Self-study Textbook
for Junior Secondary School

Algebra

First Term (for Year 8 & 9)

Published by Chiu Chang Math Books & Puzzles Co.
Provided by Chiu Chang Mathematics Education Foundation

Content

Explanation of Content

I. This book contains chapters on rational numbers, addition and subtraction of integral expressions, simple equations in one unknown, linear inequalities, simultaneous linear equations in two unknowns, multiplication and division of integral expressions, fractorization and fractions. This book is written for use by Year 8 and 9 junior secondary students.

II. The exercises in this book are classified into 3 categories, namely practice, exercise and revision exercise

(1)	**Practice**	For use in class practice.
(2)	**Exercise**	For use in homework.
(3)	**Revsion Exercise**	For use in revision. Some questions marked with "*" are challenge questions for use by able students.

III. This book is written in accordance with the secondary school syllabus set by the Mainland China Education Bureau in 1984.

How to self-study using the book

1. First plough through each Chapter and Section carefully.

2. Digest the examples, then close the book and see if you can work out the solution by yourself.

3. Work out the questions set in the Practice, Exercise and Revision Exercise in your notebook. There is no need to copy the question, but student should mark the page reference and show all working steps and details.

4. Student should form the habit of checking the solution after completing each question. After verifying that the answer is correct, place a " ∨ " mark against the question number for record.

5. When the student encounters a question that he does not understand or cannot work out the answer, he should re-read the explanation in the relevant Chapter and Section until he fully understands it.

6. It is advisable for the students to organize themselves into study group of 4-6 persons. Set target plan to monitor the progress. Meet regularly to discuss and check the progress together.

7. This book will require 3 months time (involving about 200 study hours) to complete the learning of all the topics set in the junior secondary school syllabus. Among the topics, Algebra and Geometry can be studied in parallel.

Note: The units of measurement used in this book include

Units of length:	km (kilometer), m (meter), cm (centimeter), mm (millmeter); 1 km = 1000 m, 1 m = 100 cm = 1000 mm;
Units of weight:	T (Tonne), kg (kilogram), g (gram); 1 T = 1000 kg, 1 kg = 1000 g;
Units of capacity:	kL (kiloliter), L (liter), mL (milliliter); 1 kL = 1000 L, 1 L = 1000 mL.

Chapter 9 Square Root of Number

9.1 Square Root

If the length of the side of a square is known, squaring the length will give the area of the square. Reversing the question, if the area of a square is known, how can we determine the length of its side?

For example, to make a square tabletop with area of 9 m^2, we have to find out the length of its side first. It is same as finding a number whose square is equal to 9. We have two answers, namely $3^2=9$, and $(-3)^2=9$. Since length of a side cannot be a negative number, we determine that the length of the side of this square table is 3 m.

In general, if the square of a number is equal to *a*, then this number is called **the square root of *a***. In other words, if $x^2=a$, then *x* is the square root of *a*.

Looking at the example, since $3^2=9$, and $(-3)^2=9$, both 3 and -3 are square roots of 9. Looking at another example, since $7^2=49$, and $(-7)^2=49$, therefore both 7 and -7 are square roots of 49. Similarly as $\left(\frac{2}{5}\right)^2=\frac{4}{25}$, and $\left(-\frac{2}{5}\right)^2=\frac{4}{25}$, therefore both $\frac{2}{5}$ and $-\frac{2}{5}$ are square roots of $\frac{4}{25}$.

From the computation of square, we know that the square of any non-zero number equals to the square of its opposite number. In other words, **every positive number has two square roots, one is a positive number, and the other its opposite number.**

As $0^2=0$, therefore, **the square root of zero is also zero**.

The square of a positive number is positive; the square of a negative number is positive; the square of zero is zero.

Since the square of a number is never negative, it is certain that **negative number does not have a square root**. For example, -4

does not have square root.

The process of finding the square root of a number is called **taking the square root**.

The process of squaring a number and the process of taking square root are mutually inverse operations.

As such, we can find the square root of a number by matching the radicand to the square of some other number, or we can verify the square root of a number by squaring it to see if it matches the original number. For example,

$\because\ 3^2=9$ ， $(-3)^2=9$

$\therefore$ the two square roots of 9 are 3 and -3.

The positive square root of a positive number *a* is written as "$\sqrt[2]{a}$", while the negative square root is written as 「$-\sqrt[2]{a}$」.. We use "$\pm\sqrt[2]{a}$" to represent the combined presence of the positive square root and the negative square root..

Here "$\sqrt[2]{\ }$" is the sign which reads as 2$^{\text{nd}}$ root or square root, *a* is called **radicand** and 2 is **the order of the root**.

When order of the root is 2, we usually omit the figure 2 and write the symbol of $\pm\sqrt[2]{a}$ as $\pm\sqrt{a}$, which is read as "plus and minus square root *a*".

Note: As negative numbers has no square root, therefore $\sqrt{a}$ is always greater than or equal to zero, and it implies $a\ge 0$.

【**Example 1**】Find the square root of the following numbers:

(1) 36; (2) $\frac{16}{25}$; (3) $2\frac{1}{4}$; (4) 0.49.

Solution (1) $\because\ (\pm 6)^2=36$

$\therefore$ the square root of 36 is ± 6, that is

$$\pm\sqrt{36}=\pm 6$$

(2) $\because\ \left(\pm\frac{4}{5}\right)^2=\frac{16}{25}$

$\therefore$ the square root of $\frac{16}{25}$ is $\pm\frac{4}{5}$, that is

$$\pm\sqrt{\frac{16}{25}}=\pm\frac{4}{5}$$

(3) $\because$ $2\frac{1}{4}=\frac{9}{4}$, $\left(\pm\frac{3}{2}\right)^2=\frac{9}{4}$

$\therefore$ the square root of $2\frac{1}{4}$ is $\pm\frac{3}{2}$, that is

$$\pm\sqrt{2\frac{1}{4}}=\pm\sqrt{\frac{9}{4}}=\pm\frac{3}{2}$$

(4) $\because$ $(\pm0.7)^2=0.49$

$\therefore$ the square root of 0.49 is ±0.7, that is

$$\pm\sqrt{0.49}=\pm0.7$$

【**Example 2**】Which of the following number has a square root ?

(1) 64; (2) -64; (3) 0; (4) $(-4)^2$.

Solution (1) Since $64>0$, therefore 64 has square roots;

(2) Since $-64<0$, therefore -64 does not have square root;

(3) 0 has a square root;

(4) Since $(-4)^2=16>0$, therefore $(-4)^2$ has square roots.

Practice

1. (*Mental*):

(1) What are the square roots of 81?

(2) Whar are the square roots of $\frac{4}{9}$?

(3) What are the square roots of 0.25?

2. Find the square roots of the following number:

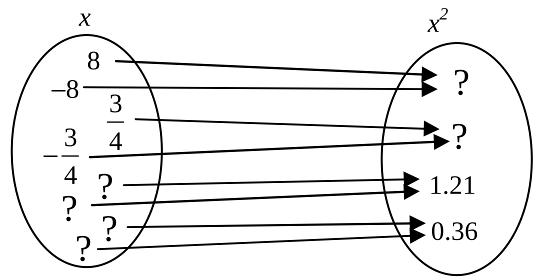
$$1,\ 64,\ 1600,\ 0,\ 0.0081,\ \frac{49}{100},\ 2.25,\ \frac{25}{144}.$$

Practice

3. Find the square roots of the following number:

$$729,\ 324,\ 0.81,\ 0.0225,\ \frac{16}{49},\ \frac{25}{64}.$$

4. In the following diagram find the value of each unknown marked with "?":



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9.2 Principal Square Root

Now, we know that a positive number has two square roots, one is positive and the other is negative, both are opposite numbers to each other.

