注意：

允許學生個人、非營利性的圖書館或公立學校合理使用 IMC 各項試題及其解答。**可直接下載而不須申請**。

重版、系統地複製或大量重製本資料的任何部分，必須獲得 IMC 行政委員會的授權許可。

申請此項授權請電郵 IMC 行政委員會主席孫文先

ccmp@seed.net.tw

Notice:

Individual students, nonprofit libraries, or schools are permitted to make fair use of the papers and its solutions. Republication, systematic copying, or multiple reproduction of any part of this material is permitted only under license from the IMC Executive Board. Requests for such permission should be made by e-mailing Mr. Wen-Hsien SUN ccmp@seed.net.tw

World Youth Mathematics Intercity Competition

Individual Contest

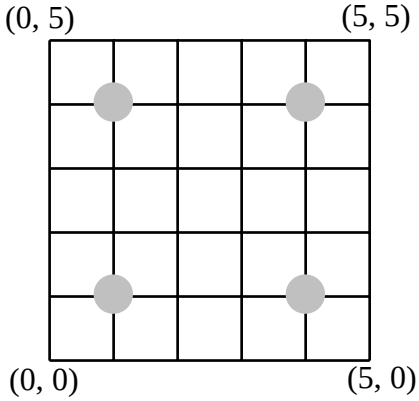
Time limit: 120 minutes 2008/10/28

Team:_____Name:_____ Score:_____

Section A.

In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

1. Starting from the southwest corner (0,0) of a 5×5 net, an ant crawls along the lines towards the northeast corner (5,5). It can only go east or north, but cannot get pass the four broken intersections at (1,1), (1,4), (4,1) and (4,4). What is the total number of different paths?



Answer：_____

2. The positive integer $a-2$ is a divisor of $3a^2-2a+10$. What is the sum of all possible values of a?

Answer：_____

3. Let a, b and c be real numbers such that $a + b + c = 11$ and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{13}{17}$. What is the value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$?

Answer：_____

World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2008/10/28

***Team:*_____*Name:*_____ *Score:*_____**

4. Let x be any real number. What is the maximum real value of $\sqrt{2008-x}+\sqrt{x-2000}$?

Answer：_____

5. How many ten-digit numbers are there in which every digit is 2 or 3, and no two 3s are adjacent?

Answer：_____

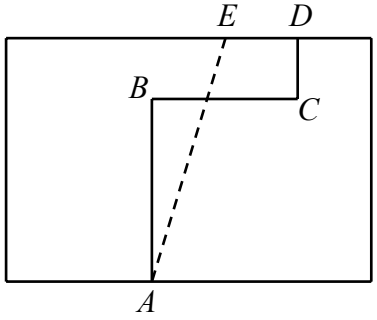
6. On a circle, there are n ($n > 3$) numbers with a total sum of 94, such that each number is equal to the absolute value of the difference between the two numbers which follow it in clockwise order. What is the possible value of n?

Answer：_____

7. If the thousands digit of a four-digit perfect square is decreased by 3 and its units digit is increased by 3, the result is another four-digit perfect square. What is the original number?

Answer：_____

8. Each segment of the broken line A-B-C-D is parallel to an edge of the rectangle, and it bisects the area of the rectangle. E is a point on the perimeter of the rectangle such that AE also bisects the area of the rectangle. If AB=30, BC=24 and CD=10, what is the length of DE?



Answer：_____

World Youth Mathematics Intercity Competition

Individual Contest

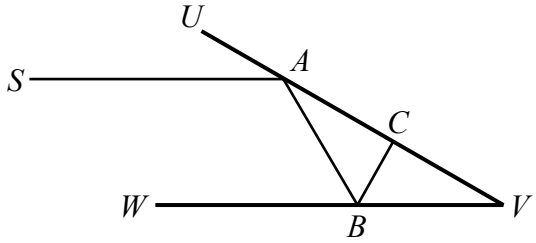
Time limit: 120 minutes 2008/10/28

***Team:*_____*Name:*_____ *Score:*_____**

9. Let $f(x)=ax^2-c$, where a and c are real numbers satisfying $-4\le f(1)\le -1$ and $-1\le f(2)\le 2$. What is the maximum value of $f(8)$?

Answer：_____

10. Two vertical mirrors facing each other form a 30° angle. A horizontal light beam from source S parallel to the mirror WV strikes the mirror UV at A, reflects to strike the mirror WV at B, and reflects to strike the mirror UV at C. After that, it goes back to S. If $SA=AV=1$, what is the total distance covered by the light beam?

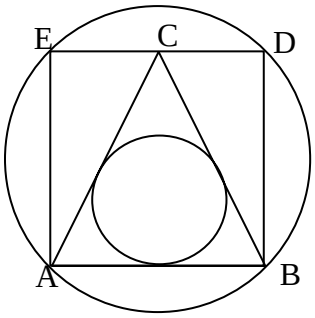


Answer：_____

11. Let n be a positive integer such that n^2-n+11 is the product of four prime numbers, some of which may be the same. What is the minimum value of n?

Answer：_____

12. ABC is an equilateral triangle, and $ABDE$ is a rectangle with DE passing through C. If the circle touching all three sides of ΔABC has radius 1, what is the diameter of the circle passing through A, B, D and E?



Answer：_____

World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2008/10/28

***Team:*_____*Name:*_____ *Score:*_____**

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. In the expression $\left[\sqrt{2008+\sqrt{2008+\sqrt{2008+\ldots+\sqrt{2008}}}}\right]$, the number 2008 appears 2008 times, and $[x]$ stands for the greatest integer not exceeding x. What is the value of this expression?

Answer：_____

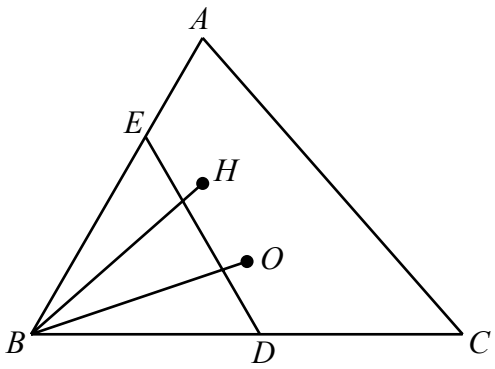
World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2008/10/28

***Team:*_____ *Name:*_____ *Score:*_____**

2. In the triangle ABC, $\angle ABC=60°$. O is its circumcentre and H is its orthocentre. D is a point on BC such that $BD=BH$. E is a point on AB such that $BE=BO$. If $BO=1$, what is the area of the triangle BDE? (The orthocenter is the intersection of the lines from each vertex of the triangle making a perpendicular with its opposite sides. The circumcenter is the center of the circle passing through each vertex of the triangle.)



World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2008/10/28

***Team:*_____*Name:*_____ *Score:*_____**

3. Let t be a positive integer such that $2^t = a^b \pm 1$ for some integers a and b, each greater than 1. What are all the possible values of t?

Answer：_____