$.

8. Solve the following equation by transposing terms, and write down the checking:
   (1) $3x=12$; (2) $-6y=6$;
   (3) $-x=0$; (4) $\frac{x}{2}=8$;
   (5) $\frac{3}{4}x=5$; (6) $-\frac{7}{12}x=-1$.

9. Solve the following equation, and do the checking mentally:
   (1) $9x=6x-6$; (2) $8z=4z+1$;
   (3) $7x-6=-5x$; (4) $\frac{3x}{100}=\frac{45}{100}$.

10. Solve the following equation:
   (1) $\frac{x}{2}+1=x$; (2) $\frac{y}{3}=y-4$;
   (3) $1=\frac{x}{2}-5$; (4) $0.48x-6=-0.02x$;
   (5) $2x: 3 = 6: 5$; (6) $8: 3 = 4x: 7$.

11. Use equation to express the following relationship between the numbers, and solve for the value of *x*:
   (1) The sum of *x* and 42 is 18;
   (2) $\frac{1}{9}$ of *x* equals $\frac{2}{3}$;
   (3) 4 times of *x* minus 10 equals 30;
   (4) 5 times of *x* equals 2 times of *x* plus 24.

12. Check whether the following equation is solved correctly? If not correct, which part is wrong? How should it be corrected?
   (1) Solve the equation $\frac{x}{2}=5$.
   Solution: $\frac{x}{2}=5$,
   $\therefore\ x=10$.
   (2) Solve the equation $2x-1=-x+5$;
   Solution: $2x-x=5-1$,
   $\therefore\ x=4$.
   (3) Solve the equation $\frac{6y}{5}=y+1$;
   Solution: $6y=5y+1$,
   $6y-5y=1$,
   $\therefore\ y=1$.

Solve the following equations（No. 13～18）:

13. (1) $2x+3=11-6x$; (2) $\frac{x-5}{3}=4$;

(3) $2x-1=5x-7$; (4) $\frac{1-3x}{2}=8$.

14. (1) $3(y+4)=12$; (2) $\frac{3}{4}x-1=7$;

(3) $1=\frac{1-z}{2}-1$; (4) $-5(x+1)=\frac{1}{2}$.

15. (1) $5(x+8)-5=6(2x-7)$;

(2) $2(3y-4)+7(4-y)=4y$;

(3) $4x-3(20-x)=6x-7(9-x)$;

(4) $4(2y+3)=8(1-y)-5(y-2)$.

16. (1) $3x-4(2x+5)=7(x-5)+4(2x+1)$;

(2) $17(2-3y)-5(12-y)=8(1-7y)$;

(3) $7(2x-1)-3(4x-1)-5(3x+2)+1=0$;

(4) $5(z-4)-7(7-z)-9=12-3(9-z)$.

17. (1) $\frac{5-3x}{2}=\frac{3-5x}{3}$; (2) $y-\frac{y-1}{2}=2-\frac{y+2}{5}$;

(3) $\frac{x+2}{4}-\frac{2x-3}{6}=1$; (4) $\frac{z-2}{5}-\frac{z+3}{10}-\frac{2z-5}{3}+3=0$.

18. (1) $2\frac{1}{2}=\frac{x+3}{4}-\frac{2-3x}{8}$;

(2) $\frac{5y+1}{6}=\frac{9y+1}{8}-\frac{1-y}{3}$;

(3) $\frac{2(x+3)}{5}=\frac{3}{2}x-\frac{2(x-7)}{3}$;

(4) $\frac{x-9}{11}-\frac{x+2}{3}=(x-1)-\frac{x-2}{2}$.

19. Check whether the following equation is solved correctly? If not correct, which part is wrong? How should it be corrected?

(1) Solve the equation $\frac{2x-1}{3}=\frac{x+2}{3}-1$.

Solution: $2x-1=x+2-1$,

$\therefore$ $x=2$.

(2) Solve the equation $\frac{x-1}{3}-\frac{x+2}{6}=\frac{4-x}{2}$;

Solution: $2x-2-x+2=12-3x$,

$4x=12$,

$\therefore$ $x=3$.

20. List the equation satisfying the following conditions, and find the value of the unknown:

(1) 5 times a number plus 3equals 7 times the number minus 5;

(2) 3 times a number minus 9 equals $\frac{1}{3}$ of the number plus 6.

21. List the equation satisfying the following conditions and find the value for the unknown:

(1) 8 times a number is larger than 5 times the number by 12;

(2) $\frac{1}{2}$ of a number plus 4, is smaller than 3 times the number by 21.

22. (1) What value of k will cause the algebraic expression $\frac{3k+5}{7}$ to take the value 2?

(2) What value of x will give equal values to the algebraic expression $\frac{x-8}{3}$ and the algebraic expression $\frac{1}{4}x+5$?

Solve the following equations（No. 23～25）:

23. (1) $\frac{17}{100}x=\frac{21}{100}(x-16)$;

(2) $\frac{65}{100}(y-1)=\frac{37}{100}(y+1)+0.1$;

(3) $3(x+1)-\frac{1}{3}(x-1)=2(x-1)-\frac{1}{2}(x+1)$;

(4) $\frac{3}{4}(z-1)-\frac{2}{5}(3z+2)=\frac{1}{10}-\frac{3}{2}(z-1)$.

