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## EXPERIMENT II Falling ball inside the liquid

## Introduction

When a metal ball is dropped into a liquid, a viscous force will drag it. Naturally, this force is not constant but depends on the ball's speed. The greater the speed, the greater the viscous force. In this experiment, a ball will be released just above the liquid surface, and after a while its speed will become constant called "terminal speed". By intuition, one would expect that this terminal speed must be greater for a bigger ball because it is heavier. In this experiment, one wants to know exactly whether the terminal speed is proportional to (i) surface area of the ball $\left(4 \pi r^{2}\right)$ or (ii) the volume of the ball $\left(4 \pi r^{3} / 3\right)$. To prove the proportionality, we need to plot a graph between two quantities on x -axis and y -axis respectively. When the plot of data has a linear tendency, i.e. fitted nicely with a linear line, one can say that they are proportional to one another.

## Equipment and accessories

| 1 | plastic bottle with unknown liquid |
| :--- | :--- |
| 1 | 18 metal balls (6 sizes, 3 balls each) |
| 2 rubber bands | 1 magnet |
| 2 graph papers | 1 ruler |
|  | Tissue paper |

## Experimental procedure

1. Wrap the plastic bottle with 2 rubber bands and set their position as the start and finish lines. Keep the start line about $60-70 \mathrm{~mm}$ below the liquid surface and the finish line as low as possible. Measure the distance between them with 1 mm precision.
2. Drop the ball less than 10 mm above the liquid surface and measure the time of falling between the start and finish lines. Leave the balls in liquid. Record your observation in the table.
3. Repeat this one more time and for all other provided ball sizes.
4. Calculate the terminal speed with two digit decimal in $\mathrm{mm} / \mathrm{s}$ unit.
5. Plot your data on graph paper so that one can make conclusion whether the terminal speed is proportional to the surface area or the volume of the ball.
6. Answer questions $1-3$ in your answer sheet.

Distance between the liquid surface and the start rubber band is $\qquad$ mm

Distance between two rubber bands is $\qquad$ mm

| No. | Surface area <br> $(A)$ | Volume <br> $(V)$ | Time (s) |  | Terminal velocity |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\left(\mathrm{mm}^{2}\right)$ | $\left(\mathrm{mm}^{3}\right)$ | Trial No. |  |  | $(\mathrm{mm} / \mathrm{s})$ |
|  |  | 1 | 2 |  |  |  |
| 1 | 12.6 | 4.2 |  |  |  |  |
| 2 | 17.8 | 7.1 |  |  |  |  |
| 3 | 28.3 | 14.1 |  |  |  |  |
| 4 | 31.7 | 16.8 |  |  |  |  |
| 5 | 50.3 | 33.5 |  |  |  |  |
| 6 | 71.2 | 56.5 |  |  |  |  |

Plot your data on graph paper so that one can make conclusion whether the terminal speed is proportional to the surface area or the volume of the ball.

Question 1 Is the terminal speed proportional to the surface area or the volume of the ball?
How do you make that conclusion? (1 point)

## Answer:

$\qquad$
$\qquad$
$\qquad$

Question 2 From the information provided in the introduction, explain clearly why does the speed finally become constant? (2 points)

Answer: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Question 3 If a student mistakenly set the start rubber band at the liquid surface, the speed will be too large or too small? Explain it. (2 points)

Answer: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Answer Key

Question 1 Is the terminal speed proportional to the surface area of the volume of the ball?
How do you make that conclusion? (1 point)
Answer: The terminal speed is proportional to $r^{2}$ because the plot yield a straight line tendency.

Question 2 From the information provided in the introduction, explain clearly why does the speed finally become constant? (2 points)

Answer: Releasing the ball slightly above the liquid surface. When the ball is fully submerged, its speed is quite small and the viscous force is also small. The combination of the weight, lift force and viscous force yield the net downward force. This makes the ball accelerate downward and the ball become faster. As the ball is getting faster, the net force become smaller in magnitude but remain in downward direction. As a consequence, the ball become faster with decreasing rate. At a certain speed, the viscous force is strong enough so that the net force becomes zero which means no acceleration. Therefore, the speed becomes constant after that situation.

Question 3 If a student mistakenly set the start rubber band at the liquid surface, the speed will be too large or too small? explain it? (2 point)

Answer: It depends on the speed of the ball when it makes first contact with liquid surface. If this speed is less than the terminal speed, it will be too small. If the this speed is more than the terminal speed, it will be too large.

## Marking Scheme

Distance between the liquid surface and start rubber band not less than 6.0 cm . (1 point)
Distance between start and stop rubber band should not be less than 6.0 cm . (1 point)
If either or both distances are not recorded with 1-millimeter resolution, deduce 0.5 point.

## Table (5 points)

Time should be recorded as X.YY s, not X:YY s. (1 point)
Average should be rounded to X.YY s also. (1 point)
Correct calculation for speed. (2 point)
Able to draw 2 graphs [1 point, 0.5 point each]

## Graph (4 point, each)

Graph has x -axis, label and unit ( 0.5 point), proper scaling ( 0.5 point) Graph has $y$-axis, label and unit ( 0.5 point), proper scaling ( 0.5 point)

Correct plot of data points (all correct $=2$ point, 3 out of $5=1$ point)


## Preliminary Result


$\mathrm{V}_{-}\{T\}=\backslash$ frac $\{2\}\{9\} \mathrm{g}(\backslash$ rho_ $0-\backslash$ rho $) \backslash$ frac $\left.\left\{\mathrm{r}^{\wedge}\{2\}\right\} \backslash \backslash \mathrm{nu}\right\}$
\nu $=\backslash$ frac $\{2\}\{9\} g(\backslash$ rho_ $\{0\}$ - $\backslash$ rho $)$ slope


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