Chapter 13 Common Logarithm

13.1 Logarithm

We have learnt about index, exponent or power, now we can move on to study logarithm which is closely related to exponent.

We know that 2 raised to the power of 4 is 16, which is written in equation form as

$$2^4 = 16,$$

Here 16 is 2 raised to the power of 4, where 2 is the base and 4 is the index or exponent.

In computation, we may experience the converse of this question: 2 raised to what power is 16? In this example, we know that the answer is 4.

To express the question what power of 2 is 16, we introduce a new function called the logarithm function, with which we can express the question in mathematics form as $\log_2 16$. When knowing that the answer is 4, we can equate it as follows:

$$\log_2 16 = 4,$$

In this format,

4 is the logarithm (abbreviated as log) of 16 to base 2.
2 is called the base.
16 is called the anti-logarithm.

In general, if a ($a > 0$, $a \neq 1$) raised to the power of b equals to N, then the equation is

$$a^b = N$$

Expressed in logarithm format, the equation is

$$\log_a N = b$$ [1],

where

b is the logarithm of N to base a.
a is the base number (in short, the base), and
N is the anti-logarithm number..

[1] 「log」 is short form notation of the latin word logarithm.

In the Real Number doman, a positive number raised to any power is also a positive number.

In $a^b = N$, we require a to be any positive number not equal to 1. When a is raised to the power of any real number b as in the equation, the result N is always a positive number and cannot be zero nor negative.

In other words, if N is zero or a negative number, we cannot find any real number b such that $a^b = N$.

Form the above analysis, we conclude that it is not possible to find the logarithm of zero or the logarithm of any negative number.

When a number N is written as a^b, we say that N is expressed in exponential notation.

In this chapter, unless it is otherwise stated, the base is always a positive number not equal to 1, and the anti-logarithm is always a positive number.

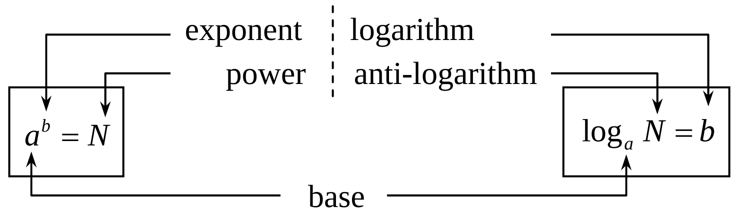
The two equations $a^b = N$ and $\log_a N = b$ are equivalent to one another. The former one is expressed in exponential form while the latter one is expressed in logarithm form.

The corresponding terms are given different names in the two equations:

(i) In $a^b = N$, a is the base, b is the index (or exponent), and N is the value of the exponential number.

(ii) In $\log_a N = b$. a is the base, b is the logarithm and N is the anti-logarithm.

The corresponding terms are depicted in the following diagrm:



In $a^b = N$, if b is substituted as $\log_a N$, then we have

$$a^{\log_a N} = N.$$

Refer the example of $2^4=16$, if 4 is substituted as $\log_2 16$, then we have

$$2^{\log_2 16}=16.$$

【Example 1】 Re-write the following exponential formula to logarithm form: (1) $5^4=625$;

(2) $3^{-2}=\frac{1}{9}$.

Solution (1) $\log_5 625=4$; (2) $\log_3 \frac{1}{9}=-2$.

【Example 2】 Re-write the following logarithm formula to exponential form:

(1) $\log_2 8=3$;

(2) $\log_{10} 10000=4$.

Solution (1) $2^3=8$; (2) $10^4=10000$.

【Example 3】 Given that $\log_{10} N=-2$, find *N*.

Solution From $\log_{10} N=-2$, we get $10^{-2}=N$.

Therefore

$$N=\frac{1}{100}.$$

【Example 4】 Find the value of the following:

(1) $\log_9 81$;

(2) $\log_3 \frac{1}{27}$.

Solution (1) Let $\log_9 81=x$, then $9^x=81$.

$\because$ $9^2=81$

$\therefore$ $x=2$

That is

$$\log_9 81=2.$$

(2) Let $\log_3 \frac{1}{27}=x$, then $3^x=\frac{1}{27}$.

$\because$ $3^{-3}=\frac{1}{27}$

$\therefore$ $x=-3$

Therefore

$$\log_3 \frac{1}{27}=-3.$$

【Example 5】 Find the value of the following:

(1) $\log_4 4$;

(2) $\log_7 1$.