To find the square roots of a number, we just need to find the positive square root, because we can obtain the negative square root by taking the opposite number of the positive square root.

The positive square root of a is also referred as **the principal square root**, written as $\sqrt{a}$ and read as "square root of a".

For example, the principal square root of 9 is 3, which can be written as $\sqrt{9}=3$. Similarly, $\sqrt{16}=4$, $\sqrt{\frac{4}{25}}=\frac{2}{5}$.

The square root of zero is zero which is neither positive nor negative. Therefore it is also called the principal square root of zero. That is, $\sqrt{0}=0$.

【**Example 1**】Compute the principal square root of the following number:

(1) 100; (2) $\frac{49}{64}$; (3) 0.81.

Solution (1) $\because$ $10^2 = 100$

$\therefore$ the principal square root of 100 is 10, that is

$$\sqrt{100} = 10$$

(2) $\because$ $\left(\frac{7}{8}\right)^2 = \frac{49}{64}$

$\therefore$ the principal square root of $\frac{49}{64}$ is $\frac{7}{8}$, that is

$$\sqrt{\frac{49}{64}} = \frac{7}{8}$$

(3) $\because$ $0.9^2 = 0.81$

$\therefore$ the principal square root of 0.81 is 0.9, that is

$$\sqrt{0.81} = 0.9$$

【**Example 2**】Find the values of the following expression:

(1) $\sqrt{10000}$; (2) $-\sqrt{144}$; (3) $\sqrt{\frac{25}{121}}$;

(4) $-\sqrt{0.0001}$; (5) $\pm\sqrt{625}$; (6) $\pm\sqrt{\frac{49}{81}}$.

Solution (1) $\because$ $100^2 = 10000$

$\therefore$ $\sqrt{10000} = 100$

(2) $\because$ $12^2 = 144$

$\therefore$ $-\sqrt{144} = -12$

(3) $\because$ $\left(\frac{5}{11}\right)^2 = \frac{25}{121}$

$\therefore$ $\sqrt{\frac{25}{121}} = \frac{5}{11}$

(4) $\because$ $(0.01)^2 = 0.0001$

$\therefore$ $-\sqrt{0.0001} = -0.01$

(5) $\because$ $25^2 = 625$

$\therefore$ $\pm\sqrt{625} = \pm 25$

(6) $\because$ $\left(\frac{7}{9}\right)^2 = \frac{49}{81}$

$\therefore$ $\pm\sqrt{\frac{49}{81}} = \pm\frac{7}{9}$

Practice

1. Determine whether the following statement is right or wrong?
   (1) 5 is the principal square root of 25;
   (2) -6 is the principal square root of 36;
   (3) 6 is the principal square root of $(-6)^2$;
   (4) 0.4 is the principal square root of 0.16.
2. Find the principal square root of the following:
   $$121,\ 0.25,\ 400,\ 0.01,\ \frac{1}{256},\ \frac{144}{169},\ 0.$$
3. (1) In the formula $c = \sqrt{a^2 + b^2}$, given $a = 6$, $b = 8$, find c.
   (2) In the formula $a = \sqrt{c^2 - b^2}$, given $c = 41$, $b = 40$, find a.
4. Find the value of the following:
   $$\sqrt{1},\ -\sqrt{\frac{4}{9}},\ \sqrt{1.21},\ -\sqrt{0.0196},\ \pm\sqrt{\frac{9}{25}},\ \pm\sqrt{\frac{36}{169}}.$$

9.3 Using Calculator to Find Square Root

We observe, from previous computation, that there are some special integers, decimals and fractions whose square roots are easy to find. But for general numbers, like 1840, $\frac{7}{11}$, 0.529, which does not conform to any pattern, it may not be easy to find their square roots. While in the past we might have to look up clumsy square root table, nowadays, we can find square roots much faster using a simple calculator.

【Example 1】 Use a calculator to find $\sqrt{1.35}$ (correct to 3 decimal places).

Solution On a calculator, the square button with the sign $\boxed{\sqrt{\ }}$ is the function key for calculating square root. Press the following buttons in sequence $\boxed{1}$, $\boxed{\bullet}$, $\boxed{3}$, $\boxed{5}$, $\boxed{\sqrt{\ }}$, the monitor will display 1.16189500386. As we are required to correct the answer to 3 decimal places, we shall select the rounding button 5/4 and choose 3 decimal places, then press the $\boxed{=}$ button, and the answer 1.162 will be displayed.

【Example 2】 Use a calculator to find $\sqrt{13.5}$ (correct to 3 decimal places).

Solution Select 5/4 and choose 3 decimal places, press the following buttons in sequence $\boxed{1}$, $\boxed{3}$, $\boxed{\bullet}$, $\boxed{5}$, $\boxed{\sqrt{\ }}$, $\boxed{=}$, the calculator will display the answer 3.674.

Note: When finding square roots, pay attention to the position of the decimal point. As the decimal point of 1.35 and 13.5 are at different positions, their square roots $\sqrt{1.35}$ and $\sqrt{13.5}$ have very different results.

Practice

1. Use a calculator to find the principal square root of the following number (correct to three decimal places):

(1) 9.73;	(2) 97.3;	(3) 38.5;	(4) 3.85;
(5) 6.8;	(6) 68;	(7) 5;	(8) 4.04.

Practice

2. Use a calculator to find the value of the following expression (correct to 3 decimal places):

(1) $\sqrt{2}$;	(2) $\sqrt{60}$;	(3) $\sqrt{95}$;	(4) $-\sqrt{9.5}$;
(5) $\sqrt{1.48}$;	(6) $\sqrt{70.4}$;	(7) $-\sqrt{47.3}$;	(8) $-\sqrt{8.47}$.

3. Use a calculator to find the principal square root of the following number (correct to 3 decimal places):

(1) 3;	(2) 7;	(3) 38.1;	(4) 1.44;
(5) 42.5;	(6) 53.8;	(7) 6.18;	(8) 83.8.

Study the following table:

n	0.04	4	400	40000
$\sqrt{n}$	0.2	2	20	200

It can be observed that when n is increased by 100 times, its principal square root is increased by 10 times; In its converse, when n is decreased to $\frac{1}{100}$ of its original value, its principal square root is only decreased to $\frac{1}{10}$ of its original value. **In other words, if the decimal point of a positive number is moved left or right by 2 places, the decimal point of its principal square root is moved left or right by only 1 place .**

【Example 3】 Given that $\sqrt{0.236} \approx 0.4858$, find the value of $\sqrt{236000}$.

Solution

$$\boxed{\sqrt{0.236}} \approx \boxed{0.4858}$$

the decimal point is moved right by 2 places ↓

the decimal point is moved right by 1 place ↓

$\sqrt{23.6} \approx 4.858$

the decimal point is moved right by 2 places ↓   ↓ the decimal point is moved right by 1 place

$\sqrt{2360} \approx 48.58$

the decimal point is moved right by 2 places ↓   ↓ the decimal point is moved right by 1 place

$\sqrt{236000} \approx 485.8$

$\therefore \quad \sqrt{236000} \approx 485.8$

Practice

1. Use a calculator to compute the following (correct to 3 decimal places):

   (1) $\sqrt{0.0415}$; (2) $-\sqrt{0.001289}$; (3) $\sqrt{0.38087}$;

   (4) $\sqrt{64090}$; (5) $-\sqrt{725}$; (6) $\sqrt{5710052}$;

   (7) $\sqrt{\dfrac{8}{25}}$; (8) $\sqrt{170\dfrac{3}{4}}$.

2. Find x from the following equation:

   (1) $x^2 = 169$; (2) $x^2 - 2.56 = 0$;

   (3) $9x^2 - 64 = 0$; (4) $3x^2 = 5$ (correct to 0.01).

Exercise 1

1. Is the following statement true or false? why?

   (1) Square of -5 is 25; (2) Square root of 25 is -5;

   (3) Squre root of 49 is ± 7; (4) Suqre root of -49 is -7.

2. (1) Fill up the following table:

n	11	12	13	14	15	16	17	18	19
n^2									

   (2) From the above, write down the square root of the following number:

   361, 289, 196, 256.

3. Find the principal square root of the following number;

   $$3600,\ \frac{1}{64},\ 10000,\ 7.29,\ 0.04,\ \frac{121}{289},\ 1\frac{11}{25}.$$

4. Find the value of the following expression:

   $$\sqrt{0},\ -\sqrt{81},\ \sqrt{0.09},\ \sqrt{(-25)^2},\ \pm\sqrt{\frac{25}{36}}.$$

5. Find x form the following equation:

   (1) $x^2 = 25$; (2) $x^2 - 81 = 0$;

   (3) $4x^2 = 49$; (4) $25x^2 - 36 = 0$.

6. Use a calculator to find the principal square root of the following number (correct to 2 decimal places):

   (1) 10; (2) 13; (3) 97.8;

   (4) 1.23; (5) 1.491; (6) 14.91;

   (7) 5.869; (8) 58.69; (9) 867;

   (10) 7590; (11) 0.759; (12) 0.003094;

   (13) 87420; (14) 0.46254; (15) 0.00035783.

7. Use a calculator to find the value of the following expression (correct to 3 decimal places):

   (1) $\sqrt{3.63}$; (2) $\sqrt{17.6}$; (3) $-\sqrt{2.248}$;

   (4) $\pm\sqrt{48.55}$; (5) $\sqrt{0.2157}$; (6) $-\sqrt{0.09286}$;

   (7) $\sqrt{278\dfrac{3}{4}}$; (8) $\pm\sqrt{6665.72}$.

9.4 Cube Root

Read the following question:

To make a cubical box with volume 125 m^3, what is the length of the side of the box?

Because the volume of a cubical is equal to the cube of its side, so if the length of a side of the cubical box is x m, then

$$x^3 = 125.$$

That is, it is required to find a number whose cube is equal to 125.

Since $5^3 = 125$, therefore, the length of the side of this cubical box is 5 m.

In general, if a number whose cube is a, this number is called **the cube root** (or **the third root** or **root of third order**) of a. In other words, if $x^3 = a$, then x is the cube root of a. The symbol "$\sqrt[3]{a}$" means the cube root of a number a, read as cube root of a, where a is the radicand, 3 is the order of the root. For example, $5^3 = 125$, 5 is the cube root of 125, the formula is $\sqrt[3]{125} = 5$.

The computation to find the cube root of a number is called **taking the cube root**. Taking cube root of a number is the inverse operation to the raising of a number to the cube power.

【Example 1】 Find the cube root of the following number:

(1) -8; (2) 8; (3) $-\frac{8}{27}$;

(4) 0.216; (5) 0.

Solution (1) $\because$ $(-2)^3 = -8$

$\therefore$ the cube root of -8 is -2, that is

$$\sqrt[3]{-8} = -2$$

(2) $\because$ $2^3 = 8$

$\therefore$ the cube root of 8 is 2, that is

$$\sqrt[3]{8} = 2$$

(3) $\because$ $\left(-\frac{2}{3}\right)^3 = -\frac{8}{27}$

$\therefore$ the cube root of $-\frac{8}{27}$ is $-\frac{2}{3}$, that is

$$\sqrt[3]{-\frac{8}{27}} = -\frac{2}{3}$$

(4) $\because$ $0.6^3 = 0.216$

$\therefore$ the cube root of 0.216 is 0.6, that is

$$\sqrt[3]{0.216} = 0.6$$

(5) $\because$ $0^3 = 0$

$\therefore$ the cube root of 0 is 0, that is

$$\sqrt[3]{0} = 0$$

From example 1 we observe that, positive numbers 8 and 0.216 have a positive cube root of 2 and 0.6 respectively; while negative numbers -8 and $-\frac{8}{27}$ have a negative cube root of -2 and $-\frac{2}{3}$ respectively; the cube root of zero is zero. **In general, a positive number has a positive cube root; a negative number has a negative cube root; the cube root of zero is zero**.

From example 1, we also observe that,

$$\because \quad \sqrt[3]{-8} = -2$$
$$\sqrt[3]{8} = 2$$
$$\therefore \quad \sqrt[3]{-8} = -\sqrt[3]{8}$$

In general, if $a > 0$, then $\sqrt[3]{-a} = -\sqrt[3]{a}$. Therefore, to find the cube root of a negative number, we move the negative sign to the front of the root sign and take the cube root of the resulting positive number.

Practice

1. (1) What are the cube values of the first nine integers 1, 2, 3, 4, 5, 6, 7, 8, 9?

   (2) From the result of (1) above, find the cube root of the following number:

$$27,\ \ -64,\ 343,\ \ -729.$$

2. Find the cube root of the following number:

$$1,\ 512,\ \ \frac{8}{227},\ \ -\frac{27}{64},\ \frac{1}{8},\ \ -\frac{1}{8},\ 0.125,\ \ -15\frac{5}{8}\ 。$$

【**Example 2**】Find the value of the following:

(1) $\sqrt[3]{27}$; (2) $\sqrt[3]{-27}$;

(3) $\sqrt[3]{-2\frac{10}{27}}$; (4) $-\sqrt[3]{-\frac{27}{64}}$ 。

Solution (1) $\sqrt[3]{27}=3$;

(2) $\sqrt[3]{-27}=-\sqrt[3]{27}=-3$;

(3) $\sqrt[3]{-2\frac{10}{27}}=\sqrt[3]{-\frac{64}{27}}=-\frac{4}{3}=-1\frac{1}{3}$;

(4) $-\sqrt[3]{-\frac{27}{64}}=\sqrt[3]{\frac{27}{64}}=\frac{3}{4}$.

Practice

1. Find the value of the following expression:

$$\sqrt[3]{1000},\ \ -\sqrt[3]{0.001},\ \ \sqrt[3]{-\frac{64}{125}},\ \ -\sqrt[3]{-216},\ \ \sqrt[3]{-1},\ \ \sqrt[3]{3\frac{3}{8}}.$$

2. Fill up the following table:

a	0.000001	0.001	1	1000	1000000
$\sqrt[3]{a}$					

Observe the movement of the decimal point (to left or right) of the cube root, when the decimal point of a moves 3 places. How does the decimal point of the cube root move?

Practice

3. Find the value of the following expression:

$$\sqrt[3]{-125},\ \ -\sqrt[3]{0.729},\ \ -\sqrt[3]{-\frac{125}{216}},\ \ \sqrt[3]{4+\frac{17}{27}},\ \ \sqrt[3]{\frac{37}{64}-1}.$$

Now we would like to generalise the concept of square root and cube root to higher order. If a number to the n^{th} power (where n is an integer greater than 1) is equal to a, then this number is called **the n^{th} root of a**. In other words, if $x^n=a$, then x is called the n^{th} root of a. The computation to find the n^{th} root of a is called **taking n^{th} root**, where a is called **the radicand** and n is called **the root index or order of the root**.

Taking root of a number to the odd order, the root is called odd order root; Taking root of a number to the even order, the root is called even order root.

We know that , $2^2=4$, $(-2)^2=4$, that means 2 and -2 are both square root of 4; $2^4=16$, $(-2)^4=16$, that means that 2 and -2 are both 4^{th} root of 16. In general, a positive number has two even roots and each is the opposite number of the other. When n is a even number, for a positive number a, its positive n^{th} root, is written as "$\sqrt[n]{a}$", and its negative n^{th} root, is written as "$-\sqrt[n]{a}$", the two can be combined and written as "$\pm\sqrt[n]{a}$".