24. (1) $\frac{x-3}{2}+\frac{6-x}{3}=\frac{2}{3}\left(1+\frac{1+2x}{4}\right)$;

(2) $\frac{1}{4}\left(1-\frac{3x}{2}\right)-\frac{1}{3}\left(2-\frac{x}{4}\right)=2$;

(3) $\frac{1}{2}\left[x-\frac{1}{2}(x-1)\right]=\frac{2}{3}(x-1)$;

(4) $\frac{3}{2}\left[\frac{2}{3}\left(\frac{x}{4}-1\right)-2\right]-x=2$.

25. (1) $\frac{x+4}{0.2}-\frac{x-3}{0.5}=-1.6$;

(2) $\frac{4-6x}{0.01}-6.5=\frac{0.02-2x}{0.02}-7.5$.

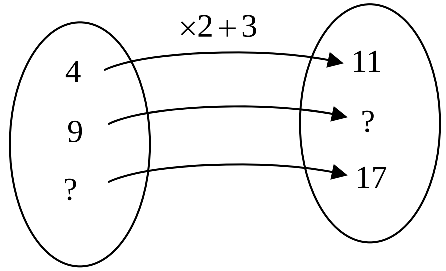
26. (1) In equation $S=2\pi r(r+h)$, given that $S=942$, $\pi=3.14$ and $r=10$, find *h*;

(2) In equation $l=l_0(1+at)$, given that $l=80.096$, $l_0=80$ and $a=0.000012$, find *t*.

27. (1) In equation $v=\frac{\pi nD}{1000}$, given that $v=120$, $D=100$, and $\pi=3.142$, find *n* (answer rounded to integer);

(2) In equation $d=2a+1.57(b+c)$, given that $d=5200$, $a=628$, and $b=500$, find *c* (answer rounded to integer).

28. In the left and right circles of the diagram, fill in appropriate values in the spaces marked with “?”:



(No. 28)

29. In accordance with the equation $v=v_0+at$, write down the appropriate values in the empty spaces in the following table:

v	v_0	a	t
	0	2	8
48		3	14
15	5		4
76	13	7	

3.4 Application of linear equation in one unknown

In Primary school, we have learnt to use arithmetic method or exhaustive listing method to solve some application problems. Here below we explain how to use listing equation method to solve applications relating to linear equation in one unknown.

【Example 1】 When wheat is grounded to flour, there is 15% loss in weight. In order to get 4250 kg of flour, what is the amount of wheat required?

Analysis: The amount of wheat (in kg), minus the amount loss in grounding (in kg), equals the amount of flour required (in kg). If we let the amount of wheat required be *x* kg, 15% of it is the amount in kg loss in grounding. Hence we can list the relationship in the form of an equation.

Solution Let the amount of wheat required be x kg, then grounding will cause a loss of $\frac{15}{100}x$ kg, according to the information of the problem, we get

$$x-\frac{15}{100}x=4250.$$

Solving this equation:

$$\frac{85}{100}x=4250$$

$$x=5000$$

Answer: 5000 kg of wheat is required.

The above example illustrates the method to solve application problem by listing equation in the form of a linear equation in one unknown: (i) first use a letter (such as x) to represent the unknown; (ii) write the required algebraic expression in terms of the letter (such as $\frac{15}{100}x$ in Example 1); (iii) then list the equation to express the relationship between the known values and unknown values according to the information in the problem; (iv) solve the equation, find the value of the unknown and write the answer (including the measuring unit).

【Example 2】 Refer to Diagram3-3, in order to make a mould of a cylindrical disk with diameter 60 mm, height 20 mm, what length of material from a circular wire of diameter 40 mm is needed to be cut (Diagram $\phi 40$ represents a wire with diameter 40 mm)?

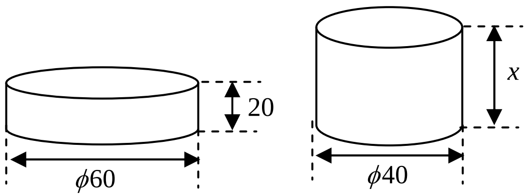


Diagram 3-3

Analysis:To prepare a mould of a cylindrical disk, although the length and cross section are different from those of the material, the volume remains unchanged. Therefore there is the following equation

Volume of circular wire= Volume of the cylindrical disk.

If we assume that the length of the circular wire to be cut is x mm, then we can specify the relationship between the volume of the cylindrical disk and the volume of the material cut from the circular wire.

Solution Let the length of the circular wire with diameter 40 mm be x mm, then the volume of steel available is $\pi \bullet \left(\frac{40}{2}\right)^2 \bullet x$ mm^3, the volume of the circular disk with diameter 60 mm, and height 20 mm is $\pi \bullet \left(\frac{60}{2}\right)^2 \bullet 20\,\text{mm}^3$, accordance to the information given in the problem, we get

$$\pi \bullet \left(\frac{40}{2}\right)^2 \bullet x=\pi \bullet \left(\frac{60}{2}\right)^2 \bullet 20$$

Solving this equation:

$$400x=18000$$

$$x=45$$

Answer: Length of circular wire required to be cut = 45 mm.

