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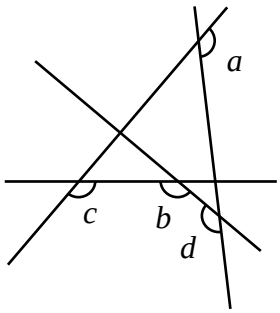
Invitational World Youth Mathematics Intercity Competition 1999

Individual Contest

Section A.
In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Find the remainder when 1223334444555556666667777777888888889999999999 is divided by 9.

2. Find the sum of the angles *a*, *b*, *c* and *d* in the following figure.



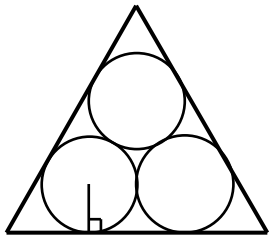
3. How many of the numbers 1^2, 2^2, . . . , 1999^2 have odd numbers as their tens-digits?

4. The height of a building is 60 metres. At a certain moment during daytime, it casts a shadow of length 40 metres. If a vertical pole of length 2 metres is erected on the roof of the building, find the length of the shadow of the pole at the same moment.

5. Calculate $1999^2 - 1998^2 + 1997^2 - 1996^2 + \cdots + 3^2 - 2^2 + 1^2$.

6. Among all four-digits numbers with 3 as their thousands-digits, how many have exactly two identical digits?

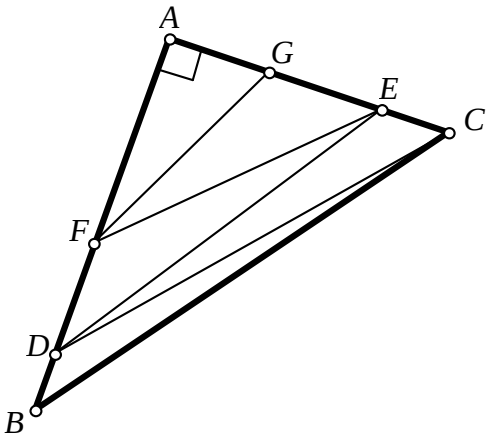
7. The diagram below shows an equilateral triangle of side 1. The three circles touch each other and the sides of the triangle. Find the radii of the circles.



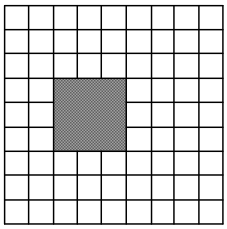
8. Let *a*, *b* and *c* be positive integers. The sum of 160 and the square of *a* is equal the sum of 5 and the square of *b*. The sum of 320 and the square of *a* is equal to the sum of 5 and the square of *b*. Find *a*.

9. Let *x* be a two-digit number. Denote by $f(x)$ the sum of *x* and its digits minus the product of its digits. Find the value of *x* which gives the largest possible value for $f(x)$.

10. The diagram below shows a triangle *ABC*. The perpendicular sides *AB* and *AC* have lengths 15 and 8 respectively. *D* and *F* are points on *AB*. *E* and *G* are points on *AC*. The segments *CD*, *DE*, *EF* and *FG* divide triangle *ABC* into five triangles of equal area. The length of only one of these segments is integral. What is that length?



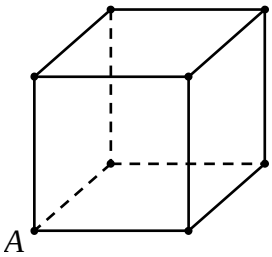
11. How many squares are formed by the grid lines in the diagram below?



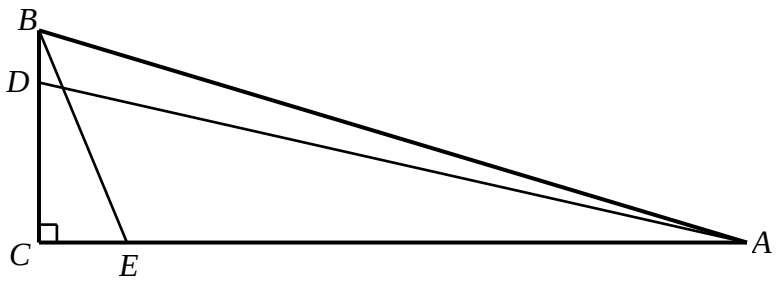
12. There are two committees A and B. Committee A had 13 members while committee B had 6 members. Each member is paid \$6000 per day for attending the first 30 days of meetings, and \$9000 per day thereafter. Committee B met twice as many days as Committee A, and the expenditure on attendance were the same for the two committees. If the total expenditure on attendance for these two committees was over \$3000000, how much was it?

Section B.
Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. The diagram below shows a cubical wire framework of side 1. An ant starts from a vertex and crawls along the sides of the framework. If it does not repeat any part of its path and finally returns to the starting vertex, what is the longest possible length of the path it has travelled?



2. In the diagram below, BC is perpendicular to AC. D is a point on BC such that $BC = 4BD$. E is a point on AC such that $AC = 8CE$. If $AD = 164$ and $BE = 52$, determine AB.



3. When a particular six-digit number is multiplied by 2, 3, 4, 5 and 6 respectively, each of the products is still a six-digit number with the same digits as the original number but in a different order. Find the original number.