Chapter 6 Multiplication and Division of Integral Expressions

I. Multiplication of Integral Expressions

6.1 Multiplication of powers of the same base

We have learnt addition and subtraction of integral expressions, now we shall proceed to learn multiplication and division of integral expressions. Thus, we first investigate multiplication of powers of the same base. Let us calculate

$$10^3 \times 10^2, \quad 2^3 \times 2^2 .$$

Following the rule of multiplication, we get

$$\begin{aligned} 10^3 \times 10^2 &= (10\times10\times10)\times(10\times10) \\ &= 10\times10\times10\times10\times10 \\ &= 10^5 \end{aligned}$$

$$\begin{aligned} 2^3 \times 2^2 &= (2\times2\times2)\times(2\times2) \\ &= 2\times2\times2\times2\times2 \\ &= 2^5 \end{aligned}$$

From the above, we observe that 10^3 is the product of three 10s multiplied together, 10^2 is two 10s multiplied together, therefore $10^3 \times 10^2$ is five 10s multiplied together. For the same reason, 2^3 is three 2s multiplied together, 2^2 is two 2s mulitipled together, therfore $2^3 \times 2^2$ is five 2s multiplied together. As such

$$10^3 \times 10^2 = 10^{3+2}$$

$$2^3 \times 2^2 = 2^{3+2}$$

For the same reason,

$$\begin{aligned} a^3 \bullet a^2 &= (aaa)(aa) \\ &= aaaaa \\ &= a^5 \end{aligned}$$

As such

$$a^3 \bullet a^2 = a^{3+2}$$

In general, if m, n are integers, then

$$a^m \bullet a^n = (\underbrace{aa\cdots a}_{m \text{ terms}})(\underbrace{a\cdots a}_{n \text{ terms}}) = \underbrace{aa\cdots a}_{(m+n) \text{ terms}} = a^{m+n}$$

that is

$$\boxed{\boldsymbol{a^m \bullet a^n = a^{m+n}}}$$

In other words, **when powers of the same base are multiplied together, the base does not change while the indices are added together.**

When three powers of the same base are multiplied together, it follows the same mechanism. For example

$$a^m \bullet a^n \bullet a^p = a^{m+n+p} \quad (m, n, p \text{ are positive integers})[6]$$

【Example 1】 Calculate: (1) $10^7 \times 10^4$; (2) $x^2 \bullet x^5$; (3) $y \bullet y^2 \bullet y^3$; (4) $-a^2 \bullet a^6$.

Solution (1) $10^7 \times 10^4 = 10^{7+4} = 10^{11}$;

(2) $x^2 \bullet x^5 = x^{2+5} = x^7$;

(3) $y \bullet y^2 \bullet y^3 = y^{1+2+3} = y^6$;

(4) $-a^2 \bullet a^6 = -a^{2+6} = -a^8$.

【Example 2】 Calculate: (1) $x^n \bullet x^2$; (2) $y^m \bullet y^{m+1}$

Solution (1) $x^n \bullet x^2 = x^{n+2}$;

(2) $y^m \bullet y^{m+1} = y^{m+(m+1)} = y^{2m+1}$.

【Example 3】 Express the following in the form of $(p+q)^n$ or $(s-t)^n$.

(1) $(p+q)^3 \bullet (p+q)^2$;

(2) $(s-t)^2 \bullet (s-t) \bullet (s-t)^4$;

(3) $(p+q)^m \bullet (p+q)^n$.

[6] All the indicies of powers in this chapter are positive integers.

Analysis: Regard $(p+q)$ or $(s-t)$ as the base *a*, then we can use the rule for product of same base to perform the calculation.

Solution (1) $(p+q)^3 \bullet (p+q)^2 = (p+q)^{3+2} = (p+q)^5$;

(2) $(s-t)^2 \bullet (s-t) \bullet (s-t)^4 = (s-t)^{2+1+4} = (s-t)^7$;

(3) $(p+q)^m \bullet (p+q)^n = (p+q)^{m+n}$.

Practice

1. (*Mental*) Calculate:

(1) $10^5 \bullet 10^6$; (2) $s^5 \bullet s^8$; (3) $a^7 \bullet a^3$; (4) $y^3 \bullet y^2$;

(5) $b^5 \bullet b$; (6) $x^4 \bullet x^4 \bullet x$; (7) $a^n \bullet a$; (8) $x^n \bullet x^n$.

2. Calculate:

(1) 10×10^8; (2) $a^4 \bullet a^6$; (3) $x^5 \bullet x^5$; (4) $y^{12} \bullet y^6$;

(5) $x^{10} \bullet x$; (6) $-b^3 \bullet b^7$; (7) $y^4 \bullet y^3 \bullet y^2 \bullet y$;

(8) $x^5 \bullet x^6 \bullet x^3$; (9) $10^2 \bullet 10^n$; (10) $a^n \bullet a^{2n}$;

(11) $y^{m+1} \bullet y^{m-1}$; (12) $(-2)^2 \bullet (-2)^3$; (13) $y^n \bullet y \bullet y^{n+1}$.

3. Express the following in the form of $(x+y)^n$:

(1) $(x+y)^2 \bullet (x+y)^2$; (2) $(x+y)^3 \bullet (x+y)$;

(3) $(x+y)^3 \bullet (x+y) \bullet (x+y)^2$; (4) $(x+y)^{m+1} \bullet (x+y)^{m+n-1}$.

4. Which of the following calculation is correct and which incorrect, why? If incorrect, how should it be corrected?

(1) $b^5 \bullet b^5 = 2b^5$; (2) $x^5 + x^5 = x^{10}$;

(3) $c \bullet c^3 = c^3$; (4) $m^3 \bullet m^2 = m^5$.

6.2 Multiplication of monomials

Having learnt the multiplication of powers of the same base, we can proceed to learn multiplication of monomials. Let us calculate

$$2x^2y \bullet 3xy^2,\quad 4a^2x^5 \bullet (-3a^3bx^2).$$

Using commutative law for multiplication, associative law of multiplication, we can combine the coefficients into one group, each same letter into one group, and multiply together, as follows

$$2x^2y \bullet 3xy^2 = (2\times 3)(x^2 \bullet x)(y \bullet y^2)$$
$$= 6x^3y^3$$
$$4a^2x^5 \bullet (-3a^3bx^2) = [4 \bullet (-3)](a^2a^3) \bullet b \bullet (x^5x^2)$$
$$= -12a^5bx^7$$

In general, **in multiplying two monomials together, the product has coefficient equal to the product of coefficients in the two monomials, and factors of all letters with each letter carrying index equal to the sum of the indices of the letter in the two monomials.**

【**Example 1**】Calculate:

(1) $4n^3 \bullet 5n^2$; (2) $(-5a^2b^3)(-3a)$;

(3) $(4\times 10^5)(5\times 10^6)(3\times 10^4)$.

Solution (1) $4n^3 \bullet 5n^2 = (4\times 5)(n^3n^2) = 20n^5$;

(2) $(-5a^2b^3)(-3a) = [(-5)\bullet(-3)]\bullet(a^2a)\bullet b^3 = 15a^3b^3$;

(3) $(4\times 10^5)(5\times 10^6)(3\times 10^4) = (4\times 5\times 3)(10^5\times 10^6\times 10^4)$
$= 60\times 10^{15}$.

NOTE: Using power of 10 to represent a number, we usually express the number in such a way that the factor before the power of 10 is a one digit integer or with only one digit before decimal. For the result of 60×10^{15} in the above computation, it can be written as $6\times 10\times 10^{15}$, which is 6×10^{16}.

【**Example 2**】Calculate:

(1) $\frac{1}{3}xy^2 \bullet 9x^2y$; (2) $\frac{2}{3}x^3y^2 \bullet \left(-\frac{3}{4}x^2y^3\right)$;

(3) $(-5a^{n+1}b)(-2a)$; (4) $(-3ab)\bullet(-a^2c)\bullet 6ab^2$.

Solution (1) $\frac{1}{3}xy^2 \bullet 9x^2y = \left(\frac{1}{3}\times 9\right)(x \bullet x^2)(y^2 \bullet y) = 3x^3y^3$;

(2) $\frac{2}{3}x^3y^2 \bullet \left(-\frac{3}{4}x^2y^3\right) = \left[\frac{2}{3}\times\left(-\frac{3}{4}\right)\right]x^5y^5 = -\frac{1}{2}x^5y^5$;

(3) $(-5a^{n+1}b)(-2a) = [(-5)\bullet(-2)](a^{n+1}\bullet a)\bullet b = 10a^{n+2}b$;

(4) $(-3ab)\bullet(-a^2c)\bullet 6ab^2 = [(-3)\bullet(-1)\bullet 6]a^4b^3c$
$= 18a^4b^3c$.

【**Example 3**】Speed of light is 3×10^5 km per second, the time it takes for the sun's ray to reach the earth is approximately 5×10^2 seconds, what is the approximate distance in km between the earth and the sun?

Solution $(3\times10^5)\times(5\times10^2) = 15\times10^7 = 1.5\times10^8$.

Answer: Distance between the earth and the sun is approximately 1.5×10^8 km.

Practice

1. Calculate:

(1) $3x^5 \bullet 5x^3$; (2) $4y \bullet (-2xy^3)$;

(3) $(-2.5x^2)\bullet(-4x)$; (4) $\frac{2}{5}x^2y^3 \bullet \frac{5}{16}xyz$;

(5) $(-6a^{n+2})\bullet 3a^nb$; (6) $8x^ny^{n+1}\bullet\frac{3}{2}x^2y$;

(7) $(-3x)\bullet 2xy^2\bullet 4y$; (8) $(-4x^2y)\bullet(-x^2y^2)\bullet\frac{1}{2}y^3$.

2. A computer performs operations at a speed of 10^{12} per second. How many operations will it perform in 5×10^2 seconds?

3. Check if the following computation is correct, why? If incorrect, How should it be corrected?

(1) $4a^3 \bullet 2a^2 = 8a^5$; (2) $2x^3 \bullet 3x^4 = 5x^7$;

(3) $3x^2 \bullet 4x^2 = 12x^2$; (4) $3y^3 \bullet 5y^3 = 15y^9$.

6.3 Multiplication of powers

Let us calculate

$$(a^4)^3,\ (a^3)^5.$$

$(a^4)^3$ is cube of a^4. If we regard a^4 as base, following the rule of multiplication of powers of the same base, we get

$$(a^4)^3 = a^4 \bullet a^4 \bullet a^4 = a^{4+4+4} = a^{4\times3}.$$

In the same way, we get

$$(a^3)^5 = a^3 \bullet a^3 \bullet a^3 \bullet a^3 \bullet a^3 = a^{3+3+3+3+3} = a^{3\times5}.$$

Therefore

$$(a^4)^3 = a^{4\times3}$$
$$(a^3)^5 = a^{3\times5}$$

In general, if *m*, *n* are positive integers, then

$$(a^m)^n = \overbrace{a^m \bullet a^m \bullet \cdots \bullet a^m}^{n\text{ terms}} = a^{\overbrace{m+m+\cdots m}^{n\text{ terms}}} = a^{mn}$$

That is

$$\boxed{\mathbf{(a^m)^n = a^{mn}}}$$

Therefore we know, **when raising power of a base by another power, the result is a power of the same base with index equal to the product of the indices.**

【**Example 1**】Calculate:

(1) $(10^7)^2$; (2) $(x^3)^2$; (3) $(z^4)^4$.