In practice when cutting the circular tube, there will be certain amount of loss. Therefore, the length cut from the tube should be longer than 45 mm.

Practice

Write down the linear equation in one unknown for the following application:

1. Buying 4 exercise books and 3 pencils cost 68 dollars. Given that the price of each pencil is 8 dollars, what is the price of each exercise book?

Practice

2. A farm produced an average of 1088 kg of rice per acre, which is 64 kg more than 4 times the average production of rice last year. Find the average production of rice last year.
3. When soybean germinates into bean sprouts, the weight is gained by 7.5 times. If we need to obtain 3400 kg of bean sprout, how many kg of soybean is required?
4. To produce a cylindrical disk with diameter 10 cm, and height 8 cm, what length of circular wire with diameter 8 cm will need to be cut?
5. To mould a cuboid of length, width and height equal to 300 mm, 300 mm and 80 mm respectively, what length of material from a circular wire with diameter 200 mm will need to be cut (answer to be rounded to the nearest mm.

   For the problems in this exercise, use the approximate value of π =3.14)?

【Example 3】 Station A, B are at a distance of 360 km apart. A slow train leaves from Station A, running at a speed of 48 km per hour; a fast train leaves from Station B, running at a speed of 72 km per hour.

(1) The two trains depart at the same time, and move towards each other, in how many hours will the two trains meet?

(2) The fast train departs 25 minutes earlier and both trains move towards each other. How many hours the fast train would have run to meet the slow train?

Analysis: (1) As the trains departs from Station A and B and moving towards each other, by the time they meet each other, they have together travelled the distance between the two stations. If the two trains meet in x hours, we can derive the distance travelled by each train, from which we can specify the relationship in the form of an equation.

(2) Consider that the fast train has departed Station B for 25 minutes to arrive at place C (Diagram 3-4) when the slow train departs from Station A and moves towards each other. Therefore when the two trains meet, the distance travelled by the fast train before reaching place C, plus the distance travelled by fast train from C to the meeting place, plus the distance travelled by the slow train to the meeting place, equals the distance between Stations A and B. If we assume that the two trains meet after the slow train has travelled x hours, then the relationship of the above distances can be specified in the form of an equation.

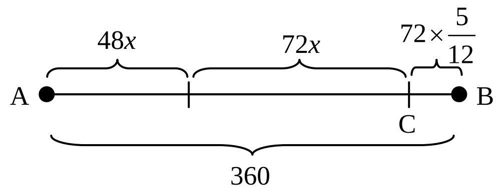


Diagram 3-4

Solution (1) Assume that the two trains meet in x hours after departure. Then the slow train has travelled $48x$ km and the fast train has travelled $72x$ km. According to the information given in the problem, we get

$$72x + 48x = 360\,.$$

Solving this equation:

$$120x = 360$$

$$x = 3$$

Answer: The trains meet in 3 hours.

(2) Assume that the slow train has travelled x hours. Then the fast train travelled $72\times\frac{25}{60}$ km in advance, travelled another $72x$ km and the slow train travelled $48x$ km. According to the information given in the problem, we get

$$72\times\frac{25}{60} + 72x + 48x = 360\,.$$

Solving this equation:

$$30+120x=360$$
$$120x=330$$
$$x=2\frac{3}{4}$$
$$\frac{25}{60}+2\frac{3}{4}=3\frac{1}{6}$$

Answer: The fast train travelled 3 hr 10 min before meeting the slow train.

【Example 4】 A company has 27 staff in Town A and 19 staff in Town B. The company would like to add a total of 20 support staff so that the number of staff in Town A is twice that in Town B. How many of the support staff will be sent to Town A and Town B respectively?

Analysis:The equality relationship in this problem can be represented as

Final number of staff in Town A
= 2 time Final number of staff in Town B

As there are a total of 20 support staff, if we assume the number of support staff sent to Town A as x, then we can list an equation to show the number of support staff sent to Town B. There is another approach: The total number of staff in Town A and Town B is 27+19+20=66, splitting the number by a ratio of 2: 1 would give the final number of staff in the two Towns.

Solution Assume that the number of support staff sent to Town A is x, then the number of support staff sent to Town B is $(20-x)$. So after sending the support staff, Town A will have staff of $27+x$. Town B will have staff of $19+(20-x)$. According to the information in the problem, we get

$$27+x=2[19+(20-x)].$$

Solving this equation:

$$27+x=2(39-x)$$
$$3x=51$$
$$x=17$$

Also, $20-x=20-17=3$.

Answer: No of support staff sent to Town A is 17, sent to Town B is 3.