Solution (1) Let $\log_4 4=x$, then $4^x=4$.

$\because$ $4^1=4$

$\therefore$ $x=1$

That is

$$\log_4 4=1.$$

(2) Let $\log_7 1=x$, then $7^x=1$.

$\because$ $7^0=1$

$\therefore$ $x=0$

That is

$$\log_7 1=0.$$

Example 5 can be generalised. Let a be a positive number not equal to 1, then

$$\mathbf{\log_a a=1}$$

$$\mathbf{\log_a 1=0}$$

Student may like to prove these equations using the definition of logarithm.

Practice

1. Re-write the following equation to logarithmic form:

(1) $2^5 = 32$; (2) $10^3 = 1000$; (3) $7^{-2} = \frac{1}{49}$;

(4) $10^0 = 1$; (5) $8^{\frac{2}{3}} = 4$; (6) $27^{-\frac{1}{3}} = \frac{1}{3}$;

2. Re-write the following equation in exponential form:

(1) $\log_3 9 = 2$; (2) $\log_2 \frac{1}{4} = -2$;

(3) $\log_{10} 0.001 = -3$; (4) $\log_8 2 = \frac{1}{3}$;

(5) $\log_5 5 = 1$; (6) $\log_{27} \frac{1}{9} = -\frac{2}{3}$.

3. Find the value of the following:

(1) $\log_5 25$; (2) $\log_2 \frac{1}{16}$;

(3) $\log_{10} 0.01$; (4) $\log_a a^5$.

4. Find x from the following equation:

(1) $\log_2 x = 5$; (2) $\log_{\frac{1}{2}} x = -3$;

(3) $\log_5 x = 1$; (4) $\log_3 x = 0$.

5. Find the value of the following expression:

(1) $\log_{15} 15$; (2) $\log_{0.4} 1$;

(3) $\log_9 1$; (4) $\log_{3.7} 3.7$.

6. Find the value of the following expression:

(1) $5^{\log_5 10}$; (2) $0.2^{\log_{0.2} 5}$.

13.2 Logarithm of Product, Quotient, Exponent and Root of anti-Logarithm Values

We have learnt the definition and operation rules of index or exponential expressions, We know that, if $a > 0$, then

$$a^p \bullet a^q = a^{p+q}$$
$$a^p \div a^q = a^{p-q}$$
$$(a^p)^n = a^{np}$$
$$\sqrt[n]{a^p} = a^{\frac{p}{n}}$$

From these equations and the definition of logarithm, we can derive the following operation rules of logarithm.

1. Let $\log_a M = p$, $\log_a N = q$.

From the definition of logarithm, we have

$$M = a^p, \quad N = a^q.$$

$\therefore \quad MN = M \times N = a^p \times a^q = a^{p+q}$

$\therefore \quad \log_a MN = p + q = \log_a M + \log_a N$

That means **the logarithm of the product of two positive numbers to a certain base is equal to the sum of the logarithm of these two numbers to the same base. Th**at is

$$\mathbf{\log_a MN = \log_a M + \log_a N}.$$

The formula is also true when there are more than two numbers multiplying together. For example

$$\log_a LMN = \log_a L + \log_a M + \log_a N.$$

2. Let $\log_a M = p$, $\log_a N = q$, then

$$M = a^p, \quad N = a^q.$$

$\therefore \quad \frac{M}{N} = \frac{a^p}{a^q} = a^{p-q}$

$\therefore \quad \log_a \frac{M}{N} = p - q = \log_a M - \log_a N$

That means **the logarithm of the quotient of two numbers to a certain base is equal to the difference of the logarithm of these numbers to the same base. T**hat is

$$\mathbf{\log_a \frac{M}{N} = \log_a M - \log_a N}.$$

3. Let $\log_a M = p$, then

$$M = a^p .$$

$\therefore \quad M^n = (a^p)^n = a^{np}$

$\therefore \quad \log_a M^n = np = n\log_a M$

That means **the logarithm of a positive number rasied to the power of a certain exponent is equal to the product of the value of the exponent times the logarithm of the positive number. That** is

$$\boldsymbol{\log_a M^n = n\log_a M} .$$

4. **The logarithm of the principal root of a certain order of a positive number is equal to the logarithm of the positive number divided by the order of the root. Th**at is

$$\boldsymbol{\log_a \sqrt[n]{M} = \frac{1}{n}\log_a M} .$$

The proof is left for the student to do.