Solution (1) $(10^7)^2 = 10^{7\times2} = 10^{14}$;

(2) $(x^3)^2 = x^{3\times2} = x^6$;

(3) $(z^4)^4 = z^{4\times4} = z^{16}$.

【**Example 2**】Calculate:

(1) $(a^m)^2$; (2) $(b^3)^n$.

Solution (1) $(a^m)^2 = a^{m\times2} = a^{2m}$;

(2) $(b^3)^n = b^{3\times n} = b^{3n}$.

【**Example 3**】Calculate: (1) $[(x+y)^2]^4$; (2) $(a^2)^4 \bullet (a^3)^3$.

Analysis: In Exercise (1), calculate with regard to $(x+y)^2$ being the power of a base; In Exercise (2), compute $(a^2)^4$, $(a^3)^3$ separately, then calculate accoding to the rule of multiplication of powers of the same base.

Solution (1) $[(x+y)^2]^4 = (x+y)^{2\times4} = (x+y)^8$;

(2) $(a^2)^4 \bullet (a^3)^3 = a^8 \bullet a^9 = a^{17}$.

Practice

1. (*Mental*) Calculate:

(1) $(x^4)^2$; (2) $x^4 \bullet x^2$; (3) $(y^5)^5$;

(4) $y^5 \bullet y^5$; (5) $(a^m)^3$; (6) $a^m \bullet a^3$.

2. (*Mental*) Calculate:

(1) $(10^3)^3$; (2) $(x^4)^3$; (3) $(a^2)^5$;

(4) $-(y^2)^4$; (5) $-(x^3)^6$; (6) $(s^m)^5$.

(7) $[(x+a)^3]^2$; (8) $[(x+y)^n]^2$; (9) $(a^2)^3 \bullet a^5$.

(10) $(a^2)^5 \bullet (a^4)^4$; (11) $(b^3)^2 \bullet (b^2)^3$; (12) $(c^2)^n \bullet c^{n+1}$.

3. Check if the following computation is correct, why? If incorrect, How should it be corrected?

(1) $(a^5)^2 = a^7$; (2) $a^5 \bullet a^2 = a^{10}$;

(3) $(x^4)^7 = x^{28}$; (4) $(a^{n+1})^2 = a^{2n+1}$.

6.4 Power of a Product

Let us calculate

$$(ab)^3, \ (ab)^4.$$

Applying Commutative Law for Multiplication, Associative Law for Multiplication to the calculation of product, we get

$$(ab)^3 = (ab)\bullet(ab)\bullet(ab) = (aaa)\bullet(bbb) = a^3b^3$$

$$(ab)^4 = (ab)\bullet(ab)\bullet(ab)\bullet(ab) = (aaaa)\bullet(bbbb) = a^4b^4$$

In general, if n is a positive number, then

$$(ab)^n = \overbrace{(ab)\bullet(ab)\bullet\cdots\bullet(ab)}^{n \text{ terms}} = \overbrace{(a\bullet a\bullet\cdots\bullet a)}^{n \text{ terms}}\bullet\overbrace{(b\bullet b\bullet\cdots\bullet b)}^{n \text{ terms}} = a^nb^n$$

That is

$$\boxed{\boldsymbol{(ab)^n = a^nb^n}}$$

That is to say, the power of a product of factors **equals the product of the power of each constituent factor**.

When there are three or more factors multiplied together, the method of taking the power of the product of factors follows the same mechanism. For example

$$(abc)^n = a^nb^nc^n.$$

【**Example 1**】Compute:

(1) $(xy)^5$; (2) $(2a)^4$; (3) $(-3x)^3$;

(4) $(-5ab)^2$; (5) $(-xy)^6$; (6) $(4xy)^2$.

Solution (1) $(xy)^5 = x^5y^5$;

(2) $(2a)^4 = 2^4a^4 = 16a^4$;

(3) $(-3x)^3 = (-3)^3x^3 = -27x^3$;

(4) $(-5ab)^2 = (-5)^2a^2b^2 = 25a^2b^2$;

(5) $(-xy)^6 = (-1)^6x^6y^6 = x^6y^6$;

(6) $(4xy)^2 = 4^2x^2y^2 = 16x^2y^2$.

【**Example 2**】Compute:

(1) $(xy^2)^2$; (2) $(a^2b^2)^4$;

(3) $(-2xy^3)^4$; (4) $\left(\frac{2}{3}a\right)^2$.

Solution (1) $(xy^2)^2 = x^2(y^2)^2 = x^2y^4$;

(2) $(a^2b^2)^4 = (a^2)^4\bullet(b^2)^4 = a^8b^8$;

(3) $(-2xy^3)^4 = (-2)^4\bullet x^4\bullet(y^3)^4 = 16x^4y^{12}$;

(4) $\left(\frac{2}{3}a\right)^2 = \left(\frac{2}{3}\right)^2\bullet a^2 = \frac{4}{9}a^2$.

【**Example 3**】Compute:

(1) $(2x)^3 \bullet (-5x^2y)$;

(2) $(3xy^2)^2 + (-4xy^3) \bullet (-xy)$.

Solution (1) $(2x)^3 \bullet (-5x^2y) = 8x^3 \bullet (-5x^2y) = -40x^5y$;

(2) $(3xy^2)^2 + (-4xy^3) \bullet (-xy) = 9x^2y^4 + 4x^2y^4 = 13x^2y^4$.

Practice

1. (*Mental*) What is the result of calculation of the following?

(1) $(ab)^6$; (2) $(xy)^4$; (3) $(2m)^3$;

(4) $(5x^2)^2$; (5) $(ab^2)^3$; (6) $(-xy)^3$.

2. Compute:

(1) $(st)^3$; (2) $(4a^3)^2$; (3) $(-2x^2y)^2$; (4) $\left(\frac{1}{2}c^2d\right)^3$;

(5) $(2\times10^2)^2$; (6) $(x^2 \bullet x \bullet x^5)^3$; (7) $(ab^2)^3 \bullet (ab^2)^2$.

(8) $(-3x^5)^3 \bullet x^2$; (9) $a \bullet (ab^2)^2$; (10) $(3y)^2 \bullet (y^2)^3$.

3. Compute:

(1) $a^2 \bullet (a^2b^3)^2$; (2) $(3m)^2 \bullet \left(-\frac{1}{2}mn\right)^3$;

(3) $2a^2 \bullet (-2a)^3 + 2a^4 \bullet 5a$; (4) $10a^3 \bullet \frac{3}{5}b + (-3.5a^2) \bullet (ab)^2$.

4. A factory would like to build a cubic oil container with side 4×10^2 cm, calculate the volume of the oil container.

5. Check if the following computation is correct, why? If incorrect, how should it be corrected?

(1) $(ab^2)^2 = ab^4$; (2) $(3xy)^3 = 9x^3y^3$;

(3) $(-2a^2)^2 = -4a^4$; (4) $\left(\frac{4}{5}x^2y^3\right)^3 = \frac{64}{125}x^6y^9$.

6.5 Multiplication of a Polynomial by a Monomial

Now let us investigate the product of a monomial and a polynomial. We calculate

$$m(a+b+c).$$

Using distributive law for multiplication, we get

$$m(a+b+c) = ma + mb + mc.$$

This result can be rationalized from Diagram 6-1[7].

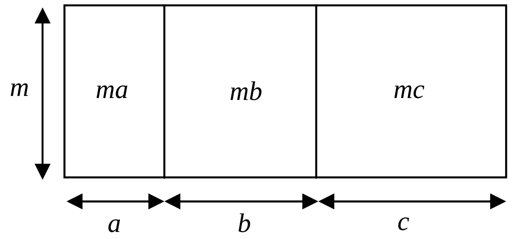


Diagram 6-1

In general, **the product of a monomial with a polynomial is the sum of the constituent products of the monomial with each constituent term of the polynomial.**

【**Example 1**】Compute:

(1) $(-4x) \bullet (2x^2 + 3x - 1)$;

(2) $\left(\frac{2}{3}a^2b - 2ab + \frac{4}{3}b\right) \bullet \frac{1}{2}ab$.

Solution (1) $(-4x) \bullet (2x^2 + 3x - 1)$

$= (-4x) \bullet (2x^2) + (-4x) \bullet (3x) + (-4x) \bullet (-1)$

$= -8x^3 - 12x^2 + 4x$

(2) $\left(\frac{2}{3}a^2b - 2ab + \frac{4}{3}b\right) \bullet \frac{1}{2}ab$

[7] In Diagram 6-1, *m*, *a*, *b*, *c* represent positive numbers. In fact, *m*, *a*, *b*, *c* in the equation $m(a+b+c) = ma + mb + mc$ can represent zero or negative numbers.

$$=\left(\frac{2}{3}a^2b\right)\cdot\left(\frac{1}{2}ab\right)+(-2ab)\cdot\left(\frac{1}{2}ab\right)+\left(\frac{4}{3}b\right)\cdot\left(\frac{1}{2}ab\right)$$
$$=\frac{1}{3}a^3b^2-a^2b^2+\frac{2}{3}ab^2$$

【**Example 2**】Simplify:

(1) $(-3xy)\cdot 5x^2y+6x^2\cdot\left(\frac{7}{2}xy^2-2y^2\right)$;

(2) $-2a^2\cdot\left(\frac{1}{2}ab+b^2\right)-5ab\cdot(a^2-1)$.

Solution (1) $(-3xy)\cdot 5x^2y+6x^2\cdot\left(\frac{7}{2}xy^2-2y^2\right)$
$$=-15x^3y^2+21x^3y^2-12x^2y^2$$
$$=6x^3y^2-12x^2y^2$$

(2) $-2a^2\cdot\left(\frac{1}{2}ab+b^2\right)-5ab\cdot(a^2-1)$
$$=-a^3b-2a^2b^2-5a^3b+5ab$$
$$=-6a^3b-2a^2b^2+5ab$$

Practice

1. Compute:

(1) $(ab)^2\cdot(2ab^2)^3$; (2) $3k\cdot(kh^2)^2$;

(3) $3(2x+1)$; (4) $-4\left(\frac{3}{2}y-5\right)$;

(5) $(x-3y)(-6x)$; (6) $5x(2x^2-3x+4)$;

(7) $\left(5a^2-\frac{4}{9}a+1\right)\cdot(-3a^2)$; (8) $\left(-x^3y-4x^2y^2+\frac{5}{6}y^4\right)\cdot\frac{3}{2}xy$

Practice

2. Simplify:

(1) $(-a)^2\cdot(-2ab)+3a^2\cdot\left(ab-\frac{1}{3}b-1\right)$;

(2) $x-\frac{1}{2}(x+1)+\frac{1}{3}(x-1)$;

(3) $3a(2a-5)+2a(1-3a)$;

(4) $x(x^2+3)+x^2(x-3)-3x(x^2-x-1)$;

(5) $3xy\left[6xy-3\left(xy-\frac{1}{2}x^2y\right)\right]$.