Practice

Write down the linear equation in one unknown according to the following problem:

1. Two teams of workers A and B work together to dig a water duct of length 1210 m from two ends. Team A digs a length of 130 m a day. Team B digs a length of 90 m a day. How many days will it take for them to work together to dig the whole length of water duct?
2. The distance between Station A and Station B is 284 km. A slow train runs from Station A to Station B at the speed of 48 km per hour. After an hour, a fast train departs from Station B, running toward Station A at the speed of 70 km per hour. How many hours does it take for the fast train to meet the slow train?
3. There are two engineering teams. The first team has 32 persons. The second team has 28 persons. Now there is a need to reshuffle the team so that the number of persons in the first team is twice the number of persons in the second team. How many persons would have to be transferred from the second team to the first team?
4. Two pools together store a total of 40 T of water. If 4 T of water is added to Pool A, and 8 T of water is added to Pool B, the amount of water (in T) in Pool A and Pool B are equal. Find the original amount of water (in T) stored in Pool A and Pool B respectively?
5. Splitting an area of 16 acres of land into two portions in the ratio of 3: 5, what is the area of land in each portion (Hint: The area is split in the ration of 3: 5, that means the total area is split into 3 parts and 5 parts. Therefore we can assume the area of one part as x acres)?

Exercise 13

Specify the linear equation in one unknown according to the following application:

1. A person buys 6 kg of apples. He pays 200 dollars and receives a change of 32 dollars. What is the price of one kg of apples?
2. Using a length of 76 cm of wire to construct a rectangular frame with width 16 cm, what is its length of the rectangle frame in cm?
3. A factory manufactured 205 cars in October last year, which is 15 more than twice the production in October two years ago. How many cars were produced in October two years ago?
4. A large box holds 36 kg of goods. When the goods from the large box is transferred to fill 4 equal-size smaller boxes, there are 2 kg of goods leftover unpacked. How many kg of goods can a smaller box hold?
5. A steel factory is required to refine 3000 T of iron from mineral ores containing 58.5% iron. How many mineral ores in T would be required (answer correct to the nearest 10 T)?
6. When alcohol is diluted by water to make alcohol solution, the volume of the alcohol solution is 4.5 times the volume of the original alcohol. If 900 mL of alcohol solution is required, how many mL of alcohol is needed?
7. When a certain dry food is mixed with water and processed into fermented food, its weight would be increased by 150%. In order to obtain 360 kg of the fermented food, how many kg of the dry food is required?
8. The area of sea surface on earth is $2\frac{13}{29}$ times that of land surface, the whole surface area of the earth is 5.1 billion square km, find the area of water surface and area of land surface respectively (correct the answers to the nearest 10,000,000 square km).

9. A factory is going to make a mould measuring 260 mm × 150 mm × 130 mm from a rectangular steel rod with cross section of 130 × 130 mm^{2}. How long of the rod is needed to be cut? (assume no extra material needed in the manufacturing process, same for all questions below)?
10. A factory is required to manufacture a cylindrical drum with diameter 300 mm and thickness 32 mm. Its raw material is a length of steel wire with diameter 120 mm. What length of steel wire in mm it should use?
11. A factory uses steel wire with diameter 90 mm as raw material to construct a steel mould. For the steel mould, the base area is 131 × 131 mm^2, and height is 81 mm. Find how long is the length of wire required (accurate to 1 mm).
12. A factory uses steel wire with diameter 4 cm as raw material. Each cm^3 of raw material weighs 7.8 g. To construct a steel mould weighing 0.62 kg, what length of wire will need to be cut (accurate to 0.1 cm)?
13. Two men A and B were 65 km apart and they cycled towards each other. They started at the same time and after 2 hours they met. It is known that A cycled 2.5 km more than B. Find the speed of B in km.
14. Two planes A and B are at two airports at a distance of 750 km apart. They flew towards each other and met after half an hour. If the speed of plane A is $1\frac{1}{2}$ times the speed of plane B. Find their speeds.
15. There are two water troughs A and B. Trough A contains 34L of water while trough B contains 18L of water. Water is drained from both troughs at a rate of 2L per minute. After how many minutes will the volume of water in trough A be 3 times that in trough B?

16. In a farm there are 108 acres of wet field and 54 acres of dry field. Now it is intended to convert some dry field into wet field. so that the area of dry field is 20% of the area of wet field. What area of dry field (in acres) is going to be converted to wet field?

17. Barn A contain 20 tons of wheat while Barn B contains 40 tons of wheat. If 80 tons of wheat is going to be delivered to these two barns, how many tons of wheat should be delivered to Barn A and Barn B respectively so that the amount of wheat in Barn A is 1.5 times of that of Barn B?

18. To manufacture a certain drink, the weight of chocolate, sugar and water is mixed in the ratio of 1:2:14. In order to manufacture 2550 kg of the drink, what amount (in kg) of chocolate, sugar and water would be required respectively?

19. A certain concrete is made up of four constituents, namely water, cement, sand and gravel. The four constituents are mixed in the ratio of 0.7 : 1 : 2 : 4.7. If 2100 kg of the concrete is produced, how much of each constituent (in kg) is mixed?

【Example 5】 A team of scouts went for an outward bound training. They travelled at a speed of 5 km/hr. After the scouts were gone for 18 minutes, the school had an urgent notice to be sent to the scout captain. A messenger started from the school, riding a bicycle at the speed of 14 km/hr to catch up with the scouts. How many hours is required for the messenger to be able to catch up with the scouts?

Analysis: In this problem, when the messenger caught up with the scout team, the distance travelled by the messenger and the scout team would be the same.
In the 18 minutes before the messenger started from the school, the distance travelled by the team can be calculated. If the time taken by the messenger to catch up with the scout team is x hours, then we can calculate the distance travelled by the scouts and the messenger as their speeds

are known, we can then express the relationship of the distances in an equation.

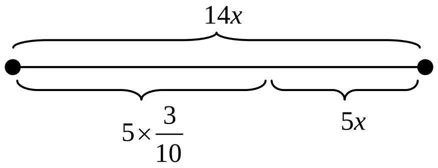


Diagram 3-5

Solution Assume that the time taken by the messenger to catch up with the scout team is x hours. Then the messenger travelled $14x$ km, the scout team travelled $5\times\frac{3}{10}$ km in 18 minutes and travelled $5x$ km in x hours. According to the information in the problem, we get

$$14x = 5\times\frac{3}{10} + 5x .$$

Solve this equation:

$$9x = \frac{3}{2}$$

$$x = \frac{1}{6}$$

Answer: The messenger took $\frac{1}{6}$h (10 minutes) to catch up with the team.

【Example 6】 There is a task to be completed. Person A working alone takes 20 hours to complete it and person B working alone takes 12 hours to complete it. Person A first works alone on the task for 4 hours, then persons A and B work together to finish the remaining part of the task. How many hours would it require for persons A and B to work together to finish remaining part of the task?

Analysis: As we know the time required for person A and person B to work individually to complete the task, we can calculate the fraction of the task each of them can do in 1 hour. If we assume the time required to finish the remaining part of the task is x hours, we can determine the fraction of the task done by person A in the first 4 hours, and the fraction of the task done by person A and B together. According to the problem, the three fractions of the task would add up to the whole (Diagram 3-6), then we can list out the following equation.

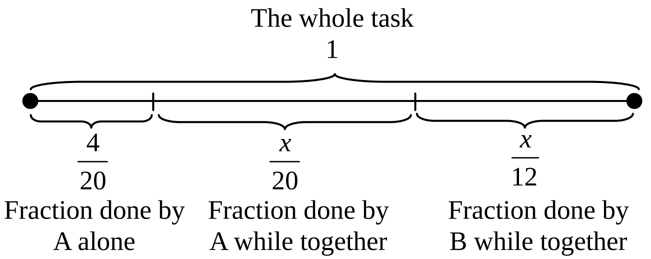


Diagram 3-6

Solution Assume that the time required for persons A and B to finish the remaining part of the task is x hours. Then representing the whole task as 1, person A performed $\frac{4}{20}$ of the task in the first 4 hours. While A and B worked together, person A and person B finished $\frac{x}{20}$ and $\frac{x}{12}$ of the task respectively. According to the information in the problem, we list the equation as $\frac{4}{20}+\frac{x}{20}+\frac{x}{12}=1$.

Solving the equation:

$$12+3x+5x=60$$
$$8x=48$$
$$x=6$$

Answer: Persons A and B would work for 6 hours to complete the remaining part of the task.

Practice

1. A and B participate in a race, A’s speed is 7 m per second and B’s speed is 6.5 m per second.. If B starts to run 1 second earlier, how many seconds would it require for A to catch up with B?

2. A and B departed from the same place in the same direction. A starts 1.5 hours earlier, and walks at the speed of 5 km per hour, while B rides a bicycle. After 50 minutes, both of them arrive the same destination at the same time. What is the speed of B in km per hour?

3. Working alone, A can pave an underground conduit in 12 days, and B can pave it in 18 days. If A and B work together to pave the conduit from both ends, how many days do they need to complete the work?

4. A certain task can be completed by A alone in 3 hours and by B alone in 5 hours. If they work together, how many hours would be needed to finish $\frac{4}{5}$ of the task?

5. A project is going to be completed in a period of time with a 3-phase development. The first phase takes 40% of the period. The second phase takes 36% of the period. The third phase takes 120 days. How long is the period to complete the whole project?

【Example 7】It is required to produce 0.15% diluted sugar solution. There are 30 kg of sugar solution at 16% concentration. How many kg of water would be required for the dilution?

Analysis: If water is added to a sugar solution, both the weight and concentration level would be changed, but the amount of sugar in the sugar solution would remain unchanged (Diagram 3-7), therefore we have the following equation

Amount of sugar in the solution before adding water
= Amount of sugar in the solution after adding water.

