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# $5^{\text {th }}$ International Mathematics $\mathcal{A}$ ssessments for Schools (2015-2016) 

Junior Division Round 2<br>Time: 120 minutes

Printed Name

Code
Score

## Instructions:

- Do not open the contest booklet until you are told to do so.
- Be sure that your name and code are written on the space provided above.
- Round 2 of IMAS is composed of three parts; the total score is 100 marks.
- Questions 1 to 5 are given as a multiple-choice test. Each question has five possible options marked as A, B, C, D and E. Only one of these options is correct. After making your choice, fill in the appropriate letter in the space provided. Each correct answer is worth 4 marks. There is no penalty for an incorrect answer.
- Questions 6 to 13 are a short answer test. Only Arabic numerals are accepted; using other written text will not be honored or credited. Some questions have more than one answer, as such all answers are required to be written down in the space provided to obtain full marks. Each correct answer is worth 5 marks. There is no penalty for incorrect answers.
- Questions 14 and 15 require a detailed solution or process in which 20 marks are to be awarded to a completely written solution. Partial marks may be given to an incomplete presentation. There is no penalty for an incorrect answer.
- Use of electronic computing devices is not allowed.
- Only pencil, blue or black ball-pens may be used to write your solution or answer.
- Diagrams are not drawn to scale. They are intended as aids only.
- After the contest the invigilator will collect the contest paper.

The following area is to be filled in by the judges; the contestants are not supposed to mark anything here.

| Question | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | 7 | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ | Total <br> Score | Signature |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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## Junior Division Round 2

## Questions 1 to 5, 4 marks each

1. Michael bought 6 pens and 3 notebooks while Wallace bought 3 pens and 6 notebooks. The pens are identical and so are the notebooks. Michael's bill is 6 dollars higher than Wallace's. How many dollars is the price of a pen higher than the price of a notebook?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Answer :
2. If all the divisors of 2016 are arranged in decreasing order, by how much is the third divisor larger than the fourth divisor?
(A) 12
(B) 48
(C) 168
(D) 672
(E) 2016

## Answer :

3. Which of the following is equal to $x r+y s$ if $r=3 x+2 y$ and $s=x y-x-y$ ?
(A ) $x^{2} y-x^{2}+2 x y+2 y^{2}$
(B) $x y^{2}+3 x^{2}+x y-y^{2}$
(C) $x^{2} y+2 x^{2}+x y$
(D) $x y^{2}+2 x^{2}+2 x y$
(E) $x^{2} y^{2}+x+y$

Answer :
4. $E$ is a variable point on the side $B C$ of a square $A B C D . D E F G$ is a rectangle with $F G$ passing through $A$. As the point $E$ moves from $B$ towards $C$, how does the area of $D E F G$ change?

(A) Steadily increasing
(B) Steadily decreasing
(C) Decreasing and then increasing
(D) Increasing and then decreasing
(E) Remaining constant
5. The diagram shows a 6 by 6 board with three barriers. An ant is at the top left corner and wishes to reach the bottom right corner. It may only crawl between squares which share a common side, and only towards the bottom or the righ. It cannot pass through any barrier. How many different paths can it follow?

(A) 88
(B) 90
(C) 92
(D) 96
(E) 112

Answer :

## Questions 6 to 13, 5 marks each

6. One square in a 3 by 3 board is to be painted black, a second square blue and a third square red. If no two of these three squares are in the same row or in the same column, how many different ways of painting them are there?


Answer :
7. The diagram shows a tile divided into regular hexagons of side length 1 cm . What is the total area, in $\mathrm{cm}^{2}$, of the parts of the tile which are shaded?


Answer :
8. Let $a, b, c$ and $d$ be real numbers such that $|a+b|,|a-b|,|c+d|$ and $|c-d|$ are $6,7,8$ and 9 in some order. What is the value of $a^{2}+b^{2}+c^{2}+d^{2}$ ?

## Answer :

9. A rectangle $A B C D$ with $B C=3 \mathrm{~cm}$ and $A B=\sqrt{3} \mathrm{~cm}$ is folded along $A C$ so that the point $B$ lands on the point $K$ symmetric to it about $A C$. What is the area, in $\mathrm{cm}^{2}$, of triangle $K D E$, where $E$ is the point on the extension of $B C$ such that $\angle C D E=30^{\circ}$ ?


Answer : $\qquad$
10. In the expression $(((10 \square 2) \square 2) \square 2) \square 2$, each $\square$ is to be replaced by a different one of,,$+- \times$ and $\div$. How many different values can this expression take?

## Answer :

11. Let $a, b$ and $c$ be real numbers such that $a b c=1,(a+1)(b+1)(c+1)=16$ and $(a+2)(b+2)(c+2)=53$. What is the value of $(a-1)(b-1)(c-1) ?$

## Answer :

12. The area of triangle $A B C$ is $120 \mathrm{~cm}^{2}$ and $B C=16 \mathrm{~cm}$. What is the minimum length, in cm , of the perimeter of $A B C$ ?
13. Let $a, b, c$ and $d$ be four different non-zero digits. The greatest common divisor of the four-digit numbers $\overline{a b c d}$ and $\overline{a c b d}$ is $n$. What is the largest possible value of $n$ ?

Answer :

## Questions 14 to 15, 20 marks each

## Detailed solutions are needed for these two problems

14. The first diagram shows a 6 by 6 board, and the second diagram shows an L-shaped piece consisting of four 1 by 1 squares. Paint as few of the squares of the 6 by 6 board black so that wherever the L-pieced piece is placed on the board covering four squares, at least one of the squares will be black. The L-shaped piece may be turned about or flipped over


Fig. 1


Fig. 2

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15. $P$ and $Q$ are points on the bisector of the exterior angle at $A$ of triangle $A B C$, with $A$ between $P$ and $Q$, such that $B P$ is parallel to $C Q . D$ is the point on $B C$ such that $D P=D Q$. Prove that $A B$ is parallel to $D Q$.