6.6 Multiplication of a Polynomial by another Polynomial

Now let us explore the mulitiplication of polynomial. Let us compute
$$(a+b)(m+n).$$

This is a multiplication of a polynomial by another polynomial. First regard $(m+n)$ as a monomial, use the rule of multiplication of monomial by polynomial to get
$$(a+b)(m+n)=a(m+n)+b(m+n).$$
Then use the use of mulitiplication of monomial by polynomial to get
$$(a+b)(m+n)=a(m+n)+b(m+n)$$
$$=am+an+bm+bn$$

That is

$$(a+b)(m+n)$$
$$=am+an+bm+bn$$

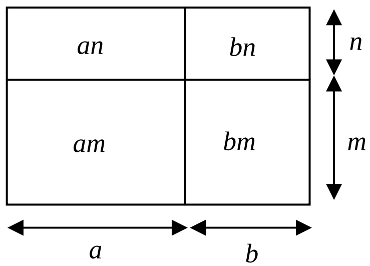


Diagram 6-2

The result can be rationalized illustratively from Diagram 6-2.

In general, **to multiply a polynomial by another polynomial, we first use each of the constituent terms of one polynomial to multiply each of the constituent terms of the other polynomial to get constituend products and then add all the constituent products together.**

【**Example 1**】Compute:

(1) $(x+2y)(5a+3b)$;

(2) $(2x-3)(x+4)$;

(3) $(3x+y)(x-2y)$.

Solution (1) $(x+2y)(5a+3b) = x\bullet 5a + x\bullet 3b + 2y\bullet 5a + 2y\bullet 3b$
$= 5ax + 3bx + 10ay + 6by$

(2) $(2x-3)(x+4) = 2x^2+8x-3x-12$
$= 2x^2+5x-12$

(3) $(3x+y)(x-2y) = 3x^2-6xy+xy-2y^2$
$= 3x^2-5xy-2y^2$

【**Example 2**】Compute:

(1) $\left(-\dfrac{a}{2}+3b^2\right)(a^2-2b)$;

(2) $(x+y)(x^2-xy+y^2)$.

Solution (1) $\left(-\dfrac{a}{2}+3b^2\right)(a^2-2b) = -\dfrac{a^3}{2}+ab+3a^2b^2-6b^3$

(2) $(x+y)(x^2-xy+y^2)$
$= x^3-x^2y+xy^2+x^2y-xy^2+y^3$
$= x^3+y^3$

Practice

1. (*Mental*) Compute:

(1) $(m+n)(u+v)$; (2) $(x+y)(a-b)$;

(3) $(p-q)(r+s)$; (4) $(a-b)(c-d)$.

Practice

2. Compute:

(1) $(2n+6)(n-3)$; (2) $(2x+3)(3x-1)$;

(3) $(2a-3b)(a+5b)$; (4) $(3x-2y)(3x+2y)$;

(5) $(2a+3)\left(\dfrac{3}{2}b-5\right)$; (6) $(2x+5)(2x+5)$.

3. Compute:

(1) $(x+1)(x^2-2x+3)$; (2) $(x-1)(x^2+x+1)$;

(3) $(4x-3)(5x^2-4x+7)$; (4) $(3x+2)(3x-2)(x^2-1)$;

(5) $(3a-2)(a-1)+(a+1)(a+2)$;

(6) $(2x^2-1)(x-4)-(x^2+3)(2x-5)$.

【**Example 3**】Compute: (1) $(x+2)(x+5)$; (2) $(y+2)(y-5)$.

Solution (1) $(x+2)(x+5) = x^2+5x+2x+10 = x^2+7x+10$;

(2) $(y+2)(y-5) = y^2-5y+2y-10 = y^2-3y-10$.

From Example 3, we observe that, when two binomials (a polynomial with two constituent terms is called a binomial) are multiplied together, if each of the binomials has a common letter, then the product will have a constituent term with power of square of the letter. From Example 3, we also observe that, when the coefficients of the letter term from each of the two multiplying binomials are both 1, then the coefficient of the constituent term with the power of square of the letter in the product is also 1. Here, since the coefficients of the letter term from each of the two multiplying binomials are both 1, then the coefficient of the consitutuent term with degree 1 of the letter in the product is the sum of the constant terms of the two binomials. The constant term of the product equals the product of the constant terms of the two multiplying binomials. Therefore, if a, b are the respective constant terms of the multiplying factors, then

$$\boxed{(x+a)(x+b) = x^2+(a+b)x+ab}$$

【**Example 4**】Solve the following equations:

(1) $(x+3)(x-4)=x^2-16$;

(2) $3x(x+2)+(x+1)(x-1)=4(x^2+8)$.

Solution (1)

$$(x+3)(x-4)=x^2-16$$
$$x^2-x-12=x^2-16$$
$$-x=-4$$
$$x=4$$

(2)

$$3x(x+2)+(x+1)(x-1)=4(x^2+8)$$
$$3x^2+6x+x^2-1=4x^2+32$$
$$6x=33$$
$$x=5\frac{1}{2}$$

Practice

1. Compute:

(1) $(x+1)(x+4)$; (2) $(m-2)(m+3)$;

(3) $(y+4)(y-5)$; (4) $(x-3)(x-5)$.

(5) $\left(y-\frac{1}{2}\right)\left(y+\frac{1}{3}\right)$; (6) $(7x+8)(6x-5)$;

(7) $\left(\frac{1}{2}x+4\right)\left(6x-\frac{3}{4}\right)$; (8) $(y^2+y+1)(y+2)$;

(9) $(x+2)(x+3)-x(x+1)-8$;

(10) $(3y-1)(2y-3)+(6y-5)(y-4)$.

2. Solve the following equations:

(1) $(2x+3)(x-4)-(x+2)(x-3)=x^2+6$;

(2) $2x(3x-5)-(2x-3)(3x+4)=3(x+4)$.

Exercise 18

1. Compute:

(1) $a^3 \bullet a^4$; (2) $x^3 \bullet x$; (3) $10^5 \bullet 10 \bullet 10^3$;

(4) $-b^3 \bullet b^2$; (5) $x^7 \bullet x \bullet x^{12}$; (6) $y^8 \bullet y^4 \bullet y \bullet y^4$.

2. Compute:

(1) $10^m \bullet 10^n$; (2) $c^2 \bullet c^m$;

(3) $x^3 \bullet x^{n+1}$; (4) $a^{n+2} \bullet a^{n+1}$;

(5) $y^n \bullet y \bullet y^{2n-1}$; (6) $b^m \bullet b^n \bullet b^s$;

(7) $a^2 \bullet a \bullet a^5 + a^3 \bullet a^2 \bullet a^3$ (8) $x^2 \bullet x^6 \bullet x^3 + x^5 \bullet x^4 \bullet x$.

3. Transform the following in the form of $(x-y)^n$:

(1) $(x-y)^2(x-y)^4$; (2) $(x-y)^3(x-y)(x-y)^{2m}$.

4. Compute:

(1) $(ax^2)(ax^n)$; (2) $(2ab^2)(-3ab)$;

(3) $(mn)(-m^2n)$; (4) $(3x^2y)(-3xy)$;

(5) $(-5a^2b^3)(2a^2b)$; (6) $(2c^3)\left(\frac{1}{4}c^2\right)(-2c)$;

(7) $\left(-\frac{3}{4}ax\right)\left(-\frac{2}{3}bx^5\right)$; (8) $(2a^nb^3)\left(-\frac{1}{6}ab^{n-1}\right)$.

(9) $-0.2xy^2+3x^2y \bullet \left(-\frac{1}{3}xy^5\right)$;

(10) $0.6m^2n \bullet \frac{1}{4}m^2n^2-(-10m) \bullet m^3n^3$.

5. The length and width of a rectangle are 2.2×10^3 cm and 1.5×10^2 cm respectively, calculate its area.

6. Light travels at the speed of approximately 3×10^5 km/second, the light emitted by a star outside our solar system takes 4 years to reach the earth, calculate the distance of the star from the earth, assuming that the number of seconds in 1 year is 3×10^7 seconds.

7. A satellite orbits the earth at a speed of 7.9×10^3 m/second, find the distance traveled by the satellite in 2×10^2 seconds.

8. Compute:

(1) $(a^3)^3$; (2) $(x^6)^5$; (3) $-(y^7)^2$;

(4) $(a^m)^3$; (5) $(b^2)^m$; (6) $[(x+3)^2]^3$;

(7) $[(ab)^n]^3$; (8) $(x^2)^3 \cdot x^4$; (9) $(y^3)^4 \cdot (y^4)^3$;

(10) $(-c)^3 \cdot (c^2)^5 \cdot c$.

9. Compute:

(1) $[(-1)^2]^3$; (2) $\left[\left(-\frac{1}{2}\right)^3\right]^2$;

(3) $(a^2)^3 + a^3 \cdot a^3$; (4) $(x^4)^2 + (x^5)^3$.

10. Compute the square of the following expression:

(1) $2xy^2$; (2) $-pq$; (3) ab^2c^3;

(4) $-3m^2$; (5) $-\frac{1}{2}st$; (6) $0.2cd^4$.

11. Compute:

(1) $(ab)^5$; (2) $(2x)^3$; (3) $(-3xy^2)^2$;

(4) $-7(m^3n)^3$; (5) $(-a^2b^3)^2$; (6) $\left(\frac{4}{5}xy^2z\right)^2$;

(7) $[1.5\times10^2]^2$; (8) $(3y^2)^3 \cdot y^4$; (9) $(3xy^2)^3 \cdot (y^3)^5$;

(10) $(a^2 \cdot a \cdot a^3 \cdot b^n)^4$.

12. Compute:

(1) $[(-2x^2y)^3]^2$; (2) $[(ab^2)^3]^3$;

(3) $\left(-\frac{2}{3}ax^3\right)^2 + (2ax^2)^2 \cdot x^2$; (4) $(a^nb)^2 + (a^2b^3)^n$;

(5) $(-2a)^6 - (-3a^3)^2 - [-(2a)^2]^3$.

13. Radius of the earth is $r = 6.4\times10^6$ m. Given that the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, calculate the volume of the earth (assume π=3.14, express answer in 2 significant figures).

14. Compute:

(1) $(3x^2y - xy^2) \cdot 3xy$; (2) $(4ab - b^2) \cdot (-2bc)$;

(3) $2x \cdot \left(x^2 - \frac{1}{2}x + 1\right)$; (4) $5ab \cdot (2a - b + 0.2)$;

(5) $(-2ab^2)^2 \cdot (3a^2b - 2ab - 4b^3)$;

(6) $\left(\frac{3}{4}x^2y - \frac{1}{2}xy^2 - \frac{5}{6}y^3\right) \cdot (-4xy^2)$.