As we know the weight and concentration of sugar solution before adding water, we can calculate the amount of sugar in the solution before adding water. Besides, if we assume the amount of water added is x kg, we can base on the weight and concentration level of the sugar solution after adding water to calculate the amount of sugar in the solution. Then we can specify the equation by equating the amount of sugar before and after adding water

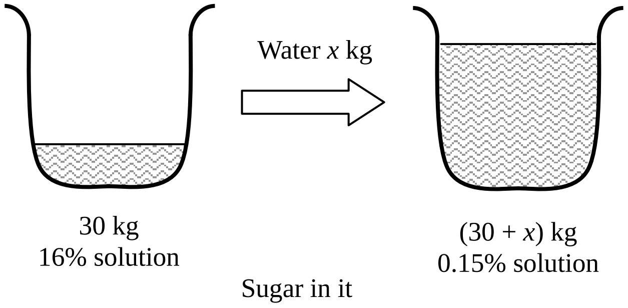


Diagram 3-7

Solution Assume that the amount of water needed to be added is x kg, then the weight of the sugar solution after adding water is $30+x$ kg and the amount of sugar in the 0.15% sugar solution is $(30+x)\times 0.15\%$ kg, the amount of sugar in the 16% sugar solution before adding water is $30\times 16\%$ kg. From the information in the problem, we get

$$(30+x)\times\frac{0.15}{100}=30\times\frac{16}{100}.$$

Solving the equation:

$$0.15(30+x)=30\times 16$$

$$30+x=3200$$

$$x=3170$$

Answer: Amount of wear required to be added is 3170 kg.

【**Example 8**】In a 2-digit number, the tens digit is less than the unit digit by 1, the sum of the tens digit and the unit digit equals $\frac{1}{5}$ of the number, Find the 2-digit number.

Analysis: Given that the tens digit is less than the unit digit by 1, if we assume that the tens digit is x, then we can express the value of the unit digit, we can further express「the sum of the two digits」and the value of the 2-digit number. According to the information that "the sum of the two digits is equal to $\frac{1}{5}$ times the 2-digit number", we can specify the equation.

Solution Assume that the tens digit is x, then the unit digit is $x + 1$, their sum is $x + (x + 1)$ and the value of the 2-digit number is $10x + (x + 1)$. According to the information of the problem, we get

$$x+(x+1)=\frac{1}{5}[10x+(x+1)].$$

Solving the equatio:

$$2x+1=\frac{1}{5}(11x+1)$$

$$10x+5=11x+1$$

$$x=4$$

$$x+1=5$$

Answer: The 2-digit number is 45.

Practice

1. Write down the answer only:
   (1) A 6% sugar solution weighs 10 kg. What is the weight (in kg) of sugar in the solution?
   (2) A 10% sugar solution weighs x kg, What is the weight (in kg) of sugar in the solution?

Practice

List the linear equation in one unknown in the following exercise:

2. To dilute 20 kg of sugar solution from 15% concentration to 10% concentration , how many kg of water is needed to be added?

3. How many kg of sugar would be required to be added to 20 kg of sugar solution to increase its concentration from 15% to 20%?

4. There is a 2-digit number, its unit digit is twice its tens digit. If the unit digit and the tens digit are interchanged, the new 2-digit number is larger than the original 2-digit number by 35, find the original 2-digit number.

5. The length of a rectangle is 16 m and it is longer than the width by 2 m. Find its area.

Exercise 14

List the linear equation in one unknown for the following exercise.

1. A group of students goes for a hiking They walk at a speed of 4 km per hour. After the students left for half an hour, the school has an urgent notice to send to the student leader. The school sends a messenger to ride a bicycle at 14 km per hour to catch up with the students. How many minutes would it require for the messenger to catch up with the students?

2. Stations A and B are separated at a distance of 243 km. A slow train departs from Station A running at a speed of 52 km per hour. At the same time, a fast train running at 70 km per hour departs from Station B. The two trains travel in the same direction with the fast train running behind the slow train. How many hours will it take for the fast train to catch up with the slow train?

3. Two trucks deliver supplies to a farm. The first truck departs from the warehouse to the farm travelling at 30 km per hour. The second truck departs 12 minutes later from the warehouse to the farm. It runs at a speed of 40 km per hour. The two trucks arrive at the farm at the same time. Find the distance between the warehouse and the farm.

4. A pool can be filled with water running from three taps A, B, and C. Tap A alone takes 45 minutes to fill up the pool. Tap B alone takes 60 minutes to fill up the pool and Tap C alone takes 90 minutes to fill up the pool. If all three taps are turned on at the same time, how many minutes will it take to fill up the pool?

5. To perform maintenance inspection work on a machine, person A working alone would take 7.5 hours to complete the task and person B working alone would take 5 hours to complete the task. If persons A and B work together for 1 hours, and person B is left alone to complete the remaining part of the task, how many hours in total would it take to complete the task?

6. A factory needs to produce 2940 kg of 10% saline solution, how many kg of 98% saline solution is required?

7. To dilute 600 g of 95% alcohol solution into 75% alcohol solution, how many g of distilled water is required to be added?

8. There is 40 kg of 8% saline solution. It is required to increase the salinity to a 20% saline solution, how many kg of salt is required to be added?