For example $\sqrt[4]{16}=2$, $-\sqrt[4]{16}=-2$, and can be combined as $\pm\sqrt[4]{16}=\pm2$.

We know that:

$2^3=8$, 2 is the cube root of 8;

$(-2)^3=-8$, -2 is the cube root of -8;

$2^5=32$, 2 is the 5^{th} root of 32;

$(-2)^5=\ -32$, -2 is the 5^{th} root of -32.

In general, the odd root of a positive number is positive, and the odd root of a negative number is negative. When n is an odd number, and the n^{th} root of a positive number a is written as "$\sqrt[n]{a}$". For example, $\sqrt[3]{27}=3$, $\sqrt[5]{-32}=-2$.

The n^{th} root of zero is written as $\sqrt[n]{0}$, $\sqrt[n]{0}=0$.

The positive n^{th} root of a positive number a is the principal n^{th} root.

Taking the n^{th} root and taking the n^{th} power are inverse computation of each other.

9.5 Use a calculator to find the cube root*

Most simple calculators do not have cube root function. Without any other supporting facilities, we can still use trial and error method to progressively narrow down the search to compute cube root. For example, find $\sqrt[3]{13}$, correct to 2 decimal places. Since $2^3=8<13<3^3=27$, therefore we know its value is greater than 2 but smaller than 3. Then we can use our calculator to narrow down the serach to 1 decimal place $2.5^3=15.625>13$, $2.4^3=13.824>13$, $2.3^3=12.167<13$, so its value is between 2.3 and 2.4. Then we use our calculator to narrow down the search to 2 decimal places $2.35^3=12.977875<13$, $2.36^3=13.144256>13$, so we know the value is between 2.35 and 2.36. To determine which value is the correct anwer, we calculate $2.355^3=13.06088875>13$. Then we can conclude $\sqrt[3]{13}=2.35$ (correct to 2 decimal places).

If you have an engineering calculator , then you can easily find the cube root of any number a

$\boxed{\text{2ndF}}$, $\boxed{\sqrt{\ }}$, $\boxed{\text{a}}$, $\boxed{=}$.

Note: $\boxed{\text{2ndF}}$ is secondary function button, press $\boxed{\text{2ndF}}$, $\boxed{\sqrt{\ }}$ buttons, means execute the secondary function of $\boxed{\sqrt{\ }}$ button, that is $\sqrt[3]{\ }$ fucntion.

* Skip this section if the calculator does not have the engineering function

【Example 1】 Use an engineering calculator to find the cube roots of the following numbers:

(1) 343; (2) -1.331.

Solution (1) Press the buttons: $\boxed{\text{2ndF}}$, $\boxed{\sqrt{\ }}$, $\boxed{3}$, $\boxed{4}$, $\boxed{3}$, $\boxed{=}$, the calculator will display the answer 7;

(2) Press the buttons: $\boxed{\text{2ndF}}$, $\boxed{\sqrt{\ }}$, $\boxed{-}$, $\boxed{1}$, $\boxed{\bullet}$, $\boxed{3}$, $\boxed{3}$, $\boxed{1}$, $\boxed{=}$, the calculator will display the answer -1.1 (or press the buttons $\boxed{\text{2ndF}}$, $\boxed{\sqrt{\ }}$, $\boxed{1}$, $\boxed{\bullet}$, $\boxed{3}$, $\boxed{3}$, $\boxed{1}$, $\boxed{=}$ and add a negative sign in front of the answer).

【Example 2】 Use an engineering calculator to find $\sqrt[3]{2}$ (correct to 3 decimal places).

Solution Press the buttons: $\boxed{\text{2ndF}}$, $\boxed{\sqrt{\ }}$, $\boxed{2}$, $\boxed{=}$, the calculator will display 1.25992105, so the answer is $\sqrt[3]{2}=1.260$.

【Example 3】 It is known that $\sqrt[3]{0.00525}\approx 0.1738$, find the value of $\sqrt[3]{5250000}$.

Solution

$$\boxed{\sqrt[3]{0.00525}} \approx \boxed{0.1738}$$

the decimal point is moved right by 3 places ↓ the decimal point is moved right by 1 place ↓

$$\boxed{\sqrt[3]{5.25}} \approx \boxed{1.738}$$

the decimal point is moved right by 3 places ↓ the decimal point is moved right by 1 place ↓

$$\boxed{\sqrt[3]{5250}} \approx \boxed{17.38}$$

the decimal point is moved right by 3 places $\downarrow$ $\qquad$ $\downarrow$ the decimal point is moved right by 1 place

$$\boxed{\sqrt[3]{5250000}} \approx \boxed{173.8}$$

$$\therefore \quad \sqrt[3]{5250000} \approx 173.8$$

Practice

1. Use a calculatior to find the cube root of the following number (accurate to 2 decimal places):

   (1) 3; (2) 8.8; (3) 98.7; (4) 0.35;

   (5) $65\frac{4}{5}$; (6) 1.847; (7) 25.738; (8) $8\frac{1}{8}$;

   (9) -80; (10) -16.5.

2. Use a calculator to find the value of the following expression (accurate to 2 decimal places):

   (1) $\sqrt[3]{5}$; (2) $\sqrt[3]{68.8}$; (3) $\sqrt[3]{0.8725}$;

   (4) $\sqrt[3]{-43.6}$; (5) $-\sqrt[3]{0.459}$; (6) $\sqrt[3]{0.00094}$;

   (7) $\sqrt[3]{0.037}$; (8) $\sqrt[3]{6570}$; (9) $\sqrt[3]{420000}$;

   (10) $\sqrt[3]{0.00005087}$; (11) $\sqrt[3]{-258}$; (12) $-\sqrt[3]{0.0062548}$.

3. Find x from the following equation:

   (1) $x^3 = 27$; (2) $x^3 = \frac{1}{8}$;

   (3) $27x^3 + 64 = 0$; (4) $4x^3 - 5 = 0$ (correct to 0.01).

9.6 Real Numbers

We have learnt that any rational number can be written as a finite decimal (integer can be treated as a decimal number trailed by zeros after the decimal point) or a recurring decimal. For example:

$$3 = 3.0$$
$$-\frac{3}{5} = -0.6$$
$$\frac{9}{11} = 0.\overline{81}$$
$$\vdots$$

The converse is also true, any finite decimal or recurring decimal is rational number.

In fact there are numbers which cannot be written as finite decimal or recurring decimal. For example, $\sqrt{2} = 1.41421356\cdots$, $0.101001000100001\cdots$ (the numer of zeros between two 1s is increased by 1 in every recurrance), the ratio of circumference to the diameter of a circle $\pi = 3.14159265\cdots$, it has infinite decimal places with no recurring pattern.

Infinte and non recurring decimals are called **irrational numbers**. Like the examples above, $\sqrt{2}$, $0.101001000100001\cdots$, π are all irrantional numbers. There are many irrational numbers, such as,

$$\sqrt{3} = 1.732050\cdots$$
$$-\sqrt{5} = -2.236067\cdots$$
$$\sqrt[3]{2} = 1.259921\cdots$$
$$\frac{\pi}{2} = 1.57079632\cdots$$

$0.232332333233332\cdots$ (the numer of threes between two 2s is increased by 1 in every recurrence)

$$\vdots$$

But $\sqrt{4}$, $-\sqrt[3]{27}$ are not irrational numbers. They are rational numbers.

There are positive irrational numbers and negative irrational numbers. For example $\sqrt{2}$, $\sqrt[5]{3}$, π , … are positive irrational numbers; $-\sqrt{2}$, $-\sqrt[5]{3}$, $-\pi$, … are negative irrational numbers.