15. Simplify:

(1) $3x^2 \cdot (-3xy)^2 - x^2(x^2y^2 - 2x)$;

(2) $5x \cdot (x^2 - 2x + 4) + x^2(x-1)$;

(3) $t^3 - 2t[t^2 - 2(t-3)]$;

(4) $x - \frac{1}{4}\left(1 - \frac{3x}{2}\right) - \frac{1}{3}x\left(2 - \frac{x}{4}\right)$.

16. Compute:

(1) $(3x+1)(x+2)$; (2) $(4y-1)(y-5)$;

(3) $(2x-3)(4x-1)$; (4) $(3a+2)(4a+1)$;

(5) $(5m+2)(4m-3)$; (6) $(5n-4)(3n-1)$;

(7) $(7x^2 - 8y^2)(x^2 + 3y^2)$; (8) $\left(\frac{2}{3}x - \frac{1}{2}y\right)\left(\frac{3}{4}x - \frac{2}{3}y\right)$;

(9) $(9m-4n)(9m+4n)$;

(10) $(x+2)(x-2)(x^2+4)$;

(11) $(1-2x+4x^2)(1+2x)$;

(12) $(x-y)(x^2+xy+y^2)$;

(13) $5x(x^2+2x+1) - (2x+3)(x-5)$;

(14) $(3x-y)(y+3x) - (4x-3y)(4x+3y)$.

17. Compute:

(1) $(x+3)(x+2)$; (2) $(a+5)(a-3)$;

(3) $(x-5)(x+3)$; (4) $(m+2)(m-8)$;

(5) $(x+7)(x-7)$; (6) $(y-3)(y+3)$;

(7) $(y-6)(y-3)$; (8) $\left(x+\dfrac{1}{2}\right)\left(x-\dfrac{1}{3}\right)$.

18. First simplify, then evaluate:

(1) $(3x+1)(2x-3)-(6x-5)(x-4)$, where $x=-2$;

(2) $(y-2)(y^2-6y-9)-y(y^2-2y-15)$, where $y=\dfrac{1}{2}$.

19. Solve the equation:

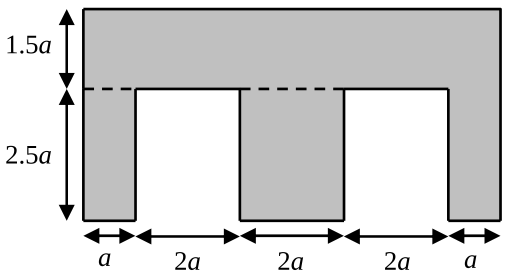
(1) $2(x^2-2)-6x(x-1)=4x(1-x)+16$;

(2) $x(x+3)-x(1-2x)=9+3x^2$;

(3) $(2x+3)(x-1)-28=(1+x)(2x+11)$;

(4) $(x-3)(x-2)+18=(x+9)(x+1)$.

20. Refer Diagram, calculate the area of the shaded portion (lengths in the Diagram is in cm).



(No. 20)

21. Length of a rectangle is $(2a+b)$ cm, width is $(a+b)$ cm, calculate its perimeter and area.

II. Multiplication Formula

In mulitiplication of polynomials, there are multiplication formula (called identities) of some special forms which are worth memorizing because when we encounter similar multiplications of polynomials, we can apply them straight away without having to do it all over again.

6.7 Difference of Two Squares Identity

Let us compute:

$$(a+b)(a-b)$$

we obtain

$$(a+b)(a-b)=a^2-ab+ab-b^2$$
$$=a^2-b^2$$

From here we get

$$\boxed{\boldsymbol{(a+b)(a-b)=a^2-b^2}}$$

That is to say, **for two numbers, the product of their sum and their difference equals the difference of squares of the two numbers.** This is **the Difference of Two Squares Identity**, which can be applied to calculate the product of the sum and difference of two numbers. For example, compute

$$(1+2x)(1-2x)$$

If we regard 1 as a, regard $2x$ as b, then

$$(1+2x)(1-2x)$$

is in the form of

$$(a+b)(a-b)$$

Therefore, we can calculate its result by applying the Difference of Two Squares Identity

$$(1+2x)(1-2x)=1^2-(2x)^2=1-4x^2$$
$$\boxed{(a+b)\ (a-b)}=\boxed{a^2-\ b^2}$$

【**Example 1**】 Compute using the Difference of Two Square Identity:

(1) $(3m+2n)(3m-2n)$;

(2) $(b^2+2a^3)(2a^3-b^2)$.

Solution (1) $(3m+2n)(3m-2n)=(3m)^2-(2n)^2=9m^2-4n^2$

(2) $$\begin{aligned}(b^2+2a^3)(2a^3-b^2)&=(2a^3+b^2)(2a^3-b^2)\\&=(2a^3)^2-(b^2)^2\\&=4a^6-b^4\end{aligned}$$

【**Example 2**】Compute using Difference of Two Squares Identity:

(1) $\left(-\frac{1}{2}x+2y\right)\left(-\frac{1}{2}x-2y\right)$;

(2) $(-4a-1)(4a-1)$.

Solution (1) $$\begin{aligned}\left(-\frac{1}{2}x+2y\right)\left(-\frac{1}{2}x-2y\right)&=\left(-\frac{1}{2}x\right)^2-(2y)^2\\&=\frac{1}{4}x^2-4y^2\end{aligned}$$

(2) $$\begin{aligned}(-4a-1)(4a-1)&=[(-1)-4a][(-1)+4a]\\&=(-1)^2-(4a)^2\\&=1-16a^2\end{aligned}$$

or

$$\begin{aligned}(-4a-1)(4a-1)&=-(4a+1)(4a-1)\\&=-[(4a)^2-1^2]\\&=-(16a^2-1)\\&=1-16a^2\end{aligned}$$

【**Example 3**】Compute using Difference of Two Squares Identity:

(1) 102×98;

(2) $(y+2)(y-2)(y^2+4)$.

Solution (1) $$\begin{aligned}102\times 98&=(100+2)(100-2)\\&=100^2-2^2\\&=10000-4\\&=9996\end{aligned}$$

(2) $$\begin{aligned}(y+2)(y-2)(y^2+4)&=(y^2-4)(y^2+4)\\&=(y^2)^2-4^2\\&=y^4-16\end{aligned}$$

Practice

1. Compute using Difference of Two Squares Identity:
   (1) $(x+a)(x-a)$; (2) $(m-n)(m+n)$;
   (3) $(a+3b)(a-3b)$; (4) $(1-5y)(1+5y)$;
   (5) $(2a+3)(2a-3)$; (6) $(-2x^2+5)(-2x^2-5)$;
   (7) $(4x-5y)(4x+5y)$; (8) $\left(\frac{2}{3}x-7y\right)\left(\frac{2}{3}x+7y\right)$.
2. Compute using Difference of Two Squares Identity:
   (1) 103×97; (2) 59.8×60.2;
   (3) $(x+3)(x-3)(x^2+9)$; (4) $\left(x-\frac{1}{2}\right)\left(x^2+\frac{1}{4}\right)\left(x+\frac{1}{2}\right)$.
3. Simplify the following:
   (1) $(x-y)(x+y)+(2x-y)(2x+y)$;
   (2) $(2a-b)(2a+b)-(3a-2b)(3a+2b)$.
4. Check if the following computation is correct? Why? If it is not correct, how can it be corrected?
   (1) $(x-6)(x+6)=x^2-6$;
   (2) $(2x+3)(x-3)=2x^2-9$;
   (3) $(5ab+1)(5ab-1)=25a^2b^2-1$.

6.8 Binomial Square Identity

Let us compute

$$(a+b)^2,\ (a-b)^2,$$

we obtain

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ab-b^2\\ &= a^2+2ab+b^2\\ (a-b)^2 &= (a-b)(a-b)\\ &= a^2-ab-ab-b^2\\ &= a^2-2ab+b^2\end{aligned}$$

From here we get

$$\boxed{\begin{aligned}\boldsymbol{(a+b)^2} &= \boldsymbol{a^2+2ab+b^2}\\ \boldsymbol{(a-b)^2} &= \boldsymbol{a^2-2ab+b^2}\end{aligned}}$$

That is to say, **the square of the sum (or difference) of two numbers is equal to the sum of squares of the two numbers plus (or minus) two times the product of the two numbers.** These are called **the Binomial Square Identities**.

We can rationalize the two identities from Diagram 6-3 and 6-4.

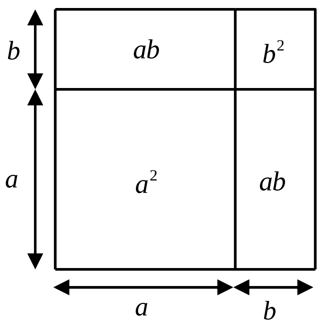


Diagram 6-3

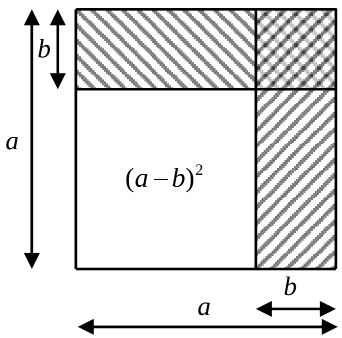


Diagram 6-4

If we need to find the square of the sum (or difference) of two numbers, we can apply the above-mentioned identities to compute it

$$(x+2y)^2,\ \ (2x-5y)^2.$$

If in $(x+2y)^2$, we regard x as a，regard $2y$ as b, and in $(2x-5y)^2$, regard $2x$ as a, regard $5y$ as b

$$(x+2y)^2,\ \ (2x-5y)^2$$

are respectively in the form of

$$(a+b)^2,\ \ (a-b)^2$$

Therefore, they can be computed using the Binomial Square Identities

$$(x+2y)^2 = x^2+2\bullet x\bullet 2y+(2y)^2 = x^2+4xy+4y^2$$

$$\boxed{(a+b)^2} = \boxed{a^2+2\ \ a\ \ \ b\ +\ b^2}$$

$$(2x-5y)^2 = (2x)^2-2\bullet 2x\bullet 5y+(5y)^2 = 4x^2-20xy+25y^2$$

$$\boxed{(a-b)^2} = \boxed{a^2\ \ -2\ \ a\ \ \ b\ +\ b^2}$$

【**Example 1**】Compute using the Binomial Square Identities:

(1) $(-b^2+4a^2)^2$;

(2) $\left(y+\dfrac{1}{2}\right)^2$.

Solution (1) $(-b^2+4a^2)^2 = (-b^2)^2+2\bullet(-b^2)\bullet(4a^2)+(4a^2)^2$

$$= b^4-8a^2b^2+16a^4$$

or

$$\begin{aligned}(-b^2+4a^2)^2 &= (4a^2-b^2)^2\\ &= 16a^4-8a^2b^2+b^4\end{aligned}$$

(2) $\left(y+\dfrac{1}{2}\right)^2 = y^2+y+\dfrac{1}{4}$

Practice

1. Compute using Binomial Square Identities:

(1) $(a+6)^2$;	(2) $(4+x)^2$;
(3) $(x-7)^2$;	(4) $(8-y)^2$;
(5) $(3a+b)^2$;	(6) $(4x+3y)^2$;

Practice

(7) $\left(\frac{1}{2}x-3y\right)^2$; (8) $(-a^2-b)^2$;

(9) $(0.4x+5y)^2$; (10) $\left(\frac{3}{4}x-\frac{2}{3}y^2\right)^2$.