【Example 1】 Use $\log_a x$, $\log_a y$, $\log_a z$ to represent the following expression:

(1) $\log_a \dfrac{xy}{z}$; (2) $\log_a x^3y^5$;

(3) $\log_a \dfrac{\sqrt{x}}{yz}$; (4) $\log_a \dfrac{x^2\sqrt{y}}{\sqrt[3]{z}}$.

Solution (1) $\log_a \dfrac{xy}{z} = \log_a xy - \log_a z$

$= \log_a x + \log_a y - \log_a z$

(2) $\log_a x^3y^5 = \log_a x^3 + \log_a y^5$

$= 3\log_a x + 5\log_a y$

(3) $\log_a \dfrac{\sqrt{x}}{yz} = \log_a \sqrt{x} - \log_a yz$

$= \dfrac{1}{2}\log_a x - (\log_a y + \log_a z)$

$= \dfrac{1}{2}\log_a x - \log_a y - \log_a z$

(4) $\log_a \dfrac{x^2\sqrt{y}}{\sqrt[3]{z}} = \log_a x^2\sqrt{y} - \log_a \sqrt[3]{z}$

$= \log_a x^2 + \log_a \sqrt{y} - \log_a \sqrt[3]{z}$

$= 2\log_a x + \dfrac{1}{2}\log_a y - \dfrac{1}{3}\log_a z$

【Example 2】 Compute: (1) $\log_{10} \sqrt[5]{100}$;

(2) $\log_2(4^7 \bullet 2^5)$.

Solution (1) $\log_{10} \sqrt[5]{100} = \dfrac{1}{5}\log_{10} 100 = \dfrac{2}{5}$;

(2) $\log_2(4^7 \bullet 2^5) = \log_2 4^7 + \log_2 2^5$

$= 7\log_2 4 + 5\log_2 2$

$= 7\times 2 + 5\times 1$

$= 19$

Practice

1. Use $\log_{10} x$, $\log_{10} y$, $\log_{10} z$, $\log_{10}(x+y)$, $\log_{10}(x-y)$ to re-write the following expression:

(1) $\log_{10} xyz$; (2) $\log_{10}(x+y)z$;

(3) $\log_{10}(x^2-y^2)$; (4) $\log_{10} \dfrac{xy^2}{z}$;

(5) $\log_{10} \dfrac{xy}{(x+y)z}$; (6) $\log_{10} \dfrac{x^2-xy}{100}$.

2. Compute:

(1) $\log_3(27\times 9^2)$; (2) $\log_{10} 100^2$;

(3) $\log_{10} 0.0001^3$; (4) $\log_7 \sqrt[3]{49}$.

Practice

3. Compute:

(1) $\log_2 6 - \log_2 3$; (2) $\log_{10} 5 + \log_{10} 2$;

(3) $\log_5 3 + \log_5 \frac{1}{3}$; (4) $\log_3 5 - \log_3 15$.

4. Check if the following equation is true or false. Why?

(1) $\log_2 (8-2) = \log_2 8 - \log_2 2$;

(2) $\log_{10} (4-2) = \frac{\log_{10} 4}{\log_{10} 2}$;

(3) $\frac{\log_2 4}{\log_2 8} = \log_2 4 - \log_2 8$.

Exercise 1

1. Find the value of x in the following expression, and point out whether x is a base, an exponent, a logarithm or a root:

(1) $3^4 = x$; (2) $x^3 = 1000$; (3) $10^x = 0.0001$.

2. Express x as the subject of the equation in terms of logarithm:

(1) $4^x = 16$; (2) $3^x = 1$;

(3) $4^x = 2$; (4) $2^x = 0.5$.

3. Re-write the following expression in exponential form and find the value of x:

(1) $\log_2 32 = x$; (2) $\log_5 625 = x$;

(3) $\log_{10} 1000 = x$; (4) $\log_8 4 = x$;

(5) $\log_3 \frac{1}{9} = x$; (6) $\log_3 3 = x$;

(7) $\log_{10} \frac{1}{1000} = x$; (8) $\log_{16} \frac{1}{2} = x$.