We use the term **Real Numbers** to describe collectively both Rational Numbers and Irrational Numbers. So far, the different number systems we have learnt can be tabulated as follows:

$$\text{Real Number}\begin{cases}\text{Rational Number}\left.\begin{cases}\text{Postiive Rational Number}\\ \text{Zero}\\ \text{Negative Rational Number}\end{cases}\right\} & \text{Finiate decimal or recurring decimal}\\ \text{Irrational Number}\left.\begin{cases}\text{Positive Irrational Number}\\ \text{Negative Irrational Number}\end{cases}\right\} & \text{Infinite non-recurring decimal}\end{cases}$$

Rational numbers and irrational numbers both can have positive and negative numbers, therefore real numbers also have positive and negative numbers. If a represent a positive real number, then $-a$ represent a negative real number. a and $-a$ are opposite number to each other. The opposite number of 0 is still 0.

The absolute value of real number has the same meaning as the absolute value of rational number: **The absolute value of a positive real number is same as itself; the absolute value of a negative real number is its opposite number; the absolute value of zero is zero.** For example, $|\sqrt{2}|=\sqrt{2}$, $|-\sqrt{2}|=\sqrt{2}$, $|\pi|=\pi$, $|-\pi|=\pi$, $|0|=0$.

We know that every rational number can be represented by a point on the number axis. However, not every point on the number axis represents a rational number. Because some of the points represent non-recurring decimals and they are irrational numbers.

After extending the concept of numbers from rational numbers to real numbers, then there is a one to one correspondence between the real numbers and the points on the number axis. That is, every real number is represented by a point on the number axis, and conversely, every point on the number axis represents a real number.

In real number computation, the laws of computation follow the same as those for rational numbers. In the domain of real numbers, we can take any order root for zero and for positive real numbers, but can only take odd order root (and not even order root) for negative real numbers.

In real number computation, an irrational number can be approximated by a decimal number, correct to any degree of accuracy as required to represent it.

【**Example**】 Compute: (1) $\sqrt{5}+\pi$ (correct to 0.01);

(2) $\sqrt{3}\bullet\sqrt{2}$ (correct to 3 significant figures).

Solution (1) $\sqrt{5}+\pi\approx 2.236+3.142\approx 5.38$;

(2) $\sqrt{3}\bullet\sqrt{2}\approx 1.732\times 1.414\approx 2.45$.

Practice

1. (*Mental*) What is an irrational number? Give two irrational numbers as examples.
2. (*Mental*) Is the following statement true or false ? If not, how can it be corrected ?
   (1) All infinite decimal numbers are irrantional numbers;
   (2) All irrational numbers are infinite decimal numbers;
   (3) All numbers with a square root sign are irrational number.
3. Which of the following numbers are rational numbers and which are irrational numbers?
   $$-\pi,\ -3.14,\ -\sqrt{3},\ 1.732,\ 0,\ 0.3,$$
   $$18,\ \sqrt{\frac{25}{36}},\ \frac{21}{31},\ \sqrt{7},\ -\sqrt{16}.$$
4. (1) Rational numbers are real numbers. Real numbers are rational numbers. Are these two statements true or false? In each case, give an example to support your answer.
   (2) Irrational numbers are real numbers. Real numbers are irrational numbers. Are these two statements true or false? In each case, give an example to support your answer.

Practice

5. Find x from the following equation:

(1) $|x|=\frac{2}{3}$; (2) $|x|=0$;

(3) $|x|=\sqrt{10}$; (4) $|x|=\pi$.

6. Compute:

(1) $\sqrt{10}+\frac{6}{7}-\pi$ (correct to 0.01);

(2) $(-4)\times\sqrt{7}+\frac{1}{2}\times\sqrt{6}$ (correct to 3 significant figures)。

Exercise 2

1. The following statements are true or false? And Why ?

(1) the cube root of -0.064 is -0.4;

(2) the cube root of 8 is ± 2;

(3) the cube root of $\frac{1}{27}$ is $\frac{1}{3}$.

2. Find the value of the following expressions:

(1) $-\sqrt[3]{0.027}$; (2) $\sqrt[3]{-343}$; (3) $\sqrt[3]{\frac{125}{27}}$;

(4) $\sqrt[3]{1+\frac{61}{64}}$; (5) $\sqrt[3]{1-\frac{19}{27}}$; (6) $-\sqrt[3]{\frac{7}{8}-1}$。

3. Find x from the following equation:

(1) $x^3=0.729$; (2) $64x^3+125=0$;

(3) $x^3-3=\frac{3}{8}$; (4) $(x-1)^3=8$.

4. Use a calculator to find the cube root of the following numbers (accurate to 2 decimal place):

(1) 0.39; (2) 48.3; (3) -2.36;

(4) -34.26; (5) 434.5; (6) 4936;

(7) -0.0532; (8) 0.007283。

5. Use a calculator to find the value of the following expressions (accurate to 2 decimal place):

(1) $\sqrt[3]{0.432}$; (2) $\sqrt[3]{-1.948}$; (3) $-\sqrt[3]{7456.3}$;

(4) $\sqrt[3]{67.5}$; (5) $\sqrt[3]{400000}$; (6) $\sqrt[3]{-8\frac{5}{9}}$;

(7) $\sqrt[3]{0.0000518}$; (8) $-\sqrt[3]{-350\frac{4}{25}}$。

6. (1) The volume of a cube is 11 m^2, find its sides (accurate to 0.01m);

(2) The volume of a cube is 11 m^2, find its surface area (accurate to 0.01m).

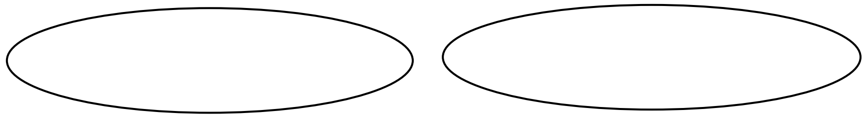
7. If $a=0.5$, $b=7.5$, $c=-0.36$, find the value of the following expressions (accurate to 0.01):

(1) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; (2) $(\sqrt[3]{b}-\sqrt[3]{a})c$.

8. Group the following numbers into the appropriate set:

$\frac{22}{7}$, 3.14159265, $\sqrt{8}$, -8, $\sqrt[3]{9}$,

0.6, $3\frac{1}{4}$, $\sqrt{36}$, 1.732, $\frac{\pi}{3}$.



The set of rational nubers The set of irrational nubers

(No. 8)

9. The following statements are right or wrong ? Why ?

(1) Every rational number is represented by a point on the number axis; the reverse, every point on the number axis represents a rational number;

(2) Every real number is represented by a point on the number axis; the reverse, every point on the number axis represents a reall number.

10. Find the absolute value of the followings:

$$-\sqrt{3},\ \sqrt[3]{-8},\ \sqrt{7},\ \frac{\sqrt{2}}{-3},\ \sqrt{3}-1.7,\ 1.4-\sqrt{2}\ 。$$

11. Compare the two numbers, which one is bigger :

(1) 15, 0; (2) $-\frac{3}{4}$, 0;

(3) -7, -9; (4) $\sqrt{2}$, $-\sqrt{3}$;

(5) $-\sqrt{3}$, -1.731; (6) π, 3.1416

12. Compute (accurate to 0.01):

(1) $\sqrt{5}-\sqrt{3}+0.415$; (2) $\sqrt[3]{6}-\pi-\sqrt{2}$.

Chapter summary

I. The main theme of this chapter is focussed on the concept of a square root number which may no longer be a rational number, and the concept of real numbers which consists of rational numbers and irrational numbers.