9. A farm needs to spray 100 ppm (ppm is a unit of concentration, 1 ppm stands for $\frac{1}{10^6}$ which is 1 in a million) of insecticide solution, how many g of 20% insecticide powder is needed to add to 50 kg of water (answer rounded to 1 g)

10. How many g of water is required to be added to 90 g of salt to prepare a 15% saline solution?

11. A tank with full load of oil weighs 8 kg. The tank with half load of oil removed weighs 4.5 kg . What is the weight of the full load of oil in kg?

12. The sum of three consecutive numbers is 18. Find their product.

13. A ferry commutes between two piers up and down the stream. When cruising down stream, it takes 4 hours. When cruising up stream, it takes 5 hours. The current of the stream is 2 km per hour. Find the speed of the ferry in still water.

14. A plane flies between two cities. The wind speed is 24 km per hour. Flying in same direction of the wind current takes $2\frac{5}{6}$ hours and flying against the direction of the wind current takes 3 hours. Find the distance between the two cities.

15. There is a 3-digit number. The sum of its three digits is 17.Tthe hundreds digit is larger than the tens digit by 7.The unit digit is 3 times the tens digit. Find the 3-digit number.

16. The lengths of three sides of a triangle are in the ratio of 2: 4: 5. The longest side is 6 cm longer than the shortest side. Find the perimeter of the triangle.

Chapter Summary

I. The main content of this chapter teaches about equations, equivalent equation axioms, linear equations in one unknown, their solving methods and applications.

II. An equality involving unknowns is called an equation. If two equations have equal solutions, the two equations are called equivalent equations. The equivalent Equation Axioms explained in this book include:

1. When both sides of an equation are added (or subtracted) by the same number or algebraic expression, the resultant equation is an equivalent equation to the original equation;
2. When both sides of an equation are multiplied (or divided) by the same non-zero number, the resultant equation is an equivalent equation to the original equation;

These two Equivalent Equation Axioms are the basis for solving equations.

III. An equation which involves only one unknown of degree 1 is called a linear equation in one unknown. To solve the linear equation in one unknown, we apply the two Equivalent Equation Axioms in steps to remove the denominators, to remove brackets, to transpose terms, to collect like terms, which transforms the equation to the simplest equation of the form $ax = b$ $(a \neq 0)$, then we divide both sides of the equation by the coefficient of the unknown to obtain the solution $x = \frac{b}{a}$.

IV. To apply linear equation in one unknown to solve problems, we proceed in steps: (i) first understand the problem, use letter (such as *x*) to represent the unknown in the problem; (ii) list the algebraic expression according to information given in the problem; (iii) formulate an equation to express the relationship between the known values and the unknown values; (iv) solve the linear equation for the value of the unknown; (v) verify the value of the unknown to see if it is a solution to the original equation; write down the final answer.

Revision Exercise 3

1. Apply knowledge learnt from Primary school to solve the following equation:

(1) $120 + x = 150$; (2) $30 - x = 16$;

(3) $0.5x = 15.5$; (4) $x \div \frac{2}{3} = \frac{7}{8}$.

2. What conditions must be satisfied for two equations to be classified as equivalent equations? Illustrate with examples.

3. What are the two Equivalent Equality Axioms? Illustrate with examples.

4. Is the following problem correctly solved? If not correct, where is the error?

(1)
$$2x+1=4x+1$$
$$2x+4x=0$$
$$6x=0$$
$$x=0$$

(2)
$$\frac{x}{2}=x+6$$
$$\frac{x}{2}-x=6$$
$$-\frac{x}{2}=6$$
$$x=12$$

(3)
$$\frac{x+1}{2}=\frac{3x-1}{2}-1$$
$$x+1=3x-1-1$$
$$2x=3$$
$$x=\frac{3}{2}$$

(3)
$$\frac{2x+1}{3}-\frac{x+1}{6}=2$$
$$4x+2-x+1=12$$
$$3x=9$$
$$x=3$$

Solve the following equations (No. 5～8) :

5. (1) $\frac{4}{3}-8x=3-\frac{11}{2}x$; (2) $0.5x-0.7=6.5-1.3x$;

(3) $\frac{1}{6}(3x-6)=\frac{2}{5}x-3$; (4) $\frac{1}{3}(1-2x)=\frac{2}{7}(3x+1)$.

6. (1) $3(8x-1)-2(5x+1)=6x(2x+3)+5(5x-2)$;

(2) $3(x-7)-2[9-4(2-x)]=22$;

(3) $\frac{x+4}{5}-x+5=\frac{x+3}{3}-\frac{x-2}{2}$;

(4) $\frac{1}{2}(y+1)+\frac{1}{3}(y+2)=3-\frac{1}{4}(y+3)$.

*7. (1) $\frac{3}{4}\left[\frac{4}{3}\left(\frac{1}{2}x-\frac{1}{4}\right)-8\right]=\frac{3}{2}x+1$;

(2) $x-\frac{1}{3}\left[x-\frac{1}{3}(x-9)\right]=\frac{1}{9}(x-9)$;

(3) $3\{2x-1-[3(2x-1)+3]\}=5$;

(4) $4(x-2)-[5(1-2x)-4(5x-1)]=0$.

