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## Mathematics Essay Problems

Country: $\qquad$ Name: $\qquad$ No.: $\qquad$ Score: $\qquad$

## Instructions:

- Write down your name and country on every page.
- You have 90 minutes to work on this test.
- Write down your detail solutions or working process in English on the space below the question.
- Each problem is worth 3 points, and partial credit may be awarded.
- Use black or blue colour pen or pencil to write your answer.

The following table is for jury use only

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | Total |
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## ESSAY PROBLEMS

Country: $\qquad$ Name: $\qquad$ No.: $\qquad$

1. Al lives in Alton and Ben lives in Benburg, the two towns are 12 km apart. They want to go to Centreville, which is 30 km from Alton and 20 km from Benburg. Ben asks Al to take a taxi from Alton to Benburg to pick up him, and then go together to Centreville. The cost of the taxi is 1000 rupiahs per km . Ben will pay the part of the cost of the taxi resulting from the extra distance caused by this detour, and will share the remaining cost equally with Al. How much is Ben's saving by sharing the taxi with Al?
2. Each of Alice and Brian has some cows. Alice says to Brian, "If I add three times the number of cows you have to what I have, then I am satisfied." Brian replies, "If I add five times the number of cows you have to what I have, then I am satisfied." If the number of cows which makes them satisfied is the same, what is the minimum value of this number?

## ESSAY PROBLEMS

Country: $\qquad$ Name: $\qquad$ No.: $\qquad$
3. Initially, a robot faces north. Whenever it stops moving, it automatically faces north. It is programmed to do the following:
(1) Turn $30^{\circ}$ to the right, move 1 km forward and stop.
(2) Turn $90^{\circ}$ to the right, move 1 km forward and stop.
(3) Turn $150^{\circ}$ to the right, move 1 km forward and stop.
(4) Turn $210^{\circ}$ to the right, move 1 km forward and stop.
(5) Turn $270^{\circ}$ to the right, move 1 km forward and stop.
(6) Turn $330^{\circ}$ to the right, move 1 km forward and stop.

What is the distance between the initial and final position of the robot?

ANSWER:
km
4. Holly's is paid 67510 rupiahs per hour while Molly is paid 32490 rupiahs per hour. Together they earn 267510 rupiahs. Had Holly worked the number of hours Molly did and Molly work the number of hours Holly did, their combined earning would have been 232490 rupiahs. How many hours Holly and Molly work?

# ESSAY PROBLEMS 

Country: $\qquad$ Name: $\qquad$ No.: $\qquad$
5. $A B C D$ is a rectangle with $A B=25 \mathrm{~cm}$ and $B C=30 \mathrm{~cm} . M$ is a point on $A D$ such that $\frac{A M}{A D}=\frac{1}{3}$ and $N$ is a point on the diagonal $A C$ such that $\frac{A N}{A C}=\frac{3}{5}$. What is the area of triangle $B M N$ ?
6. Four different positive integers are such that the sum of any two is divisible by 2 and the sum of any three is divisible by 3 . What is the minimum value of the sum of all these four integers?

## ESSAY PROBLEMS

Country: $\qquad$
$\qquad$ No.: $\qquad$
7. $A B C D$ is a square of side length $10 \mathrm{~cm} . E, F, G$ and $H$ are points on $A B, B C$, $C D$ and $D A$ respectively, such that $E G$ is parallel to $A D$ and $F H$ is parallel to $A B$. $P$ is a point on $A E$ such that $P E=2 \mathrm{~cm}$, and $Q$ is a point on $D H$ such that $H Q=3 \mathrm{~cm}$. What is the area of the quadrilateral $P F G Q$ ?

ANSWER:
$\mathrm{cm}^{2}$
8. Each of the numbers $1,2,3,4,5,6,7,8$ and 9 is to be placed into a different square in the expression

$$
\square+\frac{1}{\square+\frac{1}{\square}}+\square+\frac{1}{\square+\frac{1}{\square}}+\square+\frac{1}{\square+\frac{1}{\square}}
$$

What is the maximum value of this expression?

## ESSAY PROBLEMS

Country: $\qquad$
$\qquad$ No.: $\qquad$
9. In triangle $A B C, A D$ and $B E$ are altitudes and $A P$ and $B Q$ are angle bisectors at $A$ and $B$ respectively, where $P$ lies on $C D$ and $Q$ lies on $C E$. If $\angle P A D=6^{\circ}$ and $\angle Q B E=18^{\circ}$, what is the degree of $\angle B C A$ ?

ANSWER:
10. A 2014-digit number is the smallest positive integer such that when it is multiplied by 3 , every digit of the product is even. How many times the digit 3 appears in the original number?

## ESSAY PROBLEMS

$\qquad$
Country:
Name:
No.:
11. Give three different ways in order to divide the figure below into two parts of the same areas using one straight line.

12. Each of 18 people shakes hands with at least one other person, and no two people shake hands more than once. If X shakes hands with Y , then X does not shake hands with anyone who shakes hands with Y. If X does not shake hands with Y, then X shakes hands with everyone who shakes hands with Y. How many maximum number of handshakes and minimum number of handshakes?

ANSWER:_The maximum number of handshakes is

## ESSAY PROBLEMS

Country: $\qquad$ Name: $\qquad$ No.: $\qquad$
13. A 112 m by 75 m farm has been divided into 13 square fields, as shown in the diagram below. The field at the bottom right corner has side length 33 m . There is a straight path which serves as a boundary for several fields, and cut across two other fields along the dotted lines. What is the side length of each field?