4. Use $\log_a x$, $\log_a y$, $\log_a z$, $\log_a (x+y)$, $\log_a (x-y)$ to re-write the following expressions:

(1) $\log_a \frac{\sqrt{x}}{y^2 z}$; (2) $\log_a x\sqrt[4]{\frac{z^3}{y^2}}$;

(3) $\log_a xy^{\frac{1}{2}}z^{-\frac{2}{3}}$; (4) $\log_a \frac{xy}{x^2-y^2}$;

(5) $\log_a \left(\frac{x+y}{x-y} \bullet y\right)$; (6) $\log_a \left[\frac{y}{x(x-y)}\right]^3$.

5. Compute:

(1) $\log_a 2 + \log_a \frac{1}{2}$; (2) $\log_3 18 - \log_3 2$;

(3) $\log_{10} \frac{1}{4} - \log_{10} 25$; (4) $2\log_5 10 + \log_5 0.25$.

6. (1) Use $a = \log_{10} 5$ to re-write $\log_{10} 2$, $\log_{10} 20$;

(2) Use $a = \log_{10} 2$ and $b = \log_{10} 3$ to re-write $\log_{10} 4$, $\log_{10} 5$, $\log_{10} 6$, $\log_{10} 12$, $\log_{10} 15$.

13.3 Common Logarithm

The decimal numeral system is a base 10 numeral system. The base of 10 is commonly used probably because humans have ten fingers together on both hands. Extending the concept, it is common to take logarithm to base 10 as well. We call the system of taking logarithm to base 10 as the Common Logarithm. When using common logarithm, it is customary to omit the base 10 from the logarithm notation. So whenever the base is omitted, it is assumed that the logarithm is taken to base 10. For example, when we say log 100 is 2, we mean that the common logarithm of base 10 is being used, that is, log 100 to base 10 is 2.

We know that :

$$\begin{array}{ll} \vdots & \vdots \\ 10^3 = 1000 & \log 1000 = 3 \\ 10^2 = 100 & \log 100 = 2 \\ 10^1 = 10 & \log 10 = 1 \\ 10^0 = 1 & \log 1 = 0 \\ 10^{-1} = 0.1 & \log 0.1 = -1 \\ 10^{-2} = 0.01 & \log 0.01 = -2 \\ 10^{-3} = 0.001 & \log 0.001 = -3 \\ \vdots & \vdots \end{array}$$

From the above, we observe (i) for integral power of10, the logarithm value is an integer (ii) for a larger value of the anti-logarithm, the corresponding logarithm value is also larger..

If a positive number is not an integral power of 10 (i.e.it cannot be written as 10 to the power of a positive integer), then its logarithm is a decimal number. For example, log 72 is a decimal number between 1 and 2, and log 0.0072 is a decimal number between -3 and -2.

In the past, we used to obtain the logarithm value (with reasonable accuracy) of a positive number by looking up logarithm table. Nowadays, we can obtain the logarithm value using an engineering calculator.

13.4 Characteric and Mantissa of Logarithm

We know that, $1 < 3.408 < 10$, and the order of their logarithm values remain unchanged. Therefore $0 < \log 3.408 < 1$. In other words, $\log 3.408$ is a pure decimal number.

If we are given that $\log 3.408 = 0.5325$, then we can base on the information to calculate the logarithm values of 0.03408, 340.8, 34,080.

First re-write the above numbers in scientific notation, which will facilitate the finding of their logarithm values:

$$\begin{aligned} \log 0.03408 &= \log(3.408 \times 10^{-2}) \\ &= \log 10^{-2} + \log 3.408 \\ &= -2 + 0.5325 \\ \log 340.8 &= \log(3.408 \times 10^{2}) \\ &= \log 10^{2} + \log 3.408 \\ &= 2 + 0.5325 \\ \log 34080 &= \log(3.408 \times 10^{4}) \\ &= \log 10^{4} + \log 3.408 \\ &= 4 + 0.5325 \end{aligned}$$

In general, we know that:

1. The logarithm of every positive number can be written as the sum of an integer (positive, zero or negative) and a pure decimal (or zero).

The integer part is called **the characteristic** of the logarithm, and the decimal part is called the **mantissa. For** example:

In $\log 0.03408 = -2 + 0.5325$, the characteristic is -2, and the mantissa is 0.5325.

In $\log 340.8 = 2 + 0.5325$, the characteristic is 2, and the mantissa is 0.5325.

In $\log 34080 = 4 + 0.5325$, the characteristic is 4, and the mantissa is 0.5325.

From the above examples we also know that:

2. For numbers which have decimal points at different places, the mantissa of their logarithm is the same.

To find the characteristic of the logarithm of a positive number, use scientific notation to write this number as $a \times 10^n$, in which $1 \le a < 10$, and n is an integer, then n is the characteristic of the logarithm of this number.