2. Where is the error in the following computation? Why? What alteration is required to make them correct?

(1) $(a+b)^2=a^2+b^2$; (2) $(a-b)^2=a^2-b^2$.

【**Example 2**】Compute using the Binomial Square Identities:

(1) 102^2; (2) 199^2.

Solution (1)
$$\begin{aligned}102^2&=(100+2)^2\\&=100^2+2\times100\times2+2^2\\&=10000+400+4\\&=10404\end{aligned}$$

(2)
$$\begin{aligned}199^2&=(200-1)^2\\&=200^2-2\times200\times1+1^2\\&=40000-400+1\\&=39601\end{aligned}$$

【**Example 3**】Compute using Multiplication Rule:

(1) $(m+n)(m-n)(m^2-n^2)$;

(2) $(a+b+c)^2$;

(3) $(x+2y-3)(x-2y+3)$;

(4) $\left(\frac{x}{2}+5\right)^2-\left(\frac{x}{2}-5\right)^2$.

Solution (1)
$$\begin{aligned}(m+n)(m-n)(m^2-n^2)&=(m^2-n^2)(m^2-n^2)\\&=(m^2-n^2)^2\\&=m^4-2m^2n^2+n^4\end{aligned}$$

(2)
$$\begin{aligned}(a+b+c)^2&=[(a+b)+c]^2\\&=(a+b)^2+2(a+b)c+c^2\\&=a^2+2ab+b^2+2ac+2bc+c^2\\&=a^2+b^2+c^2+2ab+2ac+2bc\end{aligned}$$

(3)
$$\begin{aligned}(x+2y-3)(x-2y+3)&=[x+(2y-3)][x-(2y-3)]\\&=x^2-(2y-3)^2\\&=x^2-(4y^2-12y+9)\\&=x^2-4y^2+12y-9\end{aligned}$$

(4) $$\left(\frac{x}{2}+5\right)^2-\left(\frac{x}{2}-5\right)^2=\frac{x^2}{4}+5x+25-\frac{x^2}{4}+5x-25=10x$$

Practice

1. Compute using Binomial Square Identities:

(1) $(2x+1)^2$; (2) $(3y-4)^2$;

(3) 91^2; (4) 79.8^2.

2. Compute using Muliplicative Rule:

(1) $(x+3)(x-3)(x^2-9)$; (2) $(x+6)^2-(x-6)^2$;

(3) $(a+2b+c)^2$; (4) $(2a+b+1)(2a+b-1)$.

(5) $(a-2b+3c)(a+2b-3c)$;

(6) $[(x+y)^2+(x-y)^2](x^2-y^2)$;

(7) $[(x-1)(x+1)]^2$.

6.9 Sum and Difference of Cubes Identities

Let us compute

$(a+b)(a^2-ab+b^2)$, $(a-b)(a^2+ab+b^2)$, we get

$$(a+b)(a^2-ab+b^2)=a^3-a^2b+ab^2+a^2b-ab^2+b^3$$
$$=a^3+b^3$$
$$(a-b)(a^2+ab+b^2)=a^3+a^2b+ab^2-a^2b-ab^2-b^3$$
$$=a^3-b^3$$

From here we get

$$\boxed{\begin{aligned}&\boldsymbol{(a+b)(a^2-ab+b^2)=a^3+b^3}\\&\boldsymbol{(a-b)(a^2+ab+b^2)=a^3-b^3}\end{aligned}}$$

That is to say, **sum (or difference) of two numbers multiplied by the combined result of the sum of their squares minus (or plus) their product equals the sum (or difference) of their cubes.** These are called **the Sum of Cubes Identity and the Difference of Cubes Identity respectively**.

When we need to find the product of the sum of two numbers multiplied by the sum of their squares less (or plus) their product, we can use the above-mentioned identities to compute the result. For example,

$$(x+3)(x^2-3x+9),\ \ (2y-1)(4y^2+2y+1).$$

If in $(x+3)(x^2-3x+9)$, we regard x as a, regard 3 as b, in $(2y-1)(4y^2+2y+1)$, regard $2y$ as a, regard 1 as b, then

$$(x+3)(x^2-3x+9),\ \ (2y-1)(4y^2+2y+1)$$

are respectively in the form of

$$(a+b)(a^2-ab+b^2),\ \ (a-b)(a^2+ab+b^2)$$

Therefore, we can use the sum (or difference) of cubes identities to compute

$$(x+3)(x^2-3x+9)$$
$$=(x+3)(x^2-x\bullet 3+3^2)=x^3+3^3=x^3+27$$
$$\boxed{(a+b)(a^2-a\ \ b+b^2)}=\boxed{a^3+b^3}$$

$$(2y-1)(4y^2+2y+1)$$
$$=(2y-1)[(2y)^2+(2y)\bullet 1+1^2]=(2y)^3-1^3=8y^3-1$$
$$\boxed{(a-b)\ (a^2+\ \ a\ \ b+b^2)}=\boxed{a^3-b^3}$$

【Example 1】Compute using the Sum (or Difference) of Cubes Identity:

(1) $(4+a)(16-4a+a^2)$;

(2) $\left(5x-\dfrac{1}{2}y\right)\left(25x^2+\dfrac{5}{2}xy+\dfrac{1}{4}y^2\right)$.

Solution (1) $(4+a)(16-4a+a^2)=(4+a)(4^2-4\bullet a+a^2)$

$$=4^3+a^3$$
$$=64+a^3$$

(2) $\left(5x-\dfrac{1}{2}y\right)\left(25x^2+\dfrac{5}{2}xy+\dfrac{1}{4}y^2\right)$

$$=\left(5x-\frac{1}{2}y\right)\left[(5x)^2+(5x)\bullet\frac{1}{2}y+\left(\frac{1}{2}y\right)^2\right]$$
$$=(5x)^3-\left(\frac{1}{2}y\right)^3$$
$$=125x^3-\frac{1}{8}y^3$$

【Example 2】Compute using Multiplication Rule:

$$(x+1)(x-1)(x^2+x+1)(x^2-x+1).$$

Solution $(x+1)(x-1)(x^2+x+1)(x^2-x+1)$

$$=[(x+1)(x^2-x+1)][(x-1)(x^2+x+1)]$$
$$=(x^3+1)(x^3-1)$$
$$=x^6-1$$

or

$$
\begin{aligned}
&(x+1)(x-1)(x^2+x+1)(x^2-x+1) \\
&=(x^2-1)[(x^2+1)+x][(x^2+1)-x] \\
&=(x^2-1)[(x^2+1)^2-x^2] \\
&=(x^2-1)(x^4+2x^2+1-x^2) \\
&=(x^2-1)(x^4+x^2+1) \\
&=x^6-1
\end{aligned}
$$

Practice

1. Compute using multiplication rule:
   (1) $(x^2+1)(x^4-x^2+1)$; (2) $(y-3)(y^2+3y+9)$;
   (3) $(5+c)(25-5c+c^2)$; (4) $(x^2-y^2)(x^4+x^2y^2+y^4)$;
   (5) $(2x+5)(4x^2+25-10x)$;
   (6) $\left(\frac{2}{3}a-\frac{1}{2}b\right)\left(\frac{4}{9}a^2+\frac{1}{3}ab+\frac{1}{4}b^2\right)$.
2. Compute using Multiplication Rule:
   (1) $(a+2)(a-2)(a^2-2a+4)(a^2+2a+4)$;
   (2) $(a-b)(a+b)(a^2+ab+b^2)$;
   (3) $x(x-1)^2-(x^2-x+1)(x+1)$;
   (4) $(y+1)^2+(y+1)(y^2-2y+1)$.

Exercise 19

1. Compute using Difference of Two Squares Identity:
   (1) $(x+2y)(x-2y)$; (2) $(2a-3b)(2a+3b)$;
   (3) $(-1+3x)(-1-3x)$; (4) $(-2b-5)(2b-5)$;
   (5) $(2x^3+15)(2x^3-15)$; (6) $\left(\frac{2}{5}x^2-y\right)\left(\frac{2}{5}x^2+y\right)$;

   (7) $\left(4x^2-\frac{1}{2}\right)\left(4x^2+\frac{1}{2}\right)$; (8) $(0.3x-0.1)(0.3x+0.1)$;
   (9) $(x+2)(x-2)(x^2+4)$;
   (10) $(x+y)(x-y)(x^2+y^2)(x^4+y^4)$.
2. Compute using Difference of Two Squares Identity:
   (1) 69×71; (2) 503×497;
   (3) 40.5×39.5; (4) $40\frac{2}{3}\times39\frac{1}{3}$.
3. Compute:
   (1) $x(x-3)-(x+7)(x-7)$;
   (2) $(2x-5)(x-2)+(3x-4)(3x+4)$;
   (3) $\left(\frac{3}{2}x+\frac{2}{3}y\right)\left(\frac{3}{2}x-\frac{2}{3}y\right)-\left(\frac{2}{3}y+\frac{3}{2}x\right)\left(\frac{2}{3}y-\frac{3}{2}x\right)$;
   (4) $x^2(x^2+y^2)(x^2-y^2)+(x^2+y^2)(2x^4-3y^4)$.
4. Compute using Multiplication Rule:
   (1) $(6a+5b)^2$; (2) $(4x-3y)^2$;
   (3) $(-2m-1)^2$; (4) $\left(\frac{1}{4}m-2n\right)^2$;
   (5) $(4x+0.5)^2$; (6) $\left(1.5a-\frac{2}{3}b\right)^2$;
   (7) $\left(a-b+\frac{1}{2}\right)^2$; (8) $(2x+y-3)^2$;
   (9) $[(x+3y)(x-3y)]^2$;
   (10) $(3x+2y+4)(3x+2y-4)$;
   (11) $(a+3b-2)(a-3b+2)$;
   (12) $(1+x+y)(1-x-y)$;
5. Compute using Binomial Square Identities:
   (1) 63^2; (2) 895^2; (3) 9.98^2; (4) $\left(14\frac{1}{2}\right)^2$.

6. Compute:

(1) $(2a+1)^2+(1-2a)^2$; (2) $(2x)^2-3(2x+1)^2$;

(3) $3(2-y)^2-4(y+5)^2$; (4) $a^4-(1-a)(1+a)(1+a^2)$;

(5) $(3x-y)^2-(2x+y)^2+5y^2$;

(6) $3(m+1)^2-5(m+1)(m-1)+2(m-1)^2$.

7. Compute using Multiplication Rule:

(1) $(5-2y)(25+10y+4y^2)$;

(2) $(3s+2t)(9s^2+4t^2-6st)$;

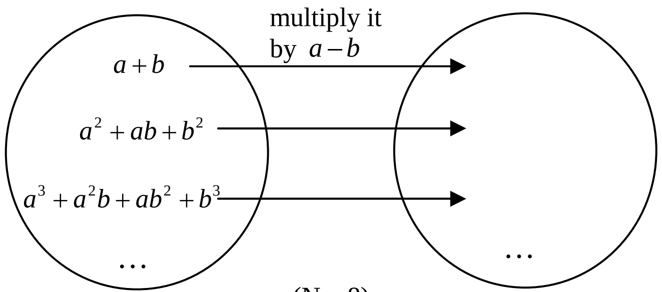
(3) $(x^2+2)(4-2x^2+x^4)$;

(4) $\left(\frac{1}{2}a+2b\right)\left(\frac{1}{4}a^2+4b^2+ab\right)$;

(5) $(x-2)(x+2)(x^4+4x^2+16)$;

(6) $(x-2)(x^2+2x+4)+(x+5)(x^2-5x+25)$.

8. For each of the algebraic expressions in the left circle, multiply it by $a-b$, then write the result in the right circle.



(No. 8)

9. Answer the following question:

(1) What algebraic expression needs to be added to a^2+b^2 to give $(a+b)^2$?

(2) What algebraic expression needs to be added to a^2+b^2 to give $(a-b)^2$?

(3) What algebraic expressin needs to be added to $(a-b)^2$ to give $(a+b)^2$?

(4) What algebraic expression needs to be added to a^2+ab+b^2 to give $(a-b)^2$?

(5) What algebraic expression needs to be multiplied to $a+b$ to give a^3+b^3?

(6) What algebraic expression will need to be multiplied to $a-b$ to give a^3-b^3?

10. Simplify and evaluate:

(1) $(x+2)(x^2-2x+4)+(x-1)(x^2+x+1)$, where $x=-\frac{2}{3}$;

(2) $(a+b)(a-b)(a^2+b^2)$, where $a=3$, $b=0.2$;

(3) $\left[\left(a+\frac{1}{2}b\right)^2+\left(a-\frac{1}{2}b\right)^2\right]\left(2a^2-\frac{1}{2}b^2\right)$, where $a=1$, $b=-2$.

11. Solve the equation:

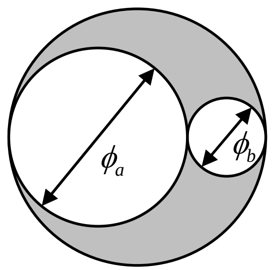
(1) $\left(x+\frac{1}{4}\right)^2-\left(x-\frac{1}{4}\right)\left(x+\frac{1}{4}\right)=\frac{1}{4}$;

(2) $(x+1)(x^2-x+1)-x(x-2)(x+2)=9$;

(3) $3(x+5)^2-2(x-3)^2-(x+9)(x-9)=180$.

12. If the length of a square is increased by 3 cm, its area is increased by 39 cm^2. Find its length.

13. If two circles with diameter a and b respectively are cut from a circle with diameter $a+b$, find the area of the remaining portion (shaded in the Diagram).



(No. 13)

III. Division of Integral Expressions

6.10 Division of Powers of the same base

Now we come to learn about division of integral expressions. We know that division is the inverse process of multiplication, therefore we would like to deduce the division rule from the multiplication rule.

In the first section, we would investigate the division of two powers with the same base. We get

$$10^5 \div 10^3, \quad 2^5 \div 2^3$$

We know that, based on the fact that division is the inverse operation of multiplication, computing the quotient of the dividend divided by the divisor is same as finding a number whose product with the divisor equals the dividend.

$\because \quad 10^2 \times 10^3 = 10^5$

$\therefore \quad 10^5 \div 10^3 = 10^2$

$\because \quad 2^2 \times 2^3 = 2^5$

$\therefore \quad 2^5 \div 2^3 = 2^2$

that is

$$10^5 \div 10^3 = 10^{5-3}$$
$$2^5 \div 2^3 = 2^{5-3}$$

In the same manner,

$\because \quad a^2 \bullet a^3 = a^5$

$\therefore \quad a^5 \div a^3 = a^2$ [8]

that is

$$a^5 \div a^3 = a^{5-3}$$

In general, if m, n are positive integers, and $m > n$, then

$$\boxed{\boldsymbol{a^m \div a^n = a^{m-n} \quad (a \neq 0)}}$$

[8] Here $a \neq 0$. If this chapter, the values of all divisors are not equal to zero.

That is to say, **when dividing two powers of the same base, the base is unchanged, and the index is the difference of indices of the two powers.**

Division of powers of the same base, if the index of the dividend is same as the index of the divisor, for example

$$3^2 \div 3^2, \quad \left(-\frac{1}{2}\right)^3 \div \left(-\frac{1}{2}\right)^3, \quad a^m \div a^m$$

Then, the quotient is 1.

That is to say, **division of powers of the same base and same index, the quotient is 1.**

【Example 1】 Compute: (1) $x^8 \div x^2$; (2) $a^9 \div a^4$; (3) $(-a)^4 \div (-a)$.

Solution (1) $x^8 \div x^2 = x^{8-2} = x^6$;

(2) $a^9 \div a^4 = a^{9-4} = a^5$;

(3) $(-a)^4 \div (-a) = (-a)^{4-1} = (-a)^3 = -a^3$.

【Example 2】 Compute:

(1) $(ab)^5 \div (ab)^2$; (2) $(a+b)^3 \div (a+b)^2$;

(3) $y^{n+2} \div y^2$; (4) $x^{n+m} \div x^{n+m}$.

Solution (1) $(ab)^5 \div (ab)^2 = (ab)^{5-2} = (ab)^3 = a^3b^3$;

(2) $(a+b)^3 \div (a+b)^2 = (a+b)^{3-2} = a+b$;

(3) $y^{n+2} \div y^2 = y^{n+2-2} = y^n$;

(4) $x^{n+m} \div x^{n+m} = 1$.

Practice

1. (*Mental*) Compute:

   (1) $x^7 \div x^5$; (2) $y^9 \div y^8$; (3) $z^{11} \div z^8$;

   (4) $a^{10} \div a^3$; (5) $b^6 \div b^6$; (6) $c^7 \div c^7$.

2. Fill in the bracket with an appropriate algebraic expression to make the following equation valid:

   (1) $x^5 \bullet (\quad) = x^9$; (2) $a^6 \bullet (\quad) = a^{12}$;

   (3) $b^3 \bullet b^3 \bullet (\quad) = b^{36}$; (4) $x^2 \bullet x^5 \bullet (\quad) = x^{20}$.

Practice

3. Is the following equation correct? If not, what changes need to be made to correct it?

(1) $x^6 \div x^3 = x^2$; (2) $z^5 \div z^4 = z$;

(3) $a^3 \div a = a^3$; (4) $(-c)^4 \div (-c)^2 = -c^2$.

4. Compute:

(1) $(xy)^5 \div (xy)^3$; (2) $(a+b)^5 \div (a+b)^4$;

(3) $a^{n+2} \div a^{n+1}$; (4) $x^{12} \div x^3 \div x^4$;

(5) $y^{10} \div (y^4 \div y^2)$; (6) $(c^{4n} \div c^{2n}) \bullet c^{3n}$.

6.11 Division of a Monomial by another Monomial

In this section, we investigate the division of two monomials. Let us compute

$$12a^3b^2x^3 \div 3ab^2$$ [9]

We already know, this is the same as finding a monomial (quotient) to enable its product with $3ab^2$ (divisor) to equal $12a^3b^2x^3$ (dividend).

$\because$ $$4a^2x^3 \bullet 3ab^2 = 12a^3b^2x^3$$

$\therefore$ $$12a^3b^2x^3 \div 3ab^2 = 4a^2x^3$$

That is to say, the quotient is $4a^2x^3$. Here the coefficient is $4 = 12 \div 3$, the index of letter *a* is $2 = 3 - 1$, the index of letter *x* is 3 and the quotient does not contain the letter *b*.

In general, **to find the quotient of division of a monomial by another monomial, (i) find the quotient of the coefficients as a factor of the quotient, (ii) for each of the letter in the divisor, find the division of powers of the letter as a factor of the quotient, (iii) for any letter not in the divisor, keep the power of the letter in the dividend as a factor of the quotient (iv) multiply all the factors together to form the quotient.**

[9] This explains why the quotient is the expression $(12a^3b^2x^3) \div (3ab^2)$.

【**Example**】 Compute: (1) $28x^4y^2 \div 7x^3y$; (2) $-5a^5b^3c \div 15a^4b$;

(3) $-a^2x^4y^3 \div \left(-\frac{5}{6}axy^2\right)$; (4) $(6x^2y^3)^2 \div (3xy^2)^2$.

Solution (1) $28x^4y^2 \div 7x^3y = (28 \div 7) \bullet x^{4-3}y^{2-1} = 4xy$;

(2) $-5a^5b^3c \div 15a^4b = [(-5) \div 15] \bullet a^{5-4}b^{3-1}c = -\frac{1}{3}ab^2c$;

(3) $-a^2x^4y^3 \div \left(-\frac{5}{6}axy^2\right) = \frac{6}{5}ax^3y$;

(4) $(6x^2y^3)^2 \div (3xy^2)^2 = 36x^4y^6 \div 9x^2y^4 = 4x^2y^2$.

Practice

1. (*Mental*) Compute:

(1) $16a \div 4$; (2) $10ab^3 \div (-5)$;

(3) $-8a^2b^2 \div 6ab^2$; (4) $6x^2y \div 3xy$;

(5) $-21x^2y^4 \div (-3x^2y^3)$; (6) $(-2ab^2)^3 \div (-2ab^2)^2$;

(7) $(6 \times 10^8) \div (3 \times 10^5)$; (8) $(4 \times 10^9) \div (-2 \times 10^3)$.

2. Compute:

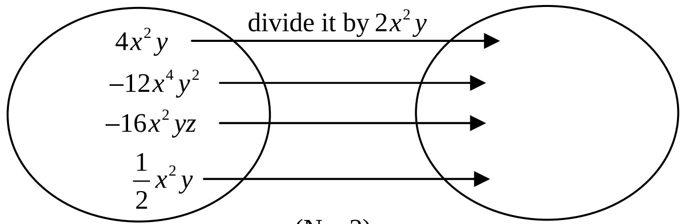
(1) $24a^3b^2 \div 8ab^2$; (2) $-14a^2x^3 \div 7ax^2$;

(3) $9c^3d^2 \div (-9c^3d^2)$; (4) $(-0.5a^2bx^2) \div \left(-\frac{2}{5}ax^2\right)$;

(5) $\left(-\frac{3}{4}a^2b^2c\right) \div 3a^2b$; (6) $(3mn^2)^3 \div 3m^2n^3$;

(7) $(4x^2y^3)^2 \div (-2xy^2)^2$; (8) $(5ab^2c)^4 \div (-5ab^2c)^2$.