II. Mathematics continuously developes new concept to meet our practical needs, which we learn progressively in stages. In primary school we learn natural numbers and zero first; then learn fractions and decimals (finite decimals and recurring decimals). We identify all the non-zero numbers under the concept of positive rational numbers. In Year 7 we extend our concept to the full range of rational numbers which include negative rational numbers, zero and positive rational numbers. Now in Year 8, we further extend our concept to real numbers which include rational numbers and irrational numbers. The following table illustrates the relationship of the various concepts:

$$\text{Real Number}\begin{cases}\text{Rational Number}\begin{cases}\text{Postiive Rational Number}\\ \text{Zero}\\ \text{Negative Rational Number}\end{cases}\\ \text{Irrational Number}\begin{cases}\text{Positive Irrational Number}\\ \text{Negative Irrational Number}\end{cases}\end{cases}$$

III. For real numbers, the laws of computation follow the same as the laws of computation for rational numbers.

Evolution is the process of taking the root of number to the required order, while involution is the power of number formed by repeatingly mulitiplying the number by itself to the required order. Evolution and involution are inverse operations. Through involution of power, we can validate the accuracy of the result of evolution, the taking root of number.

Within the domain of real numbers, we can take the n^{th} root of any positive number; but when taking the n^{th} root of a negative number, it is only allowed to take root for odd values of n and is not allowed to take root for even value of n; the n^{th} root of zero is zero.

The positive root of a positive number is called its principal root, and the principal root of zero is zero.

Revision Exercise 9

1. Compute:

(1) $-8-\left[-\frac{1}{7}+\left(-\frac{1}{6}-0.25\times\frac{2}{3}\right)\div 2\frac{1}{3}\right]\times 6$;

(2) $\left[-\frac{2}{3}-\left(-\frac{4}{5}\right)\right]\left[\left(\frac{2}{3}\right)^2+\frac{2}{3}\times\left(-\frac{4}{5}\right)+\left(-\frac{4}{5}\right)^2\right]$;

(3) $\frac{7}{9}\times(-3)^3-\frac{1}{6}\times(-3)^2+\frac{1}{12}\times(-3)-\frac{1}{4}$;

(4) $8.954^3-7.308^3-26.54^2+37.69^2$ (Compute by using a calculator, correct to 0.1).

2. Give one example to demostrative the commutative law for addition, the associative law for addition, the commutative law for multiplication, the associative law for multiplication and the distributive law for multiplication.

3. If $a \neq 0$, what is the opposite number; the reciprocal; and the absolute value of *a*?

4. Give an example to explain:
   (1) Is *a* always greater than $-a$?
   (2) Is $\frac{a}{2}$ always smaller than *a*?
   (3) Given $a^2 = b^2$, does it impliy $a = b$?
   (4) Given $|a| > |b|$, does it impliy $a = b$?

5. If *n* is a positive integer, then
   (1) Is $2n$ even or odd? Is $2n-1$ even or odd?
   (2) $(-1)^{2n} = ?$ $(-1)^{2n-1} = ?$ $(-a)^{2n} = ?$ $(-a)^{2n-1} = ?$

6. Given $A = x^3 - x + 6$, $B = x^3 - x^2 + x + 3$, compute:
   (1) $(A - x^3)(B - x^3)$;
   (2) $(A + B) \div (A - B)$;
   (3) $(A - B)^2$.

7. Factorize the following expression:
   (1) $m^3 + 12m^2n + 36mn^2$; (2) $12xy - 9x^2 - 4y^2$;
   (3) $x^4 - 13x^2 + 36$; (4) $a^6 - 7a^3 - 8$;
   (5) $a^3 - b^3 - 3ab(a - b)$;
   (6) $a^3 + b^3 + 3a^2b + 3ab^3$;
   (7) $a^3 - b^3 - a^2 + b^2$;
   (8) $a(a+1) - b(b+1)$;
   (9) $a^2 - 4ab + 4b^2 - 4c^2$;
   (10) $(x+z)(x-z) + y(y-2x)$;
   (11) $(5x^2 + x - 20)^2 - (4x^2 - x + 4)^2$;
   (12) $(a+b)^2 + (a+c)^2 - (c+d)^2 - (b+d)^2$.

8. Simplify:
   (1) $(x-1)(x+4) - \{(x+2)(x-5) - [(x+3)(x-4) - (x-1)(x+6)]\}$
   (2) $(2x^2 - 6x + 5)^2 - (2x^2 - 6x + 4)^2 - (2x+3)^2$;
   (3) $\dfrac{a^3 - b^3}{a - b} - \dfrac{a^3 + b^3}{a + b} - 2ab$;
   (4) $\dfrac{x^2 - 3x + 2}{x^2 - 9} \bullet \dfrac{x^2 - x - 12}{x^2 - 4x + 4} \div \dfrac{x^2 - 5x + 4}{x^2 - 5x + 6}$;
   (5) $\dfrac{2}{x^2 - 3x + 2} - \dfrac{2}{x^2 - 4x + 3} + \dfrac{2}{x^2 - 5x + 6} - \dfrac{1}{x^2 - 1}$;
   (6) $\left(\dfrac{a^2}{a+b} - \dfrac{a^3}{a^2 + 2ab + b^2}\right) \div \left(\dfrac{a}{a+b} - \dfrac{a^2}{a^2 - b^2}\right)$;
   (7) $\dfrac{1+\frac{x}{y}}{1-\frac{x}{y}} \div \dfrac{1+\frac{y}{x}}{1-\frac{y}{x}}$; (8) $\dfrac{1}{1-\dfrac{1}{1-\frac{1}{x}}}$.

9. (1) Given $A = 2x^4 - 3x^2 + 6x - 7$, $B = x^2 - x + 1$. For $A \div B$, find the quotient *Q* and the remainder *R*;
   (2) Use $A = BQ + R$ to verify the result obtained in (1).

10. (1) What are the Axioms of Equivalent Equations?
    (2) What are the Basic Properties of Inequalities?

11. Solve the following equations:
    (1) $3\left\{x - \dfrac{3x-1}{4} - \left[1 - 2\left(x - \dfrac{3+x}{5}\right)\right]\right\} = 5x - 2$;
    (2) $\dfrac{1}{2}(x-1)^2 - \dfrac{5}{6}(x+2)^2 + \dfrac{1}{3}(x-3)^2 = 0$;
    (3) $\dfrac{3x - 38}{4x^2 - 4} - \dfrac{4}{x+1} = \dfrac{4}{1-x}$;
    (4) $\dfrac{x+5}{5-x} + \dfrac{x-5}{5+x} = \dfrac{100}{25 - x^2}$ 。

12. Solve the following simultaneous equations:

(1) $\begin{cases} \dfrac{7x-3y}{5}=\dfrac{5x-y}{3}-\dfrac{x+y}{2} \\ 3(x-1)=5(y+1) \end{cases}$;

(2) $\begin{cases} \dfrac{3x-2y}{6}+\dfrac{2x+3y}{7}=1 \\ \dfrac{3x-2y}{6}-\dfrac{2x+3y}{7}=5 \end{cases}$;

(3) $\begin{cases} \dfrac{11}{2x-3y}+\dfrac{18}{3x-2y}=13 \\ \dfrac{27}{3x-2y}-\dfrac{2}{2x-3y}=1 \end{cases}$;

(4) $\begin{cases} 3x+14=4y+6z \\ 7y-1=2z+8x \\ 5z+18=2x+3y \end{cases}$ 。

13. Solve the following simultaneous equations:

(1) $\begin{cases} ax-by=a^2+b^2 \\ bx+ay=a^2+b^2 \end{cases}$; (2) $\begin{cases} \dfrac{x}{a+b}+\dfrac{y}{a-b}=2a \\ x-y=4ab \end{cases}$;

(3) $\begin{cases} \dfrac{a}{x}+\dfrac{b}{y}=a-b \\ \dfrac{a}{x}-\dfrac{b}{y}=a+b \end{cases}$; (4) $\begin{cases} y+z=2a \\ z+x=2b \\ x+y=2c \end{cases}$ 。

14. Given that the equation $x^3+px^2+qx+r=0$ has one root equals to 1, another root equals to 2, and the third root equals to 3. Find the value of *p*, *q*, *r*.