When the characteristic of the logarithm of a number is zero or positive, then we can combine the characteristic and the mantissa to form a decimal number. For example, $\log 340.8 = 2.5325$; $\log 3.408 = 0.5325$. When the characteristic is negative, then we would put the minus sign「－」above the characteristic and drop out the plus sign「＋」. For example, $\log 0.03408 = -2 + 0.5325$, will be written as $\log 0.03408 = \bar{2}.5325$. Please note that $\bar{2}.5325$ is equal to $-2 + 0.5325$ (that is -1.4675), and not equal to -2.5325.

【Example 1】 Write the following expression in scientific notation and find out the characteristic of its logarithm.

32.16, 7.8302, 0.0002076.

Solution $32.16 = 3.126 \times 10^1$. The characteristic is 1;

$7.8302 = 7.8302 \times 10^0$. The characteristic is 0;

$0.0002076 = 2.076 \times 10^{-4}$, The characteristic is -4.

【Example 2】 Find the characteristic and the mantissa of the following logarithm:

(1) $\log a = 0.2350$; (2) $\log b = \bar{2}.4087$;

(3) $\log c = -2.4087$.

Solution (1) The characteristic is 0, and the mantissa is 0.2350;

(2) The characteristic is -2, and the mantissa is 0.4087;

(3) $\log c = -2.4087$

$$= -2 - 0.4087$$
$$= (-2 - 1) + (1 - 0.4087)$$
$$= -3 + 0.5913$$
$$= \bar{3}.5913$$

The characteristic is -3, and the mantissa is 0.5913.

Practice

1. Rewrite the following number in scientific notation and find the characteristic and the mantissa of their logarithm.

   2570000, 354.7, 40.8, 5.06,
   9, 0.84, 0.07563, 0.00002129.

Practice

2. (*Mental*) Find the characteristic and the mantissa of the logarithm of the following number:

   6720, 3.1416, $\frac{1}{2}$, 80, 0.6428, 0.00495.

3. (*Mental*) Find the value of the following expression.

   (1) $\log 10$; (2) $\log 10000$; (3) $\log 1$;
   (4) $\log 10^6$; (5) $\log 10^{-5}$; (6) $\log 0.01$;
   (7) $\log 0.1$; (8) $\log 0.000001$.

4. Write down the characteristic and the mantissa of the logarithm of the following:

   (1) $\log a = 3.0720$; (2) $\log b = 0.0129$;
   (3) $\log c = \bar{4}.2157$; (4) $\log d = -4.2157$;
   (5) $\log 0.000432 = \bar{4}.6355$; (6) $\log 0.00574 = -2.2411$.

5. Compute:

   (1) $\bar{1}.5483 + \bar{2}.8712$; (2) $\bar{2}.7125 - \bar{1}.9418$;
   (3) $\bar{4}.5082 \times 3$; (4) $\bar{3}.6479 \times 5$;
   (5) $\bar{2}.2418 \div 2$; (6) $\bar{1}.1535 \div 3$.

6. The volume of the rectangular box is 45.8 cm^3. Its length is 3.5 cm, and width is 2.4 cm. Find its height.

7. The area of a circle is 0.6567 cm^2. Find its circumference.

8. The production of a certain item is increased by 12.5% every year. After six years, what is the percentage increase in production compared with the first year ?

Exercise 2

1. Find the characteristic and the mantissa of the following logarithm:

   (1) $\log 30 = 1.4771$; (2) $\log 8.56 = 0.9325$;
   (3) $\log 0.74 = \bar{1}.8692$; (4) $\log 0.08 = -1.0969$.

2. The characteristic of the logarithm of a number is given below. Find the following:
   (i) Determine whether the number is greater than, equal to or less than 1.
   (ii) If the number is written in scientific notation in the form $a \times 10^n$, what is the value of n?

   (1) 4; (2) -5; (3) 1; (4) 0; (5) -2; (6) 2.

3. Re-write the following characteristic and mantissa as a negative number:

   (1) $\bar{3}.2175$; (2) $\bar{1}.8998$.

4. Re-write the following in characteristic and mantissa form:

   (1) -2.3817; (2) -0.2329;
   (3) -4.1026; (4) -1.7830.

5. Given a number x. the mantissa of $\log x$ is the same as the mantissa of $\log 7409$ and its characteristic is shown below. Fnd the number x :

   (1) 5; (2) -2; (3) 0; (4) -1;
   (5) 1; (6) 3; (7) 2; (8) -3.

6. Check if the following equation is true or false? Why ?

   (1) $\log(a+b) = \log a + \log b$; (2) $\log(a-b) = \dfrac{\log a}{\log b}$;

   (3) $(\log 2)^3 = 3\log 2$; (4) $\sqrt{\log 2} = \dfrac{1}{2}\log 2$.

7. The volume of a sphere can be calculated by the formula $V = \dfrac{4}{3}\pi r^3$. Find the volume of a sphere if its radius r is equal to 13.54 cm.

8. The volume of a cylinder can be calculated by the formula $V = \pi r^2 h$, where r is the radius of the circular base, and h is the height of the cylinder. Find the weight of a copper wire whose diameter is 0.2 cm and length is 2000 m (given that the density of copper is 8.9 g/cm^3).

9. Under certain condition, a type of bacteria will grow at the rate of 1.8 times in one hour. Under this condition, how many times will the bacteria be increased to after 24 hours?

Chapter summary

I. This chapter mainly teaches the concept of logarithm, operation rules of logarithm, and the system of common logarithm.

II. If a ($a>0$, $a \neq 1$) raised to the power of b is equal to N, then $a^b = N$. This is the equation written in exponential form.

When the equation is written in the equivalent logarithm form, then $\log_a N = b$, a is the base, b is the logarithm of N to base a and N is the anti-logarithm.

In the equation, $\log_a N = b$. It is required that $a>0$, $a \neq 1$, $N>0$.

III. Operation rules for logarithm of product, quotient, exponent and root of anti-logarithm values:

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a \frac{M}{N} = \log_a M - \log_a N$$

$$\log_a M^n = n\log_a M$$

$$\log_a \sqrt[n]{M} = \frac{1}{n}\log_a M$$

Using logarithm operations,

(i) calculation of multiplication and division can be transformed to become addition and subtraction;

(ii) calculation of exponent and root can be transformed to become multiplication and division.

IV. We usually use 10 as the base of our logarithm --- this is called the common logarithm. The common logarithm of N is written as $\log N$. We can use logarithm table (or engineering calculator) to find the log of any number. Knowing the log of a number, we can use anti-log table (or engineering calculator) to find the original number. Computation of multiplication, division, exponent and root can be simplified by making use of logarithm and anti-logarithm tables.

Revision Exercise 13

1. What is the value of a if the following equation is true?

   (1) $|a| = a$; (2) $|a| = -a$;

   (3) $|a| = |-a|$; (4) $a = -a$.

2. (1) Given that $(x-3)^2 + (y+1)^2 = 0$, and x, y are real numbers, find x, y;

   (2) Given that $x^2 + 4y^2 - 2x + 4y + 2 = 0$, and x, y are real numbers, find x, y.

3. Compute:

   (1) $(a+b-c)(a-b+c)(-a+b+c)(a+b+c)$;

   (2) $(a+b-c-d)(a-b-c+d)$;

   (3) $(x+y)(2x+y)(x^2+xy+y^2)(4x^2-2xy+y^2)$;

   (4) $(x-1)(x-2)(x-3)(x-4)$;

   (5) $(a-2b)^2(a+2b)^2$;

   (6) $(a-2b)(a+2b)(a^2+2ab+4b^2)(a^2-2ab+4b^2)$.

4. Factorize:

   (1) $a^2+b^2+c^2+2ab+2bc+2ac$;

   (2) $a^4b+a^3b^2-a^2b^3-ab^4$;

   (3) x^4-5x^2+4;

   (4) $4mn+1-m^2-4n^2$;

   (5) m^4+m^2+1;

   (6) $a^3-b^3+a(a^2-b^2)+b(a-b)^2$;

   (7) $2(a^2+b^2)(a+b)^2-(a^2-b^2)^2$.

5. (1) Given that $x+y+z=a$ and $xy+yz+zx=b$, find $x^2+y^2+z^2$;

   (2) Given that $x+\frac{1}{x}=5$, find $x^2+\frac{1}{x^2}$.