3. For each algebraic expression in the left circle, divide it by $2x^2y$ and write the quotient in the right circle.



(No. 3)

6.12 Division of Polynomial by a Monomial

Let us now investigate the division of a polynomial by a mononial

$$(am+bm+cm)\div m .$$

This is the same as finding a polynomia in such a way that its product with m is

$$am+bm+cm .$$

$\because$ $$(a+b+c)m = am+bm+cm$$

$\therefore$ $$(am+bm+cm)\div m = a+b+c$$

We know,

$$am\div m + bm\div m + cm\div m = a+b+c ,$$

therefore

$$(am+bm+cm)\div m = am\div m + bm\div m + cm\div m .$$

In general, **to find the quotient of division of a polynomial by a monomial, first divide each constituent term of the polynomial by the monomial to become a constituent quotient, then add all the constituent quotients together to become the quotient of the division.**

【Example 1】 Compute $(28a^3-14a^2+7a)\div 7a$.

Solution $(28a^3-14a^2+7a)\div 7a$

$= 28a^3\div 7a - 14a^2\div 7a + 7a\div 7a$

$= 4a^2-2a+1$

【Example 2】 Compute $(36x^4y^3-24x^3y^2+3x^2y^2)\div(-6x^2y)$.

Solution $(36x^4y^3-24x^3y^2+3x^2y^2)\div(-6x^2y)$

$= -6x^2y^2+4xy-\dfrac{1}{2}y$

Practice

1. (*Mental*) Compute:

(1) $(6xy+5x)\div x$; (2) $(15x^2y-10xy^2)\div 5xy$;

(3) $(8a^2b-4ab^2)\div 4ab$; (4) $(4c^2d+c^3d^2)\div(-2c^2d)$.

Practice

2. Compute:

(1) $(16m^3-24m^2)\div(-8m^2)$; (2) $(9x^3y^2-21xy^2)\div 7xy^2$;

(3) $(12a^2b^3-9a^4b^2c)\div(3a^2b^2)$;

(4) $(25x^2+15x^3y-20x^4)\div(-5x^2)$;

(5) $(-4a^3+12a^2b-7a^3b^2)\div(-4a^2)$;

6.13 Division of a Polynomial by another Polynomial

Now we can study the division of a polynomial by another polynomial.

To find the quotient of division of polynomials, first arrange the dividend and divisor in decending order of the same letter, then follow the rule similar to the division of multiple-digit numbers in long division to compute. For example, let us compute

$$(6x^2+7x+2)\div(2x+1) ,$$

Drawing similarity to the division of multiple-digit numbers, say $672\div 21$, compute as follows:

$$\begin{array}{r} 32 \\ 21\overline{)672} \\ \underline{63} \\ 42 \\ \underline{42} \\ 0 \end{array} \qquad \begin{array}{rr} & 3x+2 \longleftarrow \text{quotient} \\ \text{divisor} \longrightarrow 2x+1\!\!\!\! & \overline{)6x^2+7x+2} \longleftarrow \text{dividend} \\ & \underline{6x^2+3x\quad\;\;} \\ & 4x+2 \\ & \underline{4x+2} \\ & 0 \longleftarrow \text{remainder} \end{array}$$

$\therefore$ $$(6x^2+7x+2)\div(2x+1) = 3x+2$$

Steps in computation is:

1. Use the first term of the divisor $2x$ to divide the first term of the dividend $6x^2$, thereby obtain the first term of the quotient of $3x$;

2. Use the first term of the dividend $3x$ to multiply each term of the divisor, write the product $6x^2+3x$ underneath the dividend (align like terms together), subtract the product from the dividend, and obtain $4x+2$;
3. Regard $4x+2$ as the new dividend, repeat the same procedure as above and continue dividing until the remainder is 0 (or the remainder is of lower order than the divisor).

In general, division of polynomials can follow the above-mentioned procedure.

【Example 1】 Compute $(5x^2+2x^3-1)\div(1+2x)$.

Solution

$$
\begin{array}{r|l}
 & x^2+2x-1 \\
\hline
2x+1 & 2x^3+5x^2 \qquad -1 \\
 & 2x^3+\ \ x^2 \\
\hline
 & \qquad 4x^2 \\
 & \qquad 4x^2+2x \\
\hline
 & \qquad\quad -2x-1 \\
 & \qquad\quad -2x-1 \\
\hline
 & \qquad\qquad\quad 0
\end{array}
$$

$\therefore \quad (5x^2+2x^3-1)\div(1+2x)=x^2+2x-1$

NOTE: Arrange dividend and divisor in descending powers of x, if the dividend has a missing term, leave a space in the dividend. Alternatively use a zero to fill in the space of the mission term in the dividend. For example, write $2x^3+5x^2-1$ as $2x^3+5x^2+0-1$.

In example 1 the remainder is zero. If a polynomial is divided by another polynomial with remainder zero, we say that the former polynomial is exactly divided by the latter polynomial. We can also say that the dividend is divided by the divisor exactly.

It is possible that division is not exact. When dividend and divisor are arranged in descending powers of a letter, and the division procedure proceeds to the stage that the remainder is not zero but is of degree lower than the divisor, the division procedure cannot proceed further. We say that the divisor cannot divide the dividend exactly.

【Example 2】 Compute $(2x^3+9x^2+3x+5)\div(x^2+4x-3)$.

Solution

$$
\begin{array}{r|r}
 & 2x+1 \\
\hline
x^2+4x-3 & 2x^3+9x^2+3x+5 \\
 & 2x^3+8x^2-6x \quad \\
\hline
 & x^2+9x+5 \\
 & x^2+4x-3 \\
\hline
 & 5x+8
\end{array}
$$

Therefore quotient $=2x+1$, remainder $=5x+8$

We know, in dividing an integer by another integer, sometimes the division is not exact and carries a remainder, for example,

$$
\begin{array}{r|r}
 & 37 \\
\hline
21 & 785 \\
 & 63 \\
\hline
 & 155 \\
 & 147 \\
\hline
 & 8
\end{array}
$$

In a division which leaves a remainder, there is the following relationship:

$$\underset{\text{dividend}}{785} = \underset{\text{divisor}}{21} \times \underset{\text{quotient}}{37} + \underset{\text{remainder}}{8}$$

In a division which leaves a remainder, there is the following relationship:

$$\underbrace{(2x^3+9x^2+3x+5)}_{\text{dividend}}=\underbrace{(x^2+4x-3)}_{\text{divisor}}\underbrace{(2x+1)}_{\text{quotient}}+\underbrace{(5x+8)}_{\text{remainder}}$$

In general, dividend, divisor, quotient and remainder have the following relationship:

Dividend = Divisor × Quotient＋Remainder

Practice

1. Compute:
   (1) $(3x^2+14x-5)\div(x+5)$; (2) $(6x^2+14x+4)\div(3x+1)$;
   (3) $(8x^2+52x-21)\div(x+7)$; (4) $(1+x^2+x^4)\div(x^2+1-x)$;
   (5) $(x^3-3x+x^2-8)\div(x-2)$;
   (6) $(3x^3-4x-5x^2+3)\div(x^2-x+5)$.

2. The Divisor, quotient and remainder are known, find the dividend:
   (1) Dividend $=3x-5$, quotient $=2x+7$, remainder $=10$;
   (2) Dividend $=x^2-2x+1$, quotient $=x^2+2x-1$, remainder $=4x$.

Exercise 20

1. Compute:
   (1) $a^7\div a^4$; (2) $x^{10}\div x^6$;
   (3) $y^8\div y^8$; (4) $(-5)^6\div(-5)^3$;
   (5) $(ax)^5\div(ax)^3$; (6) $\left(-\frac{1}{2}y\right)^4\div\left(-\frac{1}{2}y\right)$;
   (7) $a^{3n}\div a^n$; (8) $x^{2n+1}\div x^{n+1}$;
   (9) $(a^2)^m\div a^m$; (10) $t^6\div(t^4\div t^3)$;
   (11) $a^3\bullet(a^2)^3\div a^4$; (12) $(a+b)^3\div(a+b)^2\bullet(a+b)^4$.

2. Compute:
   (1) $-12a^5b^3c\div(-3a^2b)$; (2) $42x^6y^8\div(-3x^2y)^3$;
   (3) $24x^2y^5\div(-6x^2y^3)$; (4) $-25t^8k\div(-5t^5k)$;
   (5) $(-5r^2c)^2\div5r^4c$; (6) $7m^2(4m^3p)\div7m^5$;
   (7) $-45(u^3v^4)^2\div5u^5v^4$; (8) $-12(s^4t^3)^3\div\left(\frac{1}{2}s^2t^3\right)^2$;
   (9) $(-5r^2st^3)^2(-2rs^2t)^3$; (10) $(2a^2bc)^3\left(\frac{1}{2}ab^3c^2\right)^5$;
   (11) $(-38x^4y^5z)\div19xy^5\bullet\left(-\frac{3}{4}x^3y^2\right)$;
   (12) $(2ax)^2\bullet\left(-\frac{2}{5}a^4x^3y^3\right)\div\left(-\frac{1}{2}a^5xy^2\right)$.

3. The speed of a satellite is 2.88×10^7 m/hour, the speed of a jet plane is 1.8×10^6 m/hour, how many times is the speed of the satellite relative to the speed of the jet plane?

4. Compute:
   (1) $(6x^4-8x^3)\div(-2x^2)$; (2) $(8a^3b-5a^2b^2)\div4ab$;
   (3) $(12x^3-8x^2+16x)\div4x$;
   (4) $\left(\frac{2}{5}y^3-7xy^2+\frac{2}{3}y^5\right)\div\frac{2}{3}y^2$;
   (5) $(9a^3x^5-6a^2x^4+15a^4x^3)\div(-3a^2x^3)$;
   (6) $[28x^7y^3-21x^5y^5+2y(7x^3y^3)^2]\div7x^5y^3$;
   (7) $\left(0.25a^3b^2-\frac{1}{2}a^4b^5-\frac{1}{6}a^5b^3\right)\div(-0.5a^3b^2)$;
   (8) $(3a^{n+1}+6a^{n+2}-9a^n)\div3a^{n-1}$.

5. Simplify:
   (1) $[(2x+y)^2-y(y+4x)]\div2x$;
   (2) $[(3x+2y)(3x-2y)-(x+2y)(5x-2y)]\div4x$;

(3) $\left[\left(4x-\frac{1}{2}y\right)^2+4y\left(x-\frac{y}{16}\right)\right]\div 8x^2$;

(4) $\left[(-3xy)^2\bullet x^3-2x^2\bullet(3xy^2)^3\bullet\frac{1}{2}y\right]\div 9x^4y^2$.

6. Compute:
   (1) $(2x^2+23x+56)\div(2x+7)$;
   (2) $(2x^3+27x-12x^2-27)\div(x-3)$;
   (3) $(2x^3+9x^2+10x+5)\div(x^2+x+1)$;
   (4) $(2y^4-y^3+y-3)\div(3-2y+y^2)$.

7. Area of rectangle is $a^2-3ab+2b^2$. Length of one of its sides is $a-b$. Find its perimeter.

8. The Divisor, the Quotient and the Remainder are given, find the Dividend:
   (1) Divisor$=6x^2+3x-5$, Quotient$=4x-5$, Remainder$=-8$;
   (2) Divisor$=-2x^2-x+1$, Quotient$=x^2-2$, Remainder$=3x+7$.

Chapter Summary

I. The main content of this chapter teaches multiplication of integral expressions, which includes rules on operation of powers of the same base, monomial multiplied (or divided) by monomial, polynomial multiplied (or divided) by monomial, polynomial multiplied (or divided) by polynomial, etc

II. The rules of operation on powers learnt in the chapter include:

$$a^m\bullet a^n=a^{m+n}$$
$$(a^m)^n=a^{mn}$$
$$(ab)^n=a^nb^n$$
$$a^m\div a^n=a^{m-n}\quad(a\neq 0,\ m>n)$$

III. Multiplication of integral expressions includes monomial multiplied (or divided) by monomial, polynomial multiplied (or divided) by monomial, polynomial multiplied (or divided) by polynomial, etc

IV. Some special forms of polynomial multiplication have wide applications, We post them as identities and apply them where appropriate. The multiplicative identities we have learnt include

$$(a+b)(a-b)=a^2-b^2$$
$$(a\pm b)^2=a^2\pm 2ab+b^2$$
$$(a\pm b)(a^2\mp ab+b^2)=a^3\pm b^3$$

V. The result of a polynomial multiplied by another polynomial is also a polynomial.

Revision Exercise 6

1. Compute:
   (1) $x\bullet(x^2)^2\bullet(x^3)^3$; (2) $2m^4\bullet(-m^2)^2$;
   (3) $(ab)^2\bullet(-a)^2\bullet(-b)^3$; (4) $2a^3b\bullet(-3ab)^3$;
   (5) $(-0.4xy^3z)\bullet(-0.5x^2z)$; (6) $(-x^2y^n)^2\bullet(xy)^3$;
   (7) $\left(-\frac{2}{3}a^7b^5\right)\div\left(\frac{3}{2}a^5b^5\right)$; (8) $(2a)^3\bullet b^4\div 12a^3b^2$;
   (9) $[(-2a^3b)^3]^2\div(-3a^2b)^2$; (10) $(4x^{n+1}y^n)^2\div[(-xy)^2]^n$.

2. Area of Taiwan is 3.6×10^4 km^2. On average, for every km^2 of land, the energy received from the Sun a year is equivalent to the energy released by burning 1.5×10^5 T of coal. Calculate how many tonnes of coal would produce energy equivalent to that of energy received from the Sun by the whole of Taiwan in 1 year (round to 2 significant figures).

3. The velocity required by a satellite to escape from the earth into the solar system (i.e. the Second Cosmo Velocity) is 1.12×10^4 m/sec. Calculate the distance in metres traveled by the satellite in 3.6×10^3 sec. (round to 2 significant figures).

4. Compute:

(1) $2a^3b(3ab^2c-2bc)$; (2) $(0.3a^2-0.2a+0.1)\times 0.2$;

(3) $\left(-\frac{2}{3}a\right)\left(\frac{1}{2}a^2+\frac{1}{6}a-\frac{1}{4}\right)$;

(4) $(4\pi r^2h-2\pi rh)\div 6\pi rh$;

(5) $\left(\frac{6}{5}a^3x^4-0.9ax^3\right)\div\frac{3}{5}ax^3$;

(6) $(3a^{n+4}+2a^{n+1})\div(-3a^{n-1})$;

(7) $6xy\bullet[x^2(5x+3)-3x^2(-4y)]$;

(8) $[5xy^2(x^2-3xy)-(-3x^2y)^3]\div 2x^2y^2$;

(9) $2a^2b-(-3a)^2\bullet(2b)+(4a^3b^2)^2\div 4a^4b^3$;

(10) $(3xy)^2(x^2-y^2)-(4x^2y^2)^2\div 8y^2+9x^2y^4$.

5. Starting from any positive integer n, follow the calculation procedure below and write the answer in the space provided in the table. Observe the pattern, and explain why?

$\boxed{n}\rightarrow\boxed{\text{Square}}\rightarrow\boxed{+n}\rightarrow\boxed{\div n}\rightarrow\boxed{-n}\rightarrow\boxed{\text{Answer}}$

Input n	3				
Output	1				

6. Compute:

(1) $(2a+3b)(2a-4b)$; (2) $(5a-b)(-a-4b)$;

(3) $(9u-2v)(u+v)$; (4) $(x^2+3)(x^2-2)$;

(5) $(3t^2+2r)(3t+5r)$;

(6) $(-3a^2b-4ab)(-a^2+5ab^2)$;

(7) $(0.3a^2b-0.4ab^2)(0.5ab^2-0.1a^2b)$;

(8) $\left(\frac{3}{5}xy^3-\frac{2}{3}x^2y\right)\left(\frac{5}{4}x^2y^2-\frac{6}{5}xy\right)$;

(9) $(3x-5)(x^2-7x+3)$;

(10) $(5a+2b)(ab-4a^2+3b^2)$;

(11) $3(2x-1)(x+6)-5(x-3)(x+6)$;

(12) $(x^3+2xy^2-3y^3)(2x-y)-8xy(x^2-y^2)$.

7. Compute:

(1) $\left(\frac{1}{3}a^2-\frac{1}{4}b\right)\left(-\frac{1}{4}b-\frac{1}{3}a^2\right)$; (2) $5x^2(x+3)(x-3)$;

(3) $\left(2x+\frac{1}{2}\right)\left(2x-\frac{1}{2}\right)\left(4x^2+\frac{1}{4}\right)$; (4) $\left(\frac{7}{3}x+\frac{3}{2}y\right)^2$;

(5) $\left(\frac{2}{3}c^2-0.6d^2\right)^2$;

(6) $4x(x-1)^2-x(2x+5)(2x-5)$;

(7) $3(2x+1)(2x-1)-4\left(\frac{3}{2}x-3\right)\left(\frac{3}{2}x+3\right)$;

(8) $5(2x+5)^2+(3x-4)(-3x-4)$;

(9) $(x+3y)(x^2-3xy+9y^2)$;

(10) $(3a-2b)(9a^2+6ab+4b^2)$;

(11) $(x-y)^2(x+y)^2$;

(12) $(2x+y-z)^2$;

(13) $(x+y)^2(x^2-xy+y^2)^2$;

(14) $\left[\left(\frac{1}{2}x-y\right)^2-\left(\frac{1}{2}x+y\right)^2\right]\left(2x^2-\frac{1}{2}y^2\right)$;

(15) $(2x+y-z+5)(2x-y+z+5)$;

(16) $(x+y-z)(x-y+z)-(x+y+z)(x-y-z)$.

8. Compute:

(1) $(a+b)^2+(a-b)^2+(-2a-b)(a+2b)$;

(2) $5(m+n)(m-n)-2(m+n)^2-3(m-n)^2$;

(3) $(x-y)[(x+y)^2-xy]+(x+y)[(x-y)^2+xy]$;

(4) $(a+b+c)^2+(a-b)^2+(b-c)^2+(c-a)^2$.

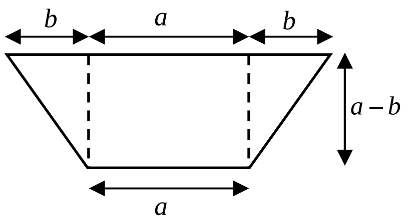
9. Simplify then evaluate:

(1) $(25y^2-5y+1)(5y+1)-5(1-4y^2)$, where $y=\dfrac{2}{5}$;

(2) $8m^2-5m(-m+3n)+4m\left(-4m-\dfrac{5}{2}n\right)$, where $m=2$, $n=-1$

(3) $x(y-z)-y(z-x)+z(x-y)$, where $x=\dfrac{1}{2}$, $y=1$, $z=-\dfrac{1}{2}$

10. The cross-section of a water duct is a trapezium with size as show in the Diagram. Find the algebraic expression of the cross-sectional area, and calculate its value when $a=2$, $b=0.8$.



(No. 10)

11. Value of A is $2a$. Value of B is 3 greater than 2 times value of A. Value of C is 3 less than 2 times value of A. Find the product of $A\times B\times C$. Find the value of the product when $a=-2.5$?

12. (1) The quotient of dividing a polynomial by x^2-4x+1 is $x+2$, find the polynomial;

(2) The product of multiplying a polynomial by $x+4$ is x^3+3x^2-4x, find the polynomial;

(3) Dividing a polynomial by x^2-4x+1 gives a quotient of $x+1$, and a remainder of $3x+1$, find the polynomial.

13. Compute:

(1) $(4x^2+4x-3)\div(2x+3)$;

(2) $(x^3-3x^2-9x+22)\div(x-2)$;

(3) $(6x^2+19x+15)\div(2x+5)$;

(4) $(x^3-x^2+x-1)\div(x^2-3x+5)$.

14. Given $A=x^3-7x+6$, $B=x^2+2x-3$, $C=x^2+x-6$, compute:

(1) $A\div(B-C)$;

(2) $A\div B-A\div C$.

15. Given $A=a^2+b^2+c^2$, $B=(a-b)^2+(b-c)^2+(c-a)^2$, compute $A-\dfrac{B}{2}$.

16. (1) What values of a, b would cause $ab>0$? $ab<0$? $ab=0$?

(2) What values of a, b would cause $a\div b$ to result in positive value? result in negative value? result in zero? result in undefined value?

17. Solve the following inequalities:

(1) $5x+4>3x-1$;

(2) $\dfrac{x-1}{3}-\dfrac{x+2}{6}<\dfrac{x}{2}-2$;

(3) $(2x-5)^2+(3x+1)^2>13(x^2-10)$;

(4) $(3x+4)(3x-4)<9(x-2)(x+3)$.

18. Solve the following equations:

(1) $x^2-(x+1)(x-5)=2(x-5)$;

(2) $(x+3)^2+2(x-1)^2=3x^2+13$;

(3) $(x-5)(x+5)-(x+1)(x+5)=24$;

(4) $(2x+3)(x-4)=(x-2)(2x+5)$;

(5) $(x-1)^2+28=\left(\dfrac{4}{3}x-12\right)\left(\dfrac{3}{4}x-12\right)$.

19. Solve the following simultaneous equations:

(1) $\begin{cases}(x+3)(y+4)-xy=17\\ x-y=3\end{cases}$

(2) $\begin{cases}(x+1)^2-(x+1)(x-1)=y\\ (y-1)^2-(y+1)(y-1)=x\end{cases}$

(3) $\begin{cases}(x+5)(y-4)-xy=0\\3x-2y=-1\end{cases}$

(4) $\begin{cases}(x+2)^2-(y-3)^2=(x+y)(x-y)\\x-3y=2\end{cases}$

20. A number of candies is to be distributed equally to a group of children. If every child is to be given 27 candies, there are 23 candies left; if every child is to be given 28 candies, there are 8 candies left. Find out how many children are there? how many candies are being distributed?

21. (1) Is the sum, difference, product and quotient (divisor not zero) respectively of two rational numbers a rational number?

(2) Is the sum, difference, and product respectively of two integral expressions an integral expression?

(This chapter is translated to English and reviewed by courtesy of Mr. SIN Wing Sang, Edward.)