15. (1) Find the value of *x* for the inquatlty $16-3(2x-5)<25-4x$ to hold?

(2) Find the value of *x* for the algebraic expression $5(x-1)-6(x-2)$ to take positive value?

(3) Find the value of *x* for the algebraic expression $\dfrac{1}{5}(x+2)-\dfrac{1}{3}(x-1)$ to take negative value?

(4) Find the integral solution of the inequality $-4<x\le-1$ and mark the value of *x* on the number axis;

(5) Find the integral solution of the inequality $|x|<3$ and mark the value of *x* on the number axis.

16. (1) If $a\ne b$, then $(a-b)^2>0$, why?

(2) If $a\ne b$, then $a^2+b^2>2ab$, why?

17. If the length of a rectangle is increased by 4 cm, but width decreased by 1 cm, its area remains unchanged; if the length of this rectangle is decreased by 2 cm, but width increased by 1 cm, its area rmains unchanged. Find the area of the rectangle.

18. A road can be repaired by team A and B working teogether for 4 days, and by team A working alone to finish the remaining task in 2 days; The same road can be repaired by team A and B working together to for 3 days, and by team B working alone to finish the remaining task in 3 days. Working alone, how many days would it take for team A and team B respectively to repair the road?

19. (1) If $x^2=a$, then what is *a* in relation to *x* and what is *x* in relation to *a*?

(2) If $y^3=b$, what is *b* in relation to *y* and what is *y* in relation to *b*?

20. Is the following statement true or false? Why?

(1) the square of -4 is 16;

(2) the square root of 16 is -4;

(3) the cube of -1 is -1;

(4) the cube root of -1 is -1;

(5) the square root of -1 is -1;

(6) the square root of 0 is 0.

21. What is principal square root of a number? Find the square root of the following number:

$$64,\ 36,\ 0.25,\ 5,\ (-3)^2,\ \left(\frac{2}{5}\right)^2,\ \left(-\frac{4}{13}\right)^2.$$

22. Use a calculator to find the value of the following expression:

(1) $\sqrt{94.3}$; (2) $\sqrt{0.5374}$; (3) $\sqrt{0.07321}$;

(4) $\sqrt{8200}$; (5) $\sqrt{527.48}$; (6) $\sqrt{4\frac{1}{4}}$;

(7) $\sqrt[3]{35.8}$; (8) $\sqrt[3]{0.43}$; (9) $\sqrt[3]{278.4}$;

(10) $\sqrt[3]{1.47}$; (11) $\sqrt[3]{0.002008}$; (12) $\sqrt[3]{4107\frac{1}{2}}$;

(13) $\sqrt{0.00895}-\sqrt{0.0234}$; (14) $\sqrt[3]{0.0695}+\sqrt[3]{0.1783}$;

(15) $3.472^2+5.089^2$; (16) $\sqrt{3.472^2+5.089^2}$ 。

23. Find the value of x in the following equation:

(1) $x^2=169$; (2) $121x^2-25=0$ (3) $9x^2=64$;

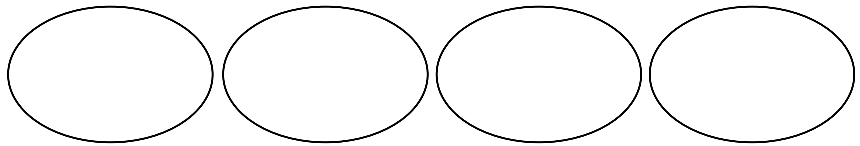
(4) $x^2-1.69=0$; (5) $x^3=64000$; (6) $x^3=-0.125$;

24. Given that the area of a square is 360 cm^2, find the length of its side (correct to 0.1 cm).

25. The volume of cube A is 5 cm. The volume of cube B is 2 times of cube. Find the length of the side of cube B (correct to 0.1 cm).

26. For a satellite to circle round the Earth, its velocity must be greater than the first cosmic velocity but less than the second cosmic velocity. The first cosmic velocity is given by this formula $V_1=\sqrt{qR}$ (m/s), while the second cosmic velocity is given by this formula $V_2=\sqrt{2qR}$ (m/s), $R=6.4\times10^6$. Find the first cosmic velocity and the second cosmic velocity (correct to 100m/s).

27. Explain what is an irrational number? In each of the circle below, list 3 numbers belonging to it.



Set of Positive Rational Numbers | Set of Negative Rational Numbers | Set of Positive Irrational Numbers | Set of Negative Irrational Numbers

(No. 27)

28. Does the smallest natural number exist? Does the smallest integer exist? Does the smallest rational number exist? Does the smallest irrational number exist? Does the smallest real number exist? Does a real number with the smallest absolute value exis?

29. Find the absolute value of the following:

$$-2\sqrt{5},\ \sqrt[3]{-7},\ \sqrt{5}-\sqrt{7},\ 3.1416-\pi.$$

30. Compare the magnitude of the following pair of real numbers:

(1) 1.574, 1.5; (2) $-\sqrt{5}$, -2.24;

(3) $-\pi$, -3.1415926; (4) $\sqrt{29}$, $5\frac{4}{13}$.

31. Compute (correct to 0.01):

(1) $\pi+\sqrt{10}-\frac{1}{3}+0.145$; (2) $\sqrt{5}+\frac{1}{7}-\left(4.375-\frac{4}{3}\right)$.

32. Read the square root table and answer the following questions:

(1) If the radicand is increased from 1 to 1.1, is its square root increased or decreased?

(2) If the radicand is gradually increased, is its square root increased or decreased?

Appendix: Manual Taking of Square Root Method

It is possible to find the square root of a positive number manually without looking up the square root table or using the square root function of a calculator.

To find the square root of a positive number, the first steip is to determine the number of digits of the resulting number after taking the square root.

We know that for two numbers, the square root of the larger number is also larger than the square root of the smaller number. We compute the square of the smallest and largest 1-digit, 2-digit and 3-digit numbers as follows,

$$\begin{array}{ll} 1^2 = 1 & 9^2 = 81 \\ 10^2 = 100 & 99^2 = 9801 \\ 100^2 = 10000 & 999^2 = 998001 \\ \vdots & \vdots \end{array}$$

From the above calculations, we observe that the square of a single digit number has 1 to 2 digits; the square of a double digit number has 3 to 4 digits; the square of 3 digit number has 5 to 6 digits. Looking it in reverse,

the square root of a 1 or 2 digit number is a single digit number,
the square root of a 3 or 4 digit number is a double digit number,
the square root of a 5 or 6 digit number is a 3 digit number,

$$\vdots$$

Hence we can chop the radicand, from right to left, into groups of 2 digits. For example, 1156 can be chopped into 11’56 two groups, its square root would be a double digit number. 85264 can be chopped into 8’52’64 three groups, its square root would be a 3-digit number.

The second step is to determine the most significant digit of the principal root of this positive number.

For two positive numbers, the square root of the larger number is also larger. Based on this principle we can determine the most

significant digit of the principal square root. For example, the leftmost group of 11’56 is 11, 11 is between 3^2 and 4^2, hence we know that, 1156 is between 30^2 and 40^2, therefore the most significant digit of the principal square root of 1156 is 3; the leftmost group of 8’52’64 is 8, 8 is between 2^2 and 3^2, hence we know that, 85264 is between 200^2 and 300^2, therefore the most significant digit of the principal square root of 85264 is 2.

The third step is to determine the other digits of the square root. Let’s explain the method through an example.

For example, find $\sqrt{1156}$.

We know that $\sqrt{1156}$ is a double digit number, and the most significant figure is 3, that is, the tens digit is 3. If we use a to represent the unit digit, then the square root can be written as $30+a$. Therefore

$$\begin{aligned} 1156 &= (30+a)^2 \\ &= 30^2 + 2\times 30a + a^2 \end{aligned}$$

From 1156 minus $30^2 = 900$, we have

$$\begin{array}{rrrrrl} & 1 & 1 & 5 & 6 & \cdots\cdots\ (30+a)^2 \\ - & & 9 & 0 & 0 & \cdots\cdots\ 30^2 \\ \hline & & 2 & 5 & 6 & \cdots\cdots\ 2\times 30a + a^2 \end{array}$$

that means, $256 = 2\times 30a + a^2 = (2\times 30 + a)a$. From this relationship, we can derive the value of a.

As a is a single digit number, 2×30 is far greater than a, so the approximate value of $2\times 30 + a$ is 2×30. Use 2×30 to divide 256, the trial quotient is 4.

To make sure that a equals 4 is correct, we can just put back $a = 4$ to the formula $(2\times 30 + a)a$ to see if its value is equal to 256. $(60+4)\bullet 4 = 256$. This verify that a is equal to 4. Hence $\sqrt{1156} = 34$.

The above calculation can be written in long form as follows:

$$\begin{array}{r|l} & 3\quad 4 \\ & \sqrt{11'56} \\ & 9 \\ \hline 2\times 30=60 & 2\ 56 \\ +\qquad 4 & \\ \hline 64 & 2\ 56 \\ \hline & \quad 0 \end{array}$$

The remainder is zero, which means the taking square root process is completed. In other words, $34^2 = 1156$ exactly. We call 1156 a perfect square. If a positive number is the square of a rational number, then this positive number is called **a perfect square**.

The key steps of the Manual Taking of Square Root Method are:

1. Chop the radicands from right to left into groups of 2 digits;
2. Take the left most group and determine the most significant digit of its principal square root;
3. The leftmost group minus the square of this most significant digit results in a difference. Append the second group of the radicand to the right side of the difference to form the first remainder;
4. Multiply the most significant digit by 20, and use this number to trial divide the first remainder. If the trial quotient is greater than 10, use 9 instead as the trial quotient cannot have more than 1-digit;
5. Add the trial quotient to 20 times the original quotient, and multiply it by the trial quotient, if the product is less that the first remainer, then this test quotient is the second digit of the square root; if the product is larger than the first remainder, then it is necessary to reduce the test quotient and repeat the trial, until the product is smaller than the first remainder;
6. Repeat the procedure, the other digits of the square root can be found.

【Example 1】 Find (1) $\sqrt{1444}$;

(2) $\sqrt{85264}$.

Solution (1)

$$\begin{array}{r|l} & 3\quad 8 \\ & \sqrt{14'44} \\ & 9 \\ \hline 2\times 30=60 & 5\ 44 \\ +\qquad 8 & \\ \hline 68 & 5\ 44 \\ \hline & \quad 0 \end{array}$$

$\therefore \quad \sqrt{1444} = 38$

(2)

$$\begin{array}{r|l} & 2\quad 9\quad 2 \\ & \sqrt{8'52'64} \\ & 4 \\ \hline 49 & 4\ 52 \\ & 4\ 41 \\ \hline 582 & 11\ 64 \\ & 11\ 64 \\ \hline & \quad 0 \end{array}$$

$\therefore \quad \sqrt{85264} = 292$

Note: In example 1, the trial quotient in step (1) obtained by dividing 544 by 60 is 9, but the product of 69×9 is greater than 544, therefore the trial quotient is reduced to 8；In example 1, the test quotient in step (2) obtained by dividing 452 by 40 is 11, but the test quotient can only have one digit, therefore the test quotient is at most equal to 9.

【Example 2】 Find $\sqrt{10404}$.

Solution

$$\begin{array}{r|l} & 1\quad 0\quad 2 \\ & \sqrt{1'04'04} \\ & 1 \\ \hline 202 & 4\ 04 \\ & 4\ 04 \\ \hline & \quad 0 \end{array}$$

$\therefore \quad \sqrt{10404} = 102$

Note： In the first step, 1 minus 1 equals 0. Remainder appended with the next subgroup of radicand is 4, Use 20 to divide 4, the quotient is less than 1, and is therefore 0, Remainder appended with the next subgroup of radicand is 404. Use 200 to divide 404, the trial quotient is 2.

To take square root of a positive decimal, it can be found using the same method, the only difference lies in chopping the radicand into subgroups, this is done from the decimal point rightward, if the last digit is single, pair it into a 2-digit number by appending a zero to it. For example the number 0.3249 is chopped into 0.32’49, the number 232.5625 is chopped into 2’32.56’25, the number 321.512 is chopped into 3’21.51’20. It is important to note that the decimal point should be aligned with the decimal point of the recticand.

【**Example 3**】Find (1) $\sqrt{0.3249}$;
(2) $\sqrt{232.5625}$.

Solution (1)

$$
\begin{array}{r|l}
 & 0.\ 5\ \ 7 \\
 & \sqrt{0.32'49} \\
 & 25 \\ \hline
107 & 7\ 49 \\
 & 7\ 49 \\ \hline
 & \quad 0
\end{array}
$$

(2)

$$
\begin{array}{r|l}
 & 1\ \ 5.\ 2\ \ 5 \\
 & \sqrt{2'32.56'25} \\
 & 1 \\ \hline
25 & 1\ 32 \\
 & 1\ 25 \\ \hline
302 & 7\ 56 \\
 & 6\ 04 \\ \hline
3045 & 1\ 52\ 25 \\
 & 1\ 52\ 25 \\ \hline
 & \qquad 0
\end{array}
$$

$\therefore \quad \sqrt{0.3249} = 0.57$ $\qquad$ $\therefore \quad \sqrt{232.5625} = 15.25$

In the first 3 examples, the radicands are perfect squares. For raticands which are not perfect squares, we can still calculate the approximate value of its square root.

【**Example 4**】(1) Find the approxminate value of $\sqrt{12.5}$ (correct to 0.01);
(2) Find the approxminate value of $\sqrt{2}$ (correct to 0.01).

Solution (1)

$$
\begin{array}{r|l}
 & 3.\ 5\ \ 3\ \ 5 \\
 & \sqrt{12.50'00'00} \\
 & 9 \\ \hline
65 & 3\ 50 \\
 & 3\ 25 \\ \hline
703 & 25\ 00 \\
 & 21\ 09 \\ \hline
7065 & 3\ 91\ 00 \\
 & 3\ 53\ 25 \\ \hline
 & \quad 37\ 75
\end{array}
$$

(2)

$$
\begin{array}{r|l}
 & 1.\ 4\ \ 1\ \ 4\ \ 2\ \ 1 \\
 & \sqrt{2.00'00'00'00'00} \\
 & 1 \\ \hline
24 & 1\ 00 \\
 & \ \ 96 \\ \hline
281 & 4\ 00 \\
 & 2\ 81 \\ \hline
2824 & 1\ 19\ 00 \\
 & 1\ 12\ 96 \\ \hline
28282 & 6\ 04\ 00 \\
 & 5\ 65\ 64 \\ \hline
282841 & 38\ 36\ 00 \\
 & 28\ 28\ 41 \\ \hline
 & 10\ 07\ 59
\end{array}
$$

$\therefore \quad \sqrt{12.5} \approx 3.54$ $\qquad$ $\therefore \quad \sqrt{2} \approx 1.4142$

In example 4, the two radicands 12.5 and 2 are not complete square numbers. In applying the manual taking of square root method, we can append zeros after the decimal to continue with the division process, until the degree of accuracy is reached. Care should be taken to note that when appending zeros, we have to ensure that the decimal portion of the radicand is properly chopped in subgroups of two, counting rightward from the decimal point.

(This chapter is translated to English by courtesy of Mr. Hymen LAM and reviewed by courtesy of Mr. SIN Wing Sang, Edward.)