6. Simplify:

   (1) $\frac{1}{x+1}-\frac{1}{x+2}-\frac{x+3}{(x+1)(x+2)}$;

   (2) $\left(\frac{a}{a+b}-\frac{a^2}{a^2+2ab+b^2}\right)\div\left(\frac{a}{a+b}-\frac{a^2}{a^2-b^2}\right)$;

   (3) $\frac{a}{1+\frac{1}{a}}-\frac{1}{a+1}+1$;

   (4) $\frac{\sqrt{20}}{8}-\sqrt{\frac{9}{5}+\frac{5}{\sqrt{45}}}$;

   (5) $\frac{1}{3+\sqrt{7}}-\frac{3}{2-\sqrt{7}}-\frac{\sqrt{7}-5}{2}$;

   (6) $\frac{\sqrt[3]{5^{-\frac{3}{2}}\left(\frac{1}{5}\right)^{-3}}}{(\sqrt{5}-1)^2}$.

7. Solve the following equation:

   (1) $42x^2+x-30=0$;

   (2) $4\left(\frac{3}{x}+1\right)^2-9=0$;

   (3) $x^2-\sqrt{3}x+\sqrt{2}x-\sqrt{6}=0$;

   (4) $\frac{1}{2-x}-1=\frac{1}{x-2}-\frac{6-x}{3x^2-12}$;

   (5) $3x^4-29x^2+18=0$;

   (6) $\sqrt{x+1}-\sqrt{x-4}=\sqrt{3x+1}$.

8. Solve the following simultaneous equations:

(1) $\begin{cases} cx+y=2c+1 \\ x-cy=2-c \end{cases}$ $(c\neq 0)$, find x ,y;

(2) $\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases}$ $(a_1b_2\neq a_2b_1)$, find x ,y;

(3) $\begin{cases} \dfrac{5x+3y}{4}+\dfrac{y+2x}{3}=3 \\ \dfrac{5x+3y}{4}-\dfrac{y+2x}{3}=1 \end{cases}$; (4) $\begin{cases} \dfrac{2}{x-3}+\dfrac{5}{2y+3}=-4 \\ \dfrac{6}{x-3}-\dfrac{2}{2y+3}=5 \end{cases}$;

(5) $\begin{cases} 5x-7y=-10 \\ 9y+4z=1 \\ 3x+8z=-4 \end{cases}$; (6) $\begin{cases} x^2-xy+2=0 \\ 2x-y=1 \end{cases}$;

(7) $\begin{cases} 3x^2+5x-8y=36 \\ 2x^2-3x-4y=3 \end{cases}$.

9. What is the value of k for the equation $(k-1)x^2-2x+3=0$ to have (i) two different real roots?
(ii) two equal real roots?
(iii) no real roots?

10. Find the sum of the square of the roots, and the sum of the cube of the roots of the equation $x^2+px+q=0$.

11. Find the value of x in the following equation:

(1) $64^x=\dfrac{1}{4}$; (2) $2^x=0.125$.

12. Find the value of x in the following equation:

(1) $\log_8 x=1$; (2) $\log_2\sqrt{2}=x$;

(3) $\log_x 8=\dfrac{3}{2}$; (4) $\log_7 x=0$.

13. Is the following equation true or false (in the equation $a>0$ and $a\neq 1$, $x>0$, $y>0$, n is an integer bigger than 1)? If it is false, give a counter example.

(1) $(\log_a x)^2=2\log_a x$; (2) $-\log_a x=\log_a\dfrac{1}{x}$;

(3) $\dfrac{\log_a x}{\log_a y}=\log_a\dfrac{x}{y}$; (4) $\dfrac{\log_a x}{n}=\log_a\sqrt[n]{x}$.

14. (1) How much is $\log 100N$ greater than $\log\dfrac{N}{100}$?

(2) How much is $\log 0.001N$ smaller than $\log 1000N$?

15. Given that $\log 2=0.3010$, $\log 3=0.4771$, find $\log 0.0015$ and $\log 750$.

16. Given that $a=35.72$, $b=28.17$, $c=30.45$, $s=\dfrac{1}{2}(a+b+c)$, find $\sqrt{s(s-a)(s-b)(s-c)}$.

17. (1) Find the value corresponding to 2^{100}, find the number of digits and find the two most significant digits of the value.

(2) For the value corresponding to 0.7^{100}, find the number of contiguous zero after the decimal point, and find the value of its first non-zero digit.

18. If in 20 years, our foreign trade amount will be increased by 4 times, what is the average annual increase in percentage?

19. The original value of some equipment is $72 million. If its value is depreciated by 5.5% per year, find the value of this equipment after 5 years.

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