*8. (1) $\frac{4x-1.5}{0.5}-\frac{5x-0.8}{0.2}=\frac{1.2-x}{0.1}$;

(2) $\frac{5x+\frac{7}{3}}{2}=\frac{6x-\frac{5}{2}}{3}-\frac{x+4}{6}$.

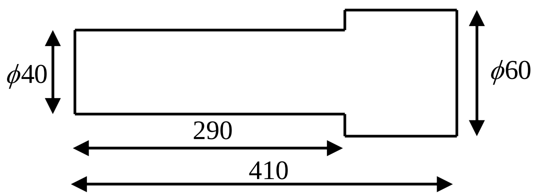
9. What value of *x*, will cause the terms $3a^2b^{2x+1}$ and $\frac{1}{4}a^2b^{3x-1}$ to become like terms?

List the linear equation in one unknown and solve the following application (No.10～23):

10. The sum of two numbers is 25. One number is larger than 2 times the other number by 4. . Find the two numbers.

11. The sum of 3 consecutive odd numbers is 69, find the three numbers.

12. The diagram below (The unit of measure in the Diagram is mm) represents the cross-section of a spare part. If we use the material from a circular wire of diameter 70 mm to construct the mould of the spare part, what length of the wire (answer rounded to 1 mm) is required to be cut?



(No. 12)

13. (China classic problem)[3] Thoroughbred horse runs 240 km a day. Ordinary horse runs 150 km a day. If the ordinary horse runs ahead 12 days, how many days will it take for the thoroughbred horse to catch up with the ordinary horse?

14. The length of the circular race track of a stadium is 400. Person A rides a bicycle at 490 m per minute. Person B runs at 250 m per minute. If the two persons depart at the same time from the same starting line and head in the same direction, how many minutes later will they come together again?

15. To manufacture 200 articles, person A works alone for 5 hours, then person A works with person B for another 4 hours, and the task is completed. Given that person A can manufacture 2 more articles than person B in one hour, find how many articles each person working alone can manufacture in one hour.

16. Spraying 50 ppm "Dong" solution can stop bacteria growth. To prepare 50 ppm "Dong" solution, it is required to add water to dilute the 2% "Dong" paste into 50 ppm concentration (1 ppm = 1 part per million). How many times of water will have to be added to the "Dong"paste?

17. An enemy plane comes to attack us, our plane detects it at a distance of 50 km. The enemy plane flees at 15 km per minute. Our plane chases it at 22 km per minute. When our plane narrows the distance to only 1km from the enemy plane, our plane shoots at the enemy plane and destroys it in half a minute. How many minutes have passed from the time the enemy plane is being detected to the time it is being destroyed?

18. There is a plane that can only fly continuously for 4 hours. The plane flies out at 950 km per hour and flies back at 850 km per hour. What is the furthest distance the plane can fly out and return within the safety 4 hours limit/ (answer rounded off to integer)?

19. A farmer ploughed $\frac{1}{3}$ of the land on the first day, and ploughed $\frac{1}{2}$ of the unfarmed land on the second day. Then there are 38 acres of land remaining unfarmed. How many acres of land are there to be farmed originally?

*20. The ratios of length versus the width of two rectangles are both 2 : 1. For the larger rectangle, the length is 3 mm larger than the width. The perimeter of the larger rectangle is 2 times the perimeter of the smaller rectangle. Find the area of each of the two rectangles.

*21. (China classic problem)[4] A rope is used to measure the depth of a well: Holding the rope in 3-fold, 4 m of rope is exposed; Holding the rope in 4-fold, 1 m of rope is exposed. Find the depth of the well and the length of the rope.

*22. An alloy of gold and silver weighs 250 gm. When immersed in water, the weight is reduced by 16 g. Given that pure gold immersed in water weighs $\frac{1}{19}$ less and that pure silver immersed in water weighs $\frac{1}{10}$ less, find the respective weight (in g) of gold and silver in the alloy.

---

[3] This problem came from the Yuen Dynasty from a book called《Introduction to Mathematics》(Year 1299). The original question is: Thoroughbred horse runs 240 km a day, ordinary horse runs150 km a day, ordinary horse starts out 12 days ahead, how many days will it take for the thoroughbred horse to catch up with the ordinary horse. Answer: 20 days.

[4] This problem came from the Ming Dynasty in a book called《Integrated Mathematics》(Year 1592) Chapter 7 . Original question is: Assume the depth of the well in unknown, probing with a rope folded 3 times, the excess is 4 feet 先, probing with a rope folded 4 times, the excess is 1 foot. Find the depth of the well and the length of the rope. Answer: Depth of well is 8 feet, length of rope is 36 feet.

*23. In an examination, there are 25 multiply choice questions. . 4 marks are credited for a correct answer and 1 mark is deducted for an incorrect answer or any question unanswered. A student scores 90 marks, how many questions has he answered correctly? If he scored 60 marks, how many questions has he answered correctly?

(The chapter is translated into English by courtesy of Mr. SIN Wing Sang, Edward and reviewed by courtesy of Ms. YIK Kwan Ying)