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## Mathematics Exploration Problems

Country: $\qquad$ Name: $\qquad$ ID: $\qquad$ Score:

## Instructions:

- Write down your name and country on every page of the answer sheet.
- Write your answer on the answer sheet.
- You have 120 minutes to work on this test.
- Each problem is worth 6 points, and partial credit may be awarded.
- Use black or blue colour pen or pencil to write your answer.

"Smart, Skilled, and Creative In a Joyful Competition for Excellence"


## EXPLORATION PROBLEMS

(1) In a $2 \times 3$ table, the three numbers in the first row are positive integers. The first number in the second row is equal to the first number in the first row. The second number in the second row is equal to the product of the first two numbers in the first row. The third number is equal to the product of all three numbers in the first row. If the sum of the three numbers in the second row is 125 , find all possible values of the sum of the three numbers in the first row.
(2) The diagram shows a $6 \times 6$ chessboard with 4 squares containing black circles and 4 squares containing white circles. Use four different colors to divide the diagram into 4 identical regions by going along the lines so that each region will be exactly of the same size, same shape; and each region contains 1 black circle and 1 white circle. The regions may be rotated but not reflected. The circles do not have to be in the same relative positions within the region.

(3) The sum of the ages of three boys is equal to the sum of the ages of three girls, and the product of the boys' ages is equal to the product of the girls' ages, too. All six children are of different ages, and all are under 20. Suppose that their ages are integers, and the age of the second oldest boy is 14 , what is the age of the second oldest girl?
(4) The diagram shows nine pieces of puzzles, including one that has the shape that looks like the letter P, and one that looks like the letter U. All nine pieces may be rotated or reflected.

(a) Consider two figures below. The figure on the left shows a $5 \times 5$ square with a hole in the shape of the letter U . The one on the right shows the double-sized letter U. Choose five different pieces, including the P-shaped one, to construct each of the figures. Find one solution for each figure.

(b) Consider other two figures below. This time, the figure on the left shows a $5 \times 5$ square with a hole in the shape of the letter P . The one on the right shows the double-sized letter P. Choose five different pieces, including all four pieces which are not used in (a) to construct each of the figures. Find one solution for each figure. (To solve this problem, you may use scissors to cut the puzzle pieces in the attached colour page.)

(5) The diagram shows nine circles joined by five solid lines and two dashed lines. Fill in the circles using each of the numbers $1,2,3,4,5,6,7,8$ and 9 once, such that the number in the middle circle of a solid line is equal to the sum of the numbers in the other circles on the line, and the number in the middle circle of a dashed line is equal to the difference of the numbers in the other circles on the line. Find all solutions.

(6) The diagram shows a bird-shaped piece consisting of two unit squares and three half-squares joined edge to edge. It may be rotated or reflected.


The next diagram shows a Greek cross enclosed by four identical bird-shaped pieces. When two pieces come into contact, they must do so along an entire edge or an entire diagonal. Find as many figures as you can that can be enclosed by the four bird-shaped pieces.

(To solve this problem, you may use scissors to cut the puzzle pieces in the attached colour page.)

You may use the scissors to cut the following shapes as you need.

For Problem 4:


For Problem 